A reduced complexity Volterra-based nonlinear equalizer for up to 100 Gb/s coherent optical communications

HICHEM MRABET[a,b,*], SOFIEN MHATLI[b]
[a]*Saudi Electronic University, College of Computation and Informatics, IT Department, Medina Branch, KSA*
[b]*Carthage University, Tunisia Polytechnic School, SERCOM-Labs 2078, La Marsa, Tunis, Tunisia*

This paper presents a comparative study in coherent optical OFDM (CO-OFDM) context using various equalizers to mitigate channel imperfections. Furthermore, two kinds of the equalizer are selected to compensate non-linearity fiber effects, such as sparse Volterra and maximum likelihood sequence estimate (MLSE). We propose a reduced complexity sparse Volterra equalizer to compensate nonlinearities up to 100 Gb/s in the CO-OFDM system. It is shown that sparse Volterra equalizer upgrades MLSE equalizer by about 2 dB factor. Likewise, it is demonstrated in this work that at a fiber length equal to 3000 km and OSNR equal to 10 dB, BER is inferior to 10E-4 and 10E-3 for the sparse Volterra and MLSE equalizer, respectively. Additionally, system performance is given as a function of constellation diagram and BER for M-QAM modulations format at 10 and 100 Gb/s. Finally, a complexity study is investigated comparing sparse Volterra and Volterra algorithm as a function of CPU time consumption.

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Keywords: Volterra, Sparse Volterra equalizer, MLSE, CO-OFDM

1. Introduction

The coherent optical orthogonal frequency division multiplexing (CO-OFDM) is a high spectral-efficient modulation technique able to virtually eliminate inter-symbol interference (ISI) caused by fiber chromatic dispersion (CD) and polarization-mode dispersion (PMD) [1]. The most disadvantage of CO-OFDM that hitherto remains unsolved is its vulnerability to fiber nonlinear effects due to its high peak-to-average power ratio (PAPR). Endeavors to surpass the Kerr non-linearity limit have been performed by either inserting an optical phase conjugator (OPC) at the middle point of the link [2] or by inverting the effects of distributed, nonlinear interaction among multiple frequencies stabilized optical signals [3]. However, OPC significantly reduces the flexibility in an optically routed network, whereas the work reported in Science [3] included a digital signal processing (DSP)-based non-linearity pre-compensator of excessive complexity. In addition, former advanced DSP nonlinearity compensation techniques include digital back propagation (DBP) [4], nonlinear hybrid pre- and post-compensation [5], phase-conjugated twin waves (PC-TW) [6], OFDM PC subcarrier coding/pilots (PC-SC/P) [7,9], and nonlinear equalizers (NLEs) based on the inverse Volterra-series transfer function (IVSTF) [8]. The main drawback of DBP is the extensive use of the fast Fourier transform (FFT) resulting in enormous DSP computational load. Pre- and post-compensation algorithms are also complex to implement and present marginal performance enhancement (< 0.5 dB in Q-factor [4]), while the PC-TW system description [5], halves the transmission capacity. In order to improve spectral efficiency, a dual PC-TW was proposed yielding 1.3 dB improvement [10]. Unfortunately, quadrature pulse shaping is required, which cannot be applied efficiently in multicarrier systems like OFDM. Hence, PC-SC/P was introduced to address this issue. However, PC-SC/P was hitless implemented in CO-OFDM for high order modulation formats, such as 16 quadrature amplitude modulation (16-QAM), restricting the maximum transmission capacity due to conjugated subcarriers or pilot symbols.

Indeed, NLE based on Volterra model has been demonstrated to compensate fiber nonlinear distortion performed with OOK, PSK and QAM modulation formats in optical communication systems [11,12]. Nonetheless, the implementation complexity of a Volterra model based on electrical equalizer prohibits its deployment in a real-life CO-OFDM system. Furthermore, a considerable amount of Volterra model coefficients is usually required to build a nonlinear system. Consequently, it may not be feasible to apply a Volterra model based on compensator in real-time signal processing applications [13]. One possible solution is to identify the most significant coefficients of a Volterra model and pass over all of the insignificant coefficients [13,14].The resulting Volterra model is referred to a sparse Volterra model by some researchers [15]. Moreover, this paper shows that the number of kernels of a Volterra model based on the equalizer can be significantly reduced using the modified Gram-Schmidt method.

On the one hand, the nonlinear effect of a high order modulation, 10/100 Gbit/s CO-OFDM system is investigated thanks to constellation diagram and BER. On the other hand, electrical equalizers based on the nonlinear equalizer, such as maximum likelihood sequence estimate

(MLSE) and sparse Volterra are designed and tested by simulations with Optisystem co-simulation with Matlab.

The rest of this paper is organized as follows: the CO-OFDM system description is performed in the second section, in terms of Volterra model, sparse Volterra model and MLSE equalizer. In the third section, the simulation results and discussions are presented. The fourth part deals with a complexity study between Volterra and sparse Volterra equalizer. Finally, the conclusion is deduced in the last section.

2. Coherent optical OFDM system description

Fig. 1 shows the block diagram of the CO-OFDM receiver equipped with NLE; the optical transmission link is set up using a single channel CO-OFDM system with and without equalizer compensation by using Matlab simulation for the transmitter and receiver blocks. Our simulation setting takes most critical optical communication systems/component parameters into account including fiber impairments.

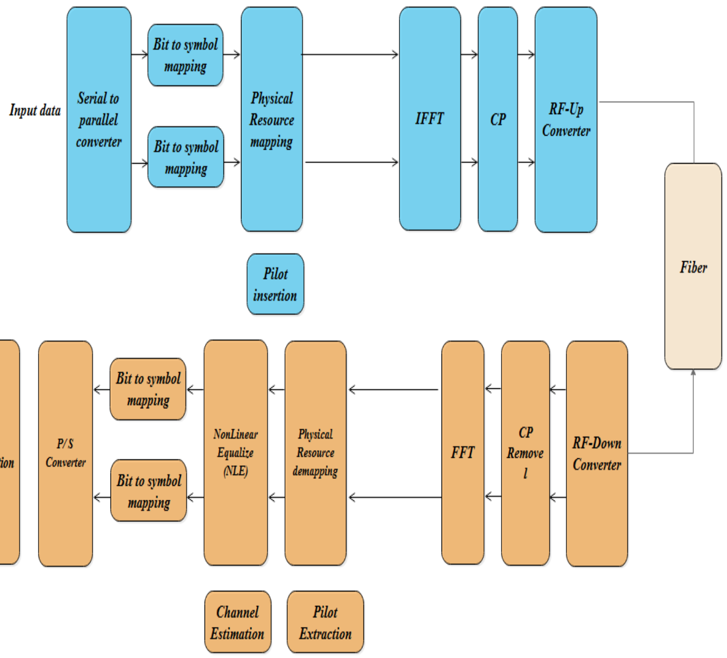

Fig. 1. Block diagram of Coherent-OFDM receiver-equipped with Nonlinear equalizer (NLE)
IFFT: inverse fast Fourier transform; CP: cyclic prefix; BER: bit error rate; FFT: fast Fourier transform; RF: radio frequency

The data transmission bit rate is equal to 10-Gb/s. On the transmitter side, a bit stream is generated using a pseudo-random binary sequence generator, and a 4-QAM encoder maps the data. The information stream is further parsed into 128 low-speed parallel data subcarriers and processed by the IFFT processor CPU is added to ensure correct data recovery.

The signal was launched into an erbium-doped fiber amplifier (EDFA) amplified transmission link with 100 km single mode fiber (SMF) per span and -2dBm signal launch power. The SMF is assumed to have a chromatic dispersion (CD) of 17 ps/km/nm and a fiber loss of 0.2 dB/km.

The optical signal to noise ratio (OSNR) of the optical signal can be varied by adjusting the attenuation of the variable optical attenuator (VOA) which is changes the amount of amplified spontaneous emission (ASE) that is added to the system, the OSNR at the end of the link is given by:

$$\mathrm{OSNR}\ \ (\mathrm{db}/0.1\mathrm{nm}) = \frac{P_0}{P_{ASE}} \qquad (1)$$

where P_0 : launched power and P_{ASE} : ASE power

The parameters adopted for the simulation are presented in Table 1.

Table 1. CO-OFDM transceiver parameters

Parameters	Values	Unit
No. of sub-carriers(Nsc)	128	—
Operating wavelength	1550	nm
Kerr non-linearity coefficient	2.6×10^{-20}	m^2/W
Light source	ideal laser diode	—
Photodetector	PIN	—
Photodetector responsivity	0.9	—
Optical launch power	-6	dBm
Fibre span	100	km
Chromatic dispersion (CD)	17	ps/nm/km
Fiber loss	0.2	dB/km
EDFA gain	20	dB
EDFA noise figure	4	dB
CP length	25	%

2.1.Volterra model

The Volterra filter of fixed order and a set memory adapt to the unknown nonlinear system using one of the various adaptive algorithms (RLS, LMS) [10]. The use of adaptive techniques for Volterra kernel estimation has been well studied. Most of the previous research considers third order Volterra filters [11]. However, the fifth order filter is becoming a compelling case to enhance the CO-OFDM systemperformance [16-17].

In a Volterra model, although the output signal is a polynomial combination of the current and past input symbol, $x(n)$ its output symbol linearly depends on the Volterra filter kernels. The input-output relations of a complex third-order adaptive Volterra model as a function of time (i.e., n) can be represented by (2), where $h_i(n)$ are the linear kernels and $h_{i,j,k}(n)$ the third order kernels at time n.

$$y[n] = \sum_{I=0}^{N} h_i[n]\, x[n-i] + \sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{k=0}^{N} h_{i,j,k}(n)x(n-i)x(n-j)x^*(n-k) \tag{2}$$

where N is defined as the model non-linearity degree.

In this section, we demonstrate how the recursive least square (RLS) algorithm can be employed to update the Volterra model coefficients. Also, a third order Volterra filter to model an unknown system is used. In the Volterra model, the input vector is given in (3):

$$\begin{aligned} u(n) &= [x(n), x(n-1), \ldots., \\ & x(n-N), x^2(n)x^*(n), x^2(n)x^*(n-1), \ldots, \\ & x^2(n-N)x^*(n-N)]^T \end{aligned} \tag{3}$$

And the desired output vector D as shown as

$$\mathrm{D} = [\mathrm{d}(1), \mathrm{d}(2), \ldots \mathrm{d}(\mathrm{i}), \ldots]$$

The coefficients vector is:

$$\begin{aligned} w = [&h_0(n), h_1(n), \ldots, h_N(n), \\ &h_{0,0,0}(n), h_{0,0,1}(n), \ldots, h_{0,N,0}(n), \\ &h_{0,N,1}(n), \ldots, h_{N,N,N}(n)]^H \end{aligned} \tag{4}$$

The difference between desired output and estimated output can be expressed by

$$e(i) = d(i) - y(i) = d(i) - w^H(n)u(i) \tag{5}$$

2.2. Sparse Volterra model

The Volterra coefficient previously cited is 30 which is the value of all RLS equalizer used to adapt the Volterra coefficient, and since the vectors basis of this algorithm is not orthogonal all time it leads to increase complexity to search all filter coefficients, so it is mandatory to remove the unwanted kernels to simplify the structure in practical applications. To get a compact Volterra structure with the orthogonal search approach, we applied the Gram Schmidt method. The nonlinear equation (2) as in [18-19]:

$$y(n) = \sum_{i=1}^{L} w_i u_i(n) + e(n) \tag{6}$$

Where L is the total number of Volterra kernels, $u_i(n)$ is the linear or cubic input terms, and $\mathrm{w_i}$ is the corresponding Volterra kernels. To represent (6) in a vector form:

$$Y = \bar{Y} + E = \sum_{i=1}^{L} w_i U_i + E \tag{7}$$

where,

$$Y = [y(1), y(2), \ldots., y(i) \ldots]^T$$
$$U = [u_1(1), u_2(2), \ldots, u_i(i), \ldots]^T$$
$$E = [e(1), e(2), \ldots, e(i) \ldots]^T$$

In an orthogonal domain, (7) is equivalent to

$$Y = \bar{Y} + E = \sum_{i=1}^{L} v_i Q_i + E \tag{8}$$

Where Q is the orthogonalized matrix of vectors U. $\mathrm{v_i}$ is the orthogonal Volterra coefficients corresponding to $\mathrm{Q_i}$. As matrix Q is orthogonal, the coefficients $\mathrm{v_i}$ can be updated by:

$$v_i = \frac{Y^T Q_i}{{Q_i}^T Q_i} \tag{9}$$

A simplified model needs to be constructed so that $\bar{\mathrm{Y}}$ approaches output Y with the minimum number of terms which is equivalent to minimizing the normalized mean square error (NMSE) of the model.

$$NMSE = \frac{E^T E}{Y^T Y} = \frac{\left(Y - \sum_{i=1}^{L} v_i Q_i\right)^T \left(Y - \sum_{i=1}^{L} v_i Q_i\right)}{Y^T Y} = 1 - \sum_{i=1}^{L} D_i \tag{10}$$

$$D_i = \frac{v_i^2 Q_i^T Q_i}{Y^T Y} \tag{11}$$

The process of converting U to Q orthogonally is completed using modified Gram-Schmidt method According to [20].

2.3. MLSE equalizer

The MLSE Equalizer block uses the Viterbi algorithm to equalize a linearly modulated signal through a dispersive channel. The block processes input frames and outputs the MLSE of the signal, using an estimate of the channel modeled as a finite input response (FIR) filter.

Viterbi Decoders are usually used as forwarding error correction (FEC) devices in many digital communications and multimedia products, including mobile communication, video-audio broadcasting receivers and modems.

Other critical applications of the Viterbi Decoders are equalization for transmission channels with a memory like multipath-fading channels and numerous applications apart from digital communications, such as pattern, text and speech recognition as well as magnetic recording [22-24].

Digital equalizers based on MLSE are considered as the most efficient electronic equalization leading to lowest penalties [20]. An MLSE receiver bases its decision on a maximum-likelihood criterion coming from the estimation of the most likely sequence among all the possible ones. Maximum Likelihood detections implemented by mean of the well-known Viterbi algorithm (VA).

This part elucidates the theory behind the so-called MLSE [21-23], which has been already widely used in RF-systems and only recently has been applied to optical communications.

The algorithm calculates each time a branch metric for each state of the convolutional encoder and each transition between the previous reports and present states. These metrics are added at each instant the previous parameter giving cumulative parameters for each state, then compares the aggregate metric leading to each state to retain only the path in the trellis give rise to small total metric, we retrace the path in the trellis smaller cumulative parameter called survivor path with the sequence of states corresponding to the sequence of states the most likely [25].

3. Simulation results and discussions

In this section simulation results are performed, in terms of output constellation diagram signal and BER of CO-OFDM system. The simulated received signal constellation diagram after 2000 km fiber transmission, with -2 dBm laser launch power is presented in Fig. 2 and Fig. 3. Due to self-phase modulation (SPM) and ASE noise, the constellation diagram becomes scattered and showing a phase and amplitude distortions according to Fig.2. As shown in the constellation diagram of Fig. 3, it is clearly proved that nonlinear equalizers outperform compensation on nonlinearities and noise.

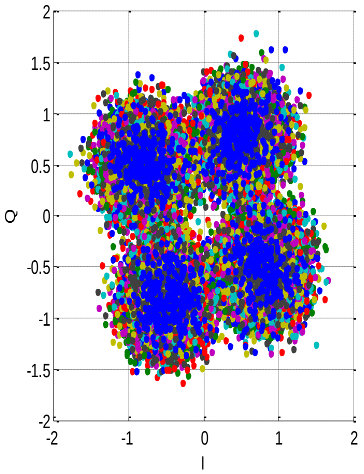

Fig. 2. Output signal constellations of the 4-QAM CO-OFDM system without equalizer

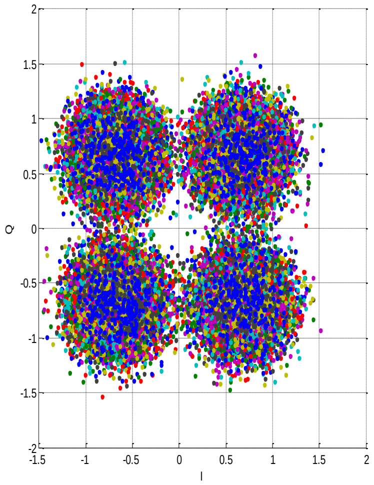

Fig. 3. Output signal constellations of the 4-QAM A CO-OFDM system with sparse Volterra equalizer

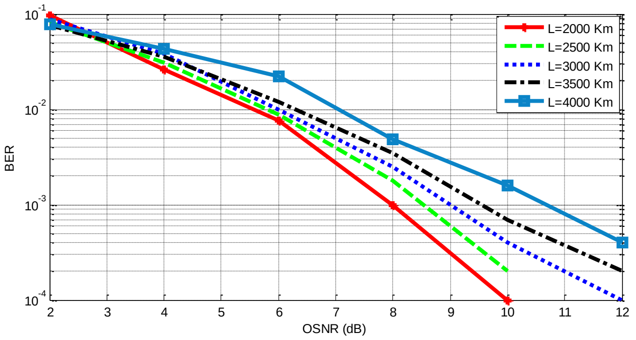

Fig. 4. BER vs fiber length for sparse Volterra equalizer and 4-QAM

The Monte Carlo simulation is repeated to calculate the BER of CO-OFDM systems with the sparse Volterra equalizer and BER curves are shown in Fig. 4. As indicated in Fig. 4, sparse Volterra equalizer achieves a bit error rate inferior to 10E-3 for 4000 km and OSNR equal to 11dB.

As demonstrated in Fig. 4, distance and performance are inversely proportional. For OSNR equal to 10 dB, BER is equal to 10E-5 and 10E-4 for L equal to 2000 km and 4000 km, respectively.

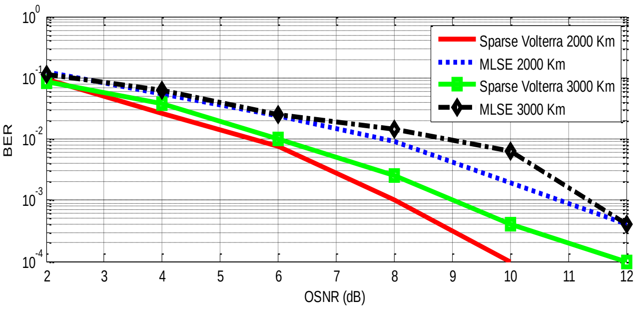

Fig. 5.Comparison between sparse Volterra and MLSE equalizer for 4-QAM

Fig. 5 shows the BERs of OFDM systems with MLSE and sparse Volterra equalizations at different OSNR under -2 dBm laser launch power. It is not surprising that by increasing OSNR, the system performance is significantly upgraded. The outperformance of nonlinear compensators becomes more evident by expanding the OSNR since the signal becomes less distorted and the compensator coefficient determination becomes more accurate. Moreover, Fig. 5 shows that sparse Volterra equalizer performs better performance compared to the MLSE one. Additionally, at a fiber length equal to 3000 km and OSNR equal to 10 dB, BER is inferior to 10E-4 and 10E-3 for sparse Volterra equalizer and MLSE equalizer, respectively. As a conclusion, the sparse Volterra equalizer enhances the system performance compared to MLSE by about 2 dB.

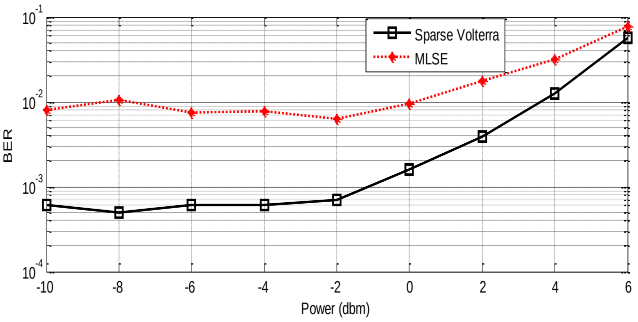

Fig. 6. BER of 4-QAM CO-OFDM systems with sparse Volterra and MLSE equalizer

The Monte Carlo simulations are conducted to evaluate the equalizer effectiveness on the CO-OFDM system after 2200 km of transmission, OSNR equal to 8 dB and the number of trellis branches is equal to 10 leading to the BER result in terms of launched power as shown in Fig. 6. At low launch powers, the sparse Volterra performance is better than MLSE corresponding to 10E-3 and 10E-2 BER value. From a launched power superior to -2 dBm, we observe a degradation of the achievements of the two equalizers and both of them are unable to compensate fiber impairments. Likewise, both equalizers reach the same performance at 6 dBm.

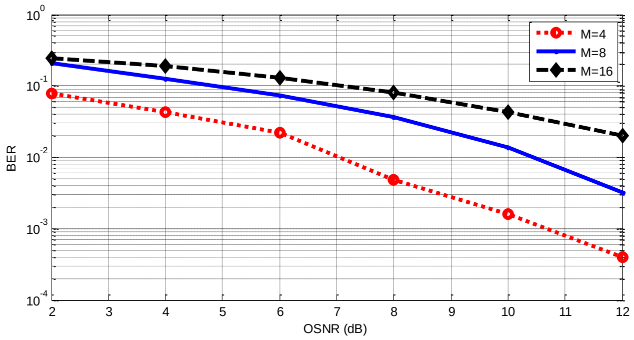

Fig. 7. CO-OFDM BER as a function of OSNR for sparse Volterra equalizer at 2200 km

Fig. 7 shows the results for the BER as a function of OSNR for all M-QAM in the range of [4-16] and under -2 dBm laser launch power. Results show deterioration of performance for optical OFDM modem under high order modulation due to the increase in the data rate. As a consequence, to achieve a high order modulation, a high SNR value should be selected.

In the next section, a simulation result is investigated for 100 Gb/s CO-OFDM based Volterra equalizer as a function of constellation diagram.

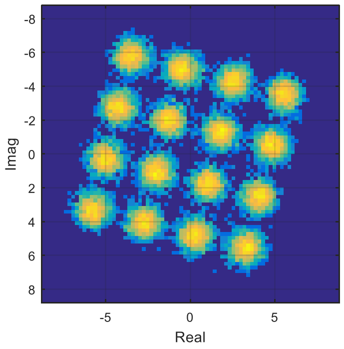

Fig. 8. Constellation before equalization for OSNR=8, 16QAM and 100 Gb/s CO-OFDM

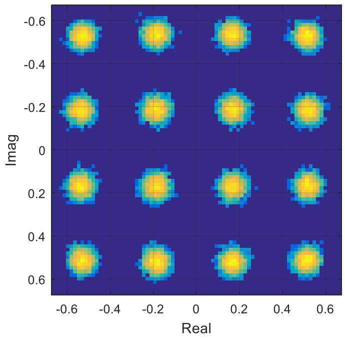

Fig. 9. Constellation for B2B, 16QAM and 100 Gb/s CO-OFDM

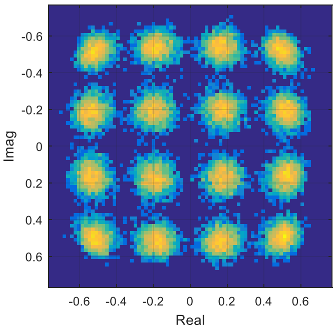

Fig. 10. Constellation after Volterra equalization for OSNR=8, 16-QAM and 100 Gb/s CO-OFDM and after 200 km

Fig. 8 shows the constellation before equalization for OSNR=8, 16QAM and 100 Gb/s CO-OFDM. Constellation diagram demonstrates the nonlinearities effect on symbols. Back to Back (B2B) constellation demonstrates a better constellation diagram to zero fiber effect on the architecture. After 200 km fiber propagation, and in comparison with constellation before equalization, Fig. 9 shows that the iterative equalizer can compensate the fiber non-linearity and we retrieve the normal constellation diagram.

Fig. 10 shows the 100 Gb/s CO-OFDM system performance as a function of constellation diagram using Volterra equalizer for OSNR equal to 8, 16-QAM high order modulation and at a fiber length equal to 200 km. Also, it's shown that Volterra equalizer can slightly enhance the 100 Gb/s CO-OFDM performance in separating symbols for decision compared to Fig. 8.

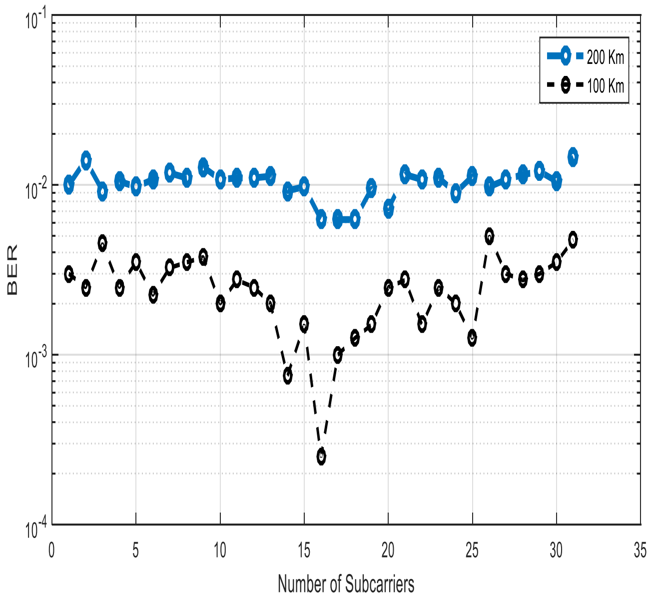

Fig. 11. BER vs. Nsc for 16QAM-100 Gb/s CO-OFDM as a function of fiber length

Fig. 11 shows the BER versus the number of subcarriers (i.e., Nsc) for 16-QAM and 100 Gb/s CO-OFDM for different fiber length equal to 100 and 200 km, respectively. As depicted in Fig. 11, it can be shown that Volterra based iterative equalizer can compensate fiber nonlinearities impairments in high data rates up to 100 Gb/s. Also, as long fiber length is increased, performance degradation is observed regarding Bit-error-rate.

Table 2. Complexity comparison between Volterra and Sparse Volterra algorithm

Complexity	**Sparse Volterra**	**Volterra**
Elapsed time consumed by the CPU(second)	0.006776	0.425097

In Table 2, we conduct a calculation of complexity in terms of CPU time consumption showing that sparse Volterra has a lower complexity in comparison with the Volterra based iterative nonlinear equalizer. The CPU consumption is equal to 0.006776 seconds and 0.425097 seconds for the sparse and the Volterra algorithm, respectively.

4. Conclusion

In this paper, system non-linearity and compensation of a single channel for 10 and 100 Gb/s with high order modulation CO-OFDM systems are investigated. It was demonstrated that sparse Volterra model and MLSE based electrical equalizer could efficiently compensate intra-channel non-linearity for the CO-OFDM system. A simple method is proposed to obtain a sparse Volterra equalizer using the modified Gram-Schmidt method. A numerical result was given indicates that sparse Volterra equalizer enhances the system performance compared to MLSE by about 2 dB factor. Additionally, a complexity study was performed in terms of CPU time consumption showing that sparse Volterra had a lower complexity compared to Volterra equalizer.

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[*] Corresponding author: h.mrabet@seu.edu.sa; sofienmhatli2017@gmail.com