

# A novel triple-band high reflector design in sum-frequency-mixing yellow laser

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A novel thickness modulation design solution based on modulation period is described for the design of triple-band high reflector in sum-frequency-mixing 593 nm yellow laser. The layer thicknesses of an arbitrary quarter-wave high reflector stack are modulated using cosine function. The relationship between modulation period and the position of stopband is analyzed. An analytical equation is derived that shows the spacing between the spectral centers of two adjacent stopbands as a function of modulation period. It is used to determine the modulation period in the triple-band high reflector design. The design of 593 nm/1064 nm/1342 nm triple-band high reflector in sum-frequency-mixing yellow laser is finished by this method. The only 32 layers laser high reflector is realized using TiO<sub>2</sub>/SiO<sub>2</sub> materials. Central wavelength reflectivity is 99.899%, 99.947% and 99.864% at 1342 nm, 1064 nm and 593 nm, respectively.

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## 1. Introduction

In recent years sum-frequency-mixing (SFM) yellow laser sources have become interesting for many technical applications in ophthalmology medicine, testing, color display etc [1, 2]. To achieve higher yellow laser power level, many resonator structures were adopted [3-5]. In resonator mirror design, high reflectivity spectra is often needed simultaneously at the fundamental two laser lines and SFM laser line [6]. So triple-band high reflector (two fundamental laser lines and SFM laser line) design is becoming urgent problem. The basic quarter-wave stack can be used to realize the standard single-band and dual-band harmonious reflector design [7]. But it is still difficult to design the non-harmonious relationship triple-band high reflector. For the 593nm yellow laser, the approach is based on SFM [8], in which, coherent frequencies of v<sub>1</sub> (1064 nm) and v<sub>2</sub> (1342 nm) is mixed, generating radiation of frequency v<sub>3</sub> (593 nm) = v<sub>1</sub>+v<sub>2</sub>. The high reflector requests 99.8% reflectivity in the three wavelengths 1342 nm, 1064 nm and 593 nm. Although the traditional design method is to combine three reflector stacks together for this non-harmonics reflector and every spectral center of stopband is the required wavelength. Unfortunately, this method always leads to too many layers and difficulty of fabrication.

In this paper a novel thickness modulation design (TMD) is analyzed for the design of triple-band high reflector in SFM laser. The layer thicknesses of an arbitrary quarter-wave stack in TMD are modulated using cosine function. The relationship between modulation period and the position of stopband is analyzed. It is founded that the spacing between the spectral centers of two adjacent stop-bands is equal when the modulation

period is fixed. An analytical equation is derived that shows the spacing between the spectral centers of two adjacent stopbands as a function of modulation period. It can be used to determine the modulation period in the triple-band high reflector design. This method is used to design triple-band high reflector in SFM 593 nm yellow laser. Compared to traditional design method, it much reduces the number of layers and computational time needed for final optimization. The final result is satisfactory. Only 32 layers stack realizes 99.899%, 99.947% and 99.864% reflectivity at 1342 nm, 1064 nm and 593 nm, respectively.

## 2. Thickness modulation theory

For a group of thin-film layers in a multilayer thin-film design, the corresponding characteristic matrix is readily determined from Eq. (1); P denotes the characteristic matrix of the group of N individual layers:

$$\begin{aligned} P &= \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} = \prod_{i=1}^n M_i \\ &= \prod_{i=1}^n \begin{bmatrix} \cos \delta_i & (i \sin \delta_i) / \eta_i \\ i \eta_i \sin \delta_i & \cos \delta_i \end{bmatrix} \end{aligned} \quad (1)$$

In Eq. (1), M is the characteristic matrix of i-th individual layer. If this period or group of layers represented by P is repeated N times, the resulting characteristic matrix for the entire periodic structure takes the general form:

$$P_1 P_2 P_3 \cdots P_N = P^N \quad (2)$$

Where the subscript of the matrix P indicated how many times it is repeated. As explained by Born and Wolf [9],

the matrix  $P$  is unimodular, it can be solved using Chebyshev polynomials of the second kind. Accordingly,  $P^N$  can be solved from Eq. (2):

$$P^N = \begin{bmatrix} p_{11}C_{N-1}(a) - C_{N-2}(a) & p_{12}C_{N-1}(a) \\ p_{21}C_{N-1}(a) & p_{22}C_{N-1}(a) - C_{N-2}(a) \end{bmatrix} C_i(a)$$

represents the Chebyshev polynomials of the i-th order and second kind, where argument a is given by

$$a = \frac{1}{2}(p_{11} + p_{22}) \quad (3)$$

The position of the stopbands rely on the value of a. A stopband exists for wavelengths where  $|a| \geq 1$  is satisfied. Otherwise, a passband exists for wavelengths where  $|a| \geq 1$  is not satisfied.

In communications, people often use modulation principle to load information on signals. Thickness modulation design transforms the modulation principle from electrical engineering communication theory into the optical thin film design [10, 11]. It differs from rugate designs and from continuously variable refractive-index films that typically have modulated and tapered refractive-index profiles, respectively [12]. TMD are limited to discrete, homogeneous multilayer thin-film structures except for one comparison with a rugate filter. Eq. (3) was used to determine the thickness of all the layers.

In this paper, the cosine function is used as the thickness modulation function. The analytical equation used to determine each layer thickness takes from  $T(L) = [1 + k \cos(2\pi fL)]$ , where  $T$  is the quarter wavelength optical thickness (QWOT) of the quarter-wave stack,  $k$  is the modulation amplitude,  $f$  is the modulation frequency, and  $L$  is the layer number. The cosine function is selected first because it is the primary refractive index modulation function for rugate filters. Cosine modulation of discrete layers produces several stopbands that are harmonics and non-harmonics of the first-order stopband of an un-modulated quarter-wave stack. To allow for full-wave, cosine modulation of the thickness of discrete layers in a multilayer thin film, the average thickness must be greater than or equal to the modulation amplitude. Example plots of layer-thickness profile and corresponding spectral reflectance for a TMD are shown in Fig. 1. The modulation period is directly defined by  $T_{base} = 1/f$ . Integer values of  $T_{base}$  correspond to periods of the modulated thin-film design that are an integer number of layers. Both the layer thickness and refractive index patterns must repeat in a characteristic matrix of the basic period. So  $T_{base}$  must be an odd integer, and the minimum of  $T_{base}$  is 2. From  $T_{base} = 1/f$ , the modulation frequency range is  $0 < f_m \leq 0.5$ .

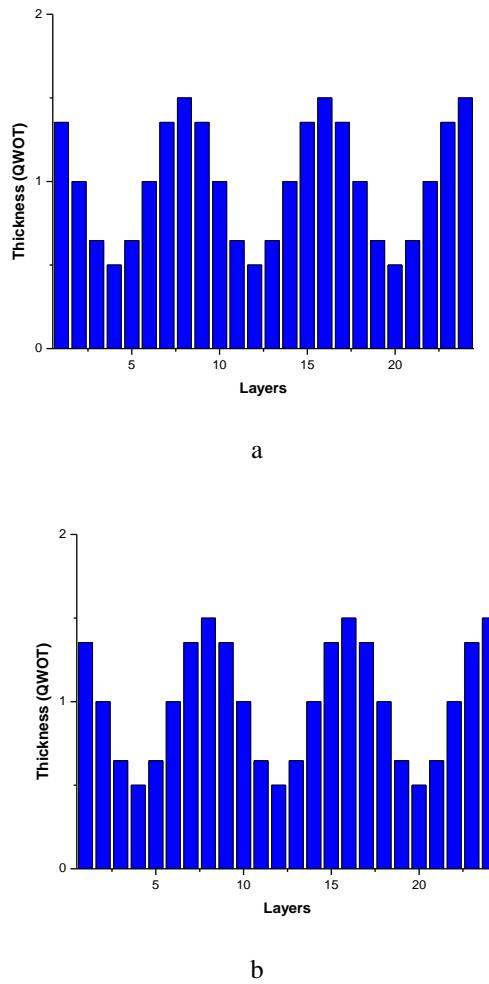


Fig. 1. Layer thickness profile (a) and the spectral reflectance (b) for an example TMD. Corresponding to TMD with base period of 4. The refractive indices for the ambient, substrate, and two films are 1.0, 1.52, 1.45, and 2.25, respectively.

Fig. 2 shows the spectral positions of all stopbands for all based periods at modulation amplitude 0.5. The refractive indices for the ambient, substrate, and two films are 1.0, 1.52, 1.45, and 2.25, respectively. The stopbands for each TMD are plotted as horizontal dashes in the figure, where each basic period represents a TMD (vertical axis) and the spectral performance (stopbands) for that TMD is shown over the spectral range of 0.0 to 5.0 wave number ( $\text{um}^{-1}$ ). The wave number of 1.0 corresponds to the average thickness of each TMD shown.

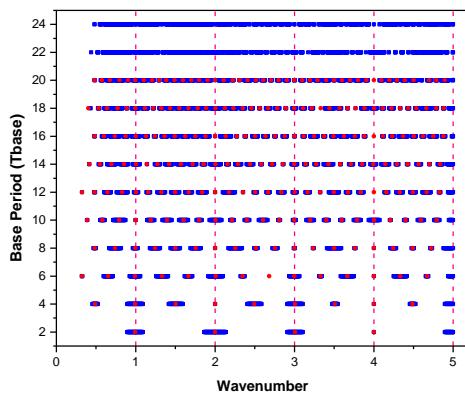


Fig. 2. Spectral positions of all stopbands for modulation amplitude of 0.5. The red point is the spectral center of each stopband.

Table 1. The space between adjacent stopbands for selected basic period.

Base Period ( $T_{base}$ )	Stop-band Positions (spectral center) [wave number $\sigma$ ]	The space ( $\Delta\sigma$ ) between two adjacent stopbands (Average)
2	0.9925, 2, 3.0075, 4.....	1=2/2
4	0.4895, 0.9925, 1.505, 2, 2.495, 3.0075, 3.5105, 4, 4.4895.....	0.5=2/4
6	0.324, 0.6545, 0.992, 1.3325, 1.666, 2.007, 2.365.....	0.3359≈2/6
$T_{base}$		$\Delta\sigma = 2/T_{base}$

It can be found that the spacing between the spectral center of two adjacent stopbands is equal when the base period ( $T_{base}$ ) is fixed, corresponding to each row in Fig. 2, and the numbers of the stopbands is increasing with the basic period increasing, or the spacing of two stopbands decreasing. Table 1 shows the detailed data of spectral center of every stopband as the base period of these a TMD structures increases from 2 to 10, with modulation amplitude  $k = 0.5$ . And the spacing between two adjacent stopbands of each base period is directly calculated and also contained in Table 1. The relationship between the

spacing  $\Delta\sigma$  and base period  $T_{base}$  is plot in Fig. 3 for the above examples and other base periods. In Fig. 3, the points on the graph represent the spectral center spacing calculated from Table 1. The solid curve is a hyperbolic curve where

$$\sigma_1 - \sigma_0 = \Delta\sigma = 2/T_{base} \quad (4)$$

Eq. (4) is solved for  $T_{base}$  where

$$T_{base} = 2 / \Delta\sigma = 2 / (\sigma_1 - \sigma_0) \quad (5)$$

$\sigma_0, \sigma_1$  represent the spectral center of the first order stop band and the adjacent stop band, respectively. That is to say there are stopbands at  $\sigma_0, \sigma_1$ .

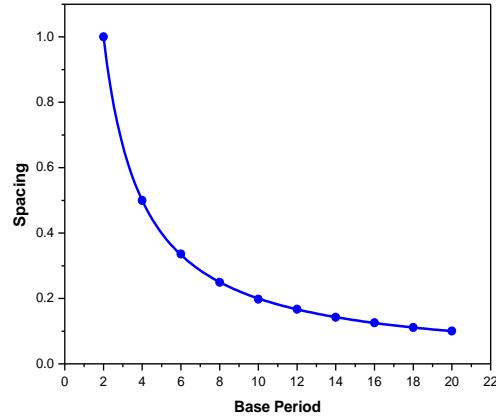


Fig. 3. The base period and the spacing between the spectral centers of two adjacent stopbands. The line is determined by  $\Delta\sigma = 2/T_{base}$ .

Because the spacing between the spectral centers of two adjacent stopbands is equal when the modulation period is fixed. If another wave number  $\sigma_2$  satisfies  $\sigma_2 - \sigma_0 = n * (\sigma_1 - \sigma_0)$ , where  $n$  is an integer, there will exit stopbands at  $\sigma_0, \sigma_1$  and  $\sigma_2$  with  $T_{base}$  calculated from  $T_{base} = 2 / (\sigma_1 - \sigma_0)$  as the modulation period. For example, set  $\sigma_0 = 1, \sigma_1 = 2, \sigma_2 = 3$ ,

$$\begin{aligned} \sigma_1 - \sigma_0 &= 1 \\ T_{base} &= 2 / (\sigma_1 - \sigma_0) = 2 \\ \sigma_2 - \sigma_0 &= 2 = 2 * (\sigma_1 - \sigma_0) \end{aligned} \quad (6)$$

From Fig. 2, it can be found that there really have stopbands at wave number 1, 2, 3. Deeply studying these relationships, we can obtain interesting information: Supposing there are three wave number,  $\sigma_1, \sigma_2$  and  $\sigma_3$  ( $\sigma_1 < \sigma_2 < \sigma_3$ )

$$\begin{aligned}\sigma_2 - \sigma_1 &= \Delta\sigma_{21} \\ \sigma_3 - \sigma_1 &= \Delta\sigma_{31}\end{aligned}\quad (7)$$

If we can find  $\Delta\sigma$  satisfy:

$$\Delta\sigma = \Delta\sigma_{21} / n = \Delta\sigma_{31} / m,$$

Here  $n$  and  $m$  can be any integer, there will be stopband at  $\sigma_1$ ,  $\sigma_2$  and  $\sigma_3$  with  $T_{base} = 2 / \Delta\sigma$  as the modulation period.

The reason for this is that the spacing between spectral centers of two adjacent stopbands is equal when the modulation period is fixed.

With  $T_{base} = 2 / \Delta\sigma$ , it can get:

$$T_{base} = nT'_{base} = mT''_{base} \quad (8)$$

So  $T_{base}$  is the common multiple of  $T'_{base}$  and  $T''_{base}$ .

Back stepping the above process, a new solution to design triple-band high reflector can be realized. It can be started from this: the three required reflected wavelength are  $\lambda_1$ ,  $\lambda_2$  and  $\lambda_0$  ( $\lambda_2 < \lambda_1 < \lambda_0$ ). Normalizing these to  $\lambda_0$ , can get:

$$\sigma_0 = \frac{\lambda_0}{\lambda_0}, \sigma_1 = \frac{\lambda_0}{\lambda_1}, \sigma_2 = \frac{\lambda_0}{\lambda_2} \quad (9)$$

So with  $T_{base} = 2 / (\sigma_1 - \sigma_0)$

$$\begin{aligned}T_1 &= 2 / (\sigma_1 - \sigma_0) \\ T_2 &= 2 / (\sigma_2 - \sigma_0).\end{aligned}\quad (10)$$

Next find  $T_{base}$ , the least common multiple of  $T_1$  and  $T_2$ . So  $T_{base}$  of course fix:

$$T_{base} = nT_1 = mT_2 \quad (11)$$

If we choose this  $T_{base}$  to be the modulation period, according to preceding saying, the TMD will exit stopbands at  $\lambda_1$ ,  $\lambda_2$  and  $\lambda_0$ .

Eq. (9)-(11) constitutes a complete recipe for calculating the modulation period in triple-band film design. With some limited help by computer, it is expected to introduce a triple-band high reflector with any required wavelength. In addition, actually, the spectral center of the three stopbands which we need is not accurately at required wavelength  $\lambda_1$ ,  $\lambda_2$  and  $\lambda_0$ , because of the approximation in several steps. But this shift of stopbands is very limited. It is impossible that this limited shift can make the design unsuccessful after many tests. This method also can be used to design multi-band high reflector in the same way.

### 3. Yellow laser triple-band high reflector design

Here we use the above method to design a normal-incidence, high-reflectance coating for the wavelengths of 1342 nm, 1064 nm and 593 nm. The triple-band reflector is used in SFM laser resonator mirror to improve the 593 nm output power. The aim is to gain 99.8% reflectivity at the three wavelengths. As previously mentioned, a triple quarter-wave stack can readily accomplish this task. The Fig. 4 shows a conventional triple stacks high reflector spectra for 593 nm yellow laser, the whole layers are 45 layers, the whole physical thickness is 6750 nm, and the central wavelength reflectivity is 99.891%, 99.924% and 99.804% at 1342 nm, 1064 nm and 593 nm, respectively.

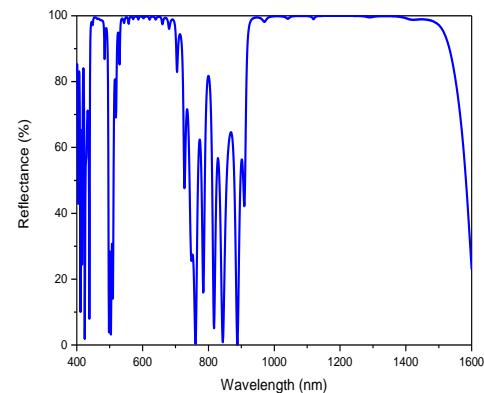


Fig. 4. Reflectance of the traditional triple quarter-wave stack. Total layers=45, Basic stack (0.59H0.59L)^7(1.06H1.06L)^8(1.34H1.34L)^7H. Film refractive indices = 2.35 and 1.45, normal incidence, central wavelength 1000 nm.

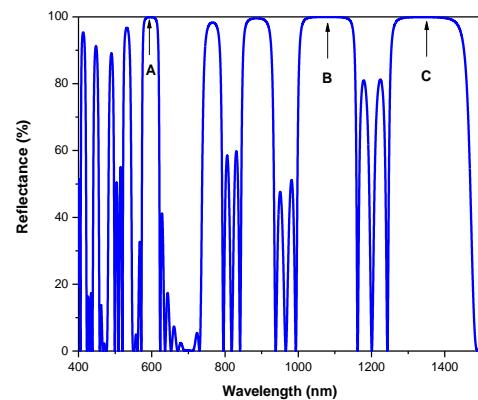


Fig. 5. Reflectance of the triple-band TMD. The TMD parameters are  $T = 8$ ,  $k = 0.4$ , Total layers = 32, and film refractive indices = 2.35 and 1.45, normal incidence. Arrows are placed at the three design central wavelengths: 593 nm, 1064 nm, and 1342 nm. The A, B and C stopbands are the required stopbands.

The TMD can provide a new design solution. First, normalizing these three wavelengths to  $\lambda_0$ , getting  $\sigma_0$ ,  $\sigma_1$  and  $\sigma_2$ . By Eq. (10), we can calculate that  $T_1 = 8$ ,  $T_2 = 2$ . Next, the modulation period ( $T_{base}$ ) is calculated to be 8, which is the least common multiple of  $T_1$ ,  $T_2$ . The selected film materials are  $TiO_2$  and  $SiO_2$ , which have refractive indices of 2.35 and 1.45. The refractive indices of ambient and substrate are 1.0 and 1.52. All of this can be easily accomplished by the computer. Because the modulation period or frequency determines the positions of the stopbands and the modulation amplitude just affect the reflection strengths of the stopbands. We use the computer to find the best value of the modulation amplitude  $k$  in its region  $0 < k \leq 1$ , by testing all the values of  $k$  from 0 until the reflectance at each wavelength was near or reached the target, on condition that keep the number of layers minimum.

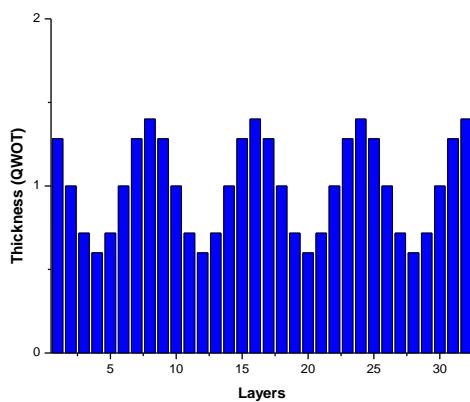


Fig. 6. Layer-thickness profile of cosine thickness modulation  
593 nm yellow laser triple-band high reflector.

The reflectance spectra using TMD is shown in Fig. 5. At last we get 99.8996%, 99.9477% and 99.8641% reflectivity at 1342 nm, 1064 nm and 593 nm, respectively. The total layers are 32. It is much less than others using the traditional method. Fig. 6 shows the layer-thickness profiles of this design. It is easy to find that the thickness of layer is modulated by the function cosine and the modulation period is 8. The layer's physical thickness of each layer is realizable. The whole physical thickness of this design is just 5700 nm and all of them can be fabricated easily, because no one is too thin or too thick.

#### 4. Conclusions

In this study, we further analyze the thickness modulation design theory and provide a new design solution for multi-bands high reflector based on searching modulation period. The design of 593 nm/1064 nm/1342 nm triple-band high reflector in sum-frequency-mixing yellow laser is realized by this method. The only 32 layers laser high reflector is achieved using  $TiO_2/SiO_2$  materials. Central wavelength reflectivity is 99.899%, 99.947% and 99.864% at 1342 nm, 1064 nm and 593 nm, respectively.

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