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A robust chaos synchronization method under parameter mismatch

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This paper proposes a new method of chaos synchronization for chaotic system under parameter mismatch. Based on Lyapunov stability theory, the proposed synchronization method is robust to parameter mismatch. The slave system can synchronize to the master system even the parameter error is large. This can overcome the parameter mismatch problem when using the circuit components to realize the synchronization. Finally, numerical simulations are given to demonstrate the effectiveness of the proposed method.

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1. Introduction

Chaos synchronization which was first introduced by Pecora and Carroll [1], has attracted much attention in recent years, especially its applications to electronics [2], optical communication [3-6], and radar [7-11]. However, the conventional synchronization method is easily influenced by parameter mismatch, since no two coupled systems can be identical in reality due to the errors of circuit components (usually nearly 5% error or more) or the other artificial factors. Therefore, it is desirable to design an alternative synchronization method that can overcome parameter mismatch synchronization problem of the conventional method.

Parameter mismatch synchronization problem has been researched by many researchers [12-15]. In [12], an adaptive control method online estimation method is proposed for adaptive synchronization of two uncertain chaotic systems. It is an effectiveness synchronization method. However, in this method, all the parameters are changed by the same proportion factor. This is a special condition for electronic engineering. In [13], a method of parameter mismatch on anticipating synchronization of chaotic systems with time delay in the framework of the master-slave configuration is proposed. In [14,15], a synchronization method of coupled chaotic systems with time delay in the presence of parameter mismatches by using intermittent linear state feedback control is proposed. In the methods of [13-15], the parameters are not changed just by the same proportion factor. They (the parameters) are changed randomly, and this is common in practical engineering. However, the parameter error in [13-15] maller than 5%.

In this paper, we propose a parameter robust synchronization method, which can make the slave system synchronize to the master system under parameter mismatch. In the proposed method, the parameters are changed randomly, and the synchronization still can be realized even the parameter error is large. The new synchronization method in this paper is based on Lyapunov stability theory and the design procedure is demonstrated by the well known Lorenz chaotic system. Though we focus on the Lorenz chaotic system in this paper, the analysis and the thought still can be extended to other chaotic systems.

This paper is organized as follows. In section 2, a new synchronization method which is robust to parameter mismatch is proposed. In section 3, simulations on the Lorenz system is given to illustrate the effect of the proposed method. Brief conclusion of this paper is drawn in section 4.

2. Synchronization method under parameter mismatch

The chaotic signal generated from the typical Lorenz system can be expressed as:

$$\begin{cases} \dot{x}_m = s(y_m - x_m) \\ \dot{y}_m = rx_m - y_m - x_m z_m \\ \dot{z}_m = x_m y_m - bz_m \end{cases} \qquad (1)$$

where x_m, y_m, z_m are the state variables. s, r and b are the parameters. We assume that the system (1) freely "moves" about, so it is called the master system. The parameter mismatch synchronization problem is to design a controller, such that the trajectories of the slave system (2) asymptotically follow those of the master system even the parameter is not identical with the master system. In this paper, the slave system is designed as

$$\begin{cases} \dot{x}_s = (s+\Delta s)(y_s - x_s) + u_1 \\ \dot{y}_s = (r+\Delta r)x_s - y_s - x_s z_s + u_2 \\ \dot{z}_s = x_s y_s - (b+\Delta b) z_s + u_3 \end{cases} \tag{2}$$

where Δs , Δr , Δb are the errors of the parameters. The synchronization aim is desired that

$$\begin{aligned} \lim_{t\to\infty} |x_s - x_m| = 0 \\ \lim_{t\to\infty} |y_s - y_m| = 0 \\ \lim_{t\to\infty} |z_s - z_m| = 0 \end{aligned} \tag{3}$$

The main aim is to design the suitable control component $\mathbf{u} = (u_1, u_2, u_3)$. In this paper, the control component is given by Eq.(4).

$$\begin{cases} u_1 = k_1 e_1 + a_1 \\ u_2 = k_2 e_2 + a_2 \\ u_3 = k_3 e_3 + a_3 \end{cases} \tag{4}$$

where $k_1, k_{2,} k_3$ and $a_1, a_{2,} a_3$ are the constants which need to be designed. Hence, next we will show how to design $k_1, k_{2,} k_3$ and $a_1, a_{2,} a_3$ to make the slave system synchronize to the master system. First we define the synchronization error as

$$\begin{cases} e_1 = x_s - x_m \\ e_2 = y_s - y_m \\ e_3 = z_s - z_m \end{cases} \tag{5}$$

So that differentiates from Eq.(5), we define

$$\begin{cases} \dot{e}_1 = (s+\Delta s)(e_2 - e_1) + k_1 e_1 + a_1 \\ \qquad - \Delta s(x_m - y_m) \\ \dot{e}_2 = (r+\Delta r)e_1 - e_2 - x_m e_3 - z_m e_1 \\ \qquad - e_1 e_3 - \Delta r x_m + k_2 e_2 + a_2 \\ \dot{e}_3 = x_m e_2 + y_m e_1 + e_1 e_2 - (b+\Delta b) e_3 \\ \qquad + \Delta b z_m + k_3 e_3 + a_3 \end{cases} \tag{6}$$

Choosing the Lyapunov function

$$L = (1/2)(e_1^2 + e_2^2 + e_3^2) \tag{7}$$

Then we have

$$\dot{L} = \dot{e}_1 e_1 + \dot{e}_2 e_2 + \dot{e}_3 e_3 \le -\beta^T \mathbf{Q} \beta + \xi \tag{8}$$

where

$$\beta = [|e_1| \quad |e_2| \quad |e_3|]^T \tag{9}$$

and

$$\begin{aligned} \xi = [a_1 - \Delta s(y_m - x_m)]e_1 + (a_2 - \Delta r x_m)e_2 \\ + (a_3 + \Delta b z_m) e_3 \end{aligned} \tag{10}$$

and

$$\mathbf{Q} = \begin{pmatrix} -k_1 + (s+\Delta s) & -C/2 & -B_2/2 \\ -C/2 & -k_2 + 1 & 0 \\ -B_2/2 & 0 & -k_3 + (b+\Delta b) \end{pmatrix} \tag{11}$$

$$C = (s+\Delta s) + (r+\Delta r) + B_3 \tag{12}$$

Assuming that

$$\begin{cases} B_1 = \max(|x_m|) \\ B_2 = \max(|y_m|) \\ B_3 = \max(|z_m|) \end{cases} \tag{13}$$

and let

$$\begin{cases} a_1 = -\Delta s(B_1 + B_2) sgn(e_1) sgn(\Delta s) \\ a_2 = -\Delta r B_1 sgn(e_2) sgn(\Delta r) \\ a_1 = -\Delta b B_3 sgn(e_3) sgn(\Delta b) \end{cases} \tag{14}$$

In order to make the slave system synchronize to the master system, $\dot{L} < 0$ should be satisfied based on Lyapunov stability theory. This is equal to make $\mathbf{Q}$ be positive matrix. The reason is $\xi < 0$. If the parameters (s, r, b) is given and the error bound of the parameters are given, we can find the suitable (k_1, k_2, k_3) to make the two systems synchronize.

3. Simulations and analysis

In order to illustrate the effect of the proposed parameter robust synchronization method, in this section, we do simulations on the Lorenz chaotic system. For comparing the performance of the traditional chaotic synchronization method in [14] is also given.

The simulations are operated as follows. Let the master system is defined by Eq.(1), the slave system by using the proposed method in this paper is defined by Eq.(2).

Let $s = 16, r = 45.6, b = 4.$ The parameter error is given as $\Delta s = 4, \Delta r = 7, \Delta b = 2$ (the parameter error is 25%, 15.35%,50% respectively). The initial condition for the master system is given by (2, 3, 7) and that of the slave system is given by (10, 14, 12). By using the Eq.(12) $B_1 = 20, B_2 = 28, B_3 = 49$ is got and $k_1 = -46, k_2 = -65, k_3 = -14$ is got by utilizing the Matlab linear matrix inequality (LMI) control Toolbox [16]. The synchronization performance is given by Fig. 1. For comparing, we also use the conventional synchronization method as [14] under the same condition. The synchronization performance is given by Fig. 2.

We can see from Fig. 1 and Fig. 2, the proposed synchronization method is more robust to parameter mismatch and the synchronization error is smaller than that of the method in [14]. In the proposed method, after 0.37s, the slave system can almost synchronize to the master system when the parameters are mismatched.

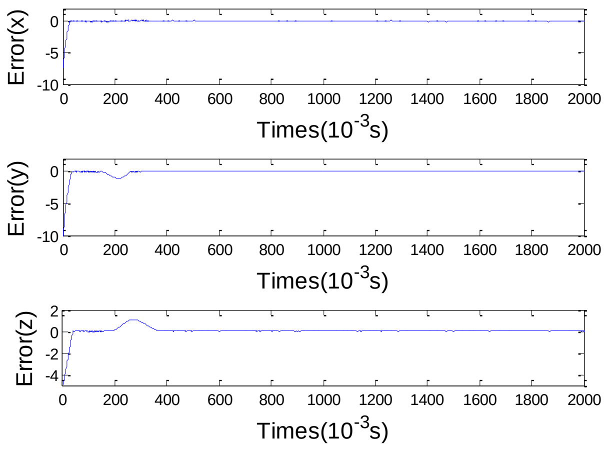

Fig. 1. The Synchronization error of the method in this paper versus time.

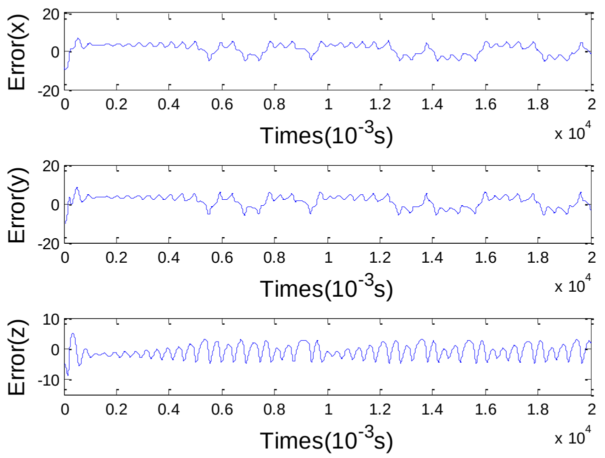

Fig. 2. The Synchronization error of the method in [14] versus time.

4. Conclusion

A method of parameter robust synchron -ization is proposed in this paper. We have investigated the properties of synchronization between master–slave chaotic systems under parameter mismatch. We find that the slave system can nearly synchronize to the master system after a short time even the parameter error is more than 20%. This can overcome the parameter mismatch problem when using the circuit components to realize the synchronization since in reality the parameter error in circuit components can not be avoided.

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