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A simple chaotic circuit with a light-emitting diode

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A novel simple autonomous optoelectronic chaotic circuit is presented in this work. Interestingly, the circuit is based on a single optoelectronic element, a light-emitting diode. The mathematical model of the circuit is described by three first-order ordinary differential equations which contain six terms with two parameters and an exponential nonlinearity. The proposed circuit is easy to be implemented by using common cheap components. The circuit can exhibit complex dynamical behavior like chaos despite of its simplicity. Therefore, it is a potential candidate for chaos-based engineering applications or is intended for educational purposes.

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1. Introduction

Chaotic circuits have received considerable interest in the literature because they have been applied in numerous areas such as secure communications, robotics, image processing or random bit generator [1-20]. One of the most important research directions is constructing robust chaotic oscillators with simple structures [21-23]. There are two kinds of simple chaotic oscillators: non-autonomous and autonomous oscillators [22]. Some typical discovered examples of simple non-autonomous chaotic oscillators are Linsay's anharmonic oscillator with a resistor, an inductor, and a varactor diode [24]; Dean's circuit with a capacitor, a linear resistor, and a resistor including ohmic losses in the inductor winding [25]; Lakshmanan's sinusoidally driven second-order circuit with three linear elements and a Chua's diode [26]; or Lindberg's chaotic oscillator with a transistor, two capacitors, and two resistors [27]. In the realm of simple autonomous chaotic oscillators, noticeable examples are Hartley's oscillator based on a junction field effect transistor and a tapped coil [28]; two-element memristive time-delay system [29]; three-element circuit with a nonlinear active memristor [30]; four-element Chua's circuit [31]; simple current tunable chaotic oscillators using floating or virtually-grounded diodes [32]; RLCC-Diode-Opamp chaotic oscillator with six electronic components [33]; or Piper's circuits using only op-amps and linear time-invariant passive components [22].

It is noting that optoelectronic chaotic circuits are the potential candidates for generating chaos [34-43]. On one hand, optoelectronic chaotic circuits have been applied in the field of optical communications with significant advantages such as larger communication channels and high bit rates [34,39]. On the other hand, optoelectronic chaotic circuits are quite simple due to the fact that optoelectronic elements provide internal nonlinear characteristics [37]. Hanias et al. introduced a simple non-autonomous chaotic circuit including an AC-voltage source, a resistor, an inductor and a light-emitting diode (LED) [38]. Authors simulated the circuit by Multisim to show chaos. A non-autonomous optocoupling circuit was presented in [37]. Authors used a 4N25 optocoupler in a typical common emitter configuration along with an emitter degeneration resistor and a collector resistor. An input sinusoidal voltage was applied through an inductor which was connected in series to the driver LED. Optoelectronic Duffing-Holmes circuit was also considered in [37]. Rocha and Medrano proposed an antiparallel blue LED configuration of a Chua's diode [44].

Motivated by published works, the aim of our work is to propose a simple chaotic system with a light-emitting diode (LED). The rest of our paper is organized as follows. In the next section, description of the new circuit and its dimensionless model are introduced. Dynamics properties of the model are investigated in section 3. Experimental results are reported in section 4. Finally, conclusion remarks are presented in section 5.

2. Description of the circuit

In this work, we consider a simple autonomous circuit as shown in Fig. 1. The circuit consists of six resistors, three capacitors, four operational amplifiers ($U_1 - U_4$) and a light-emitting diode (D). Three operational amplifiers ($U_1 - U_3$) are configured as integrators. The light-emitting diode is considered as a simple nonlinear device with an exponential function. The diode current is described by the following Shockley diode equation [45-48]:

$$I_D = I_S\left(\exp\left(\frac{V_D}{nV_T}\right)-1\right) \tag{1}$$

where I_S is a reverse bias saturation current, n is a diode ideality factor, V_T is a thermal voltage and V_D is a voltage over the diode. Conventional diodes are made from a variety of semiconductor materials and their ideality factors are not constant [49-50].

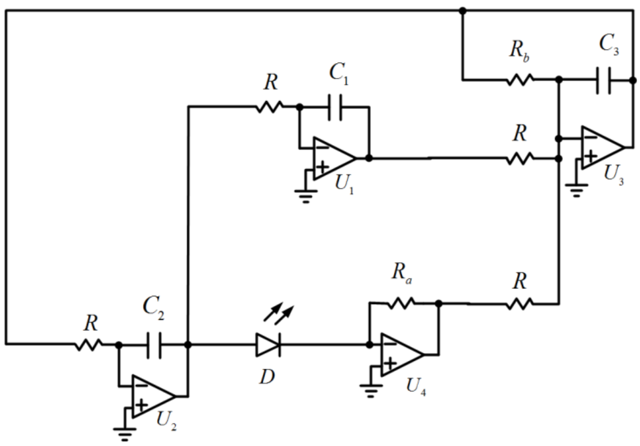


Fig. 1. Schematic of the circuit in which the light-emitting diode is denoted as D

By applying the Kirchhoff's laws into the circuit, we obtain its mathematical model given by three differential equations

$$\begin{cases} \dfrac{dv_{C_1}}{dt} = -\dfrac{1}{RC_1}v_{C_2} \\ \dfrac{dv_{C_2}}{dt} = -\dfrac{1}{RC_2}v_{C_3} \\ \dfrac{dv_{C_3}}{dt} = -\dfrac{1}{RC_3}v_{C_1} + \dfrac{1}{RC_3}R_aI_S\left(\exp\left(\dfrac{v_{C_2}}{nV_T}\right)-1\right) \\ \qquad -\dfrac{1}{R_bC_3}v_{C_3} \end{cases} \tag{2}$$

in which v_{C_1}, v_{C_2}, v_{C_3} are the voltages across three capacitors C_1, C_2 and C_3, respectively. It is noting that these capacitors have same values $C_1 = C_2 = C_3 = C$ in this work. System (2) is normalized by using dimensionless variables and parameters given by:

$$\begin{aligned} &x = \frac{v_{C_1}}{nV_T},\ y = \frac{v_{C_2}}{nV_T},\ z = \frac{v_{C_3}}{nV_T}, \\ &\tau = \frac{t}{RC},\ a = \frac{R_aI_S}{nV_T},\ b = \frac{R}{R_b}. \end{aligned} \tag{3}$$

As a result, system (2) is rewritten as

$$\begin{cases} \dot{x} = -y \\ \dot{y} = -z \\ \dot{z} = -x + a\left(\exp(y)-1\right) - bz \end{cases} \tag{4}$$

where x, y, z are state variables while a, b are two positive parameters. As seen in (4), the system contains six terms with two parameters and one exponential nonlinearity. Although the value of the parameter a depends on the light-emitting diode, but we can get a desired value of a conveniently by changing the resistor R_a. For instant, the value of a is fixed as 1.9231×10^{-4} by choosing R_a = 10 kΩ for I_S = 1 nA, V_T = 26 mV and n = 2. It is worth noting that dynamic behavior of system (4) can be changed by varying the parameter b, which does not affect the diode equation.

System (4) is dissipative because of the general condition of dissipativity

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = -b < 0. \tag{5}$$

The equilibrium point of system (4) is found by solving

$$\begin{cases} y = 0 \\ z = 0 \\ -x + a\left(\exp(y)-1\right) - bz = 0 \end{cases} \tag{6}$$

Thus, system (4) has a single equilibrium point $E(0,0,0)$. The Jacobian matrix of system (4) at the equilibrium point E is derived by

$$\mathbf{J}_E = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ -1 & a & -b \end{bmatrix}. \tag{7}$$

Therefore, the characteristic equation of system (4) at the equilibrium point E can be found as

$$\lambda^3 + b\lambda^2 + a\lambda + 1 = 0. \tag{8}$$

The eigenvalues can be obtained from (8) to determine system stability. Obviously, the stability of the equilibrium point of system (4) depends on system parameters. The system (6) is stable at the equilibrium E if the parameters satisfy $a > 0$, $b > 0$ and $ab > 1$.

3. Dynamics properties of the system

When selecting $a = 1.9231\times10^{-4}$, $b = 1$, and the initial conditions $\left(x(0), y(0), z(0)\right) = (0, 0.1, 0)$, the calculated

Lyapunov exponents of system (4) are $L_1 = 0.0983$, $L_2 = 0$, and $L_3 = -1.0994$. By applying the Routh-Hurwitz criterion to the characteristic equation (8), we see that system (4) is unstable at the equilibrium E because $ab < 1$. In this case, system (4) is chaotic because it has one positive Lyapunov exponent. Chaotic behavior of system (4) is displayed in Fig. 2.

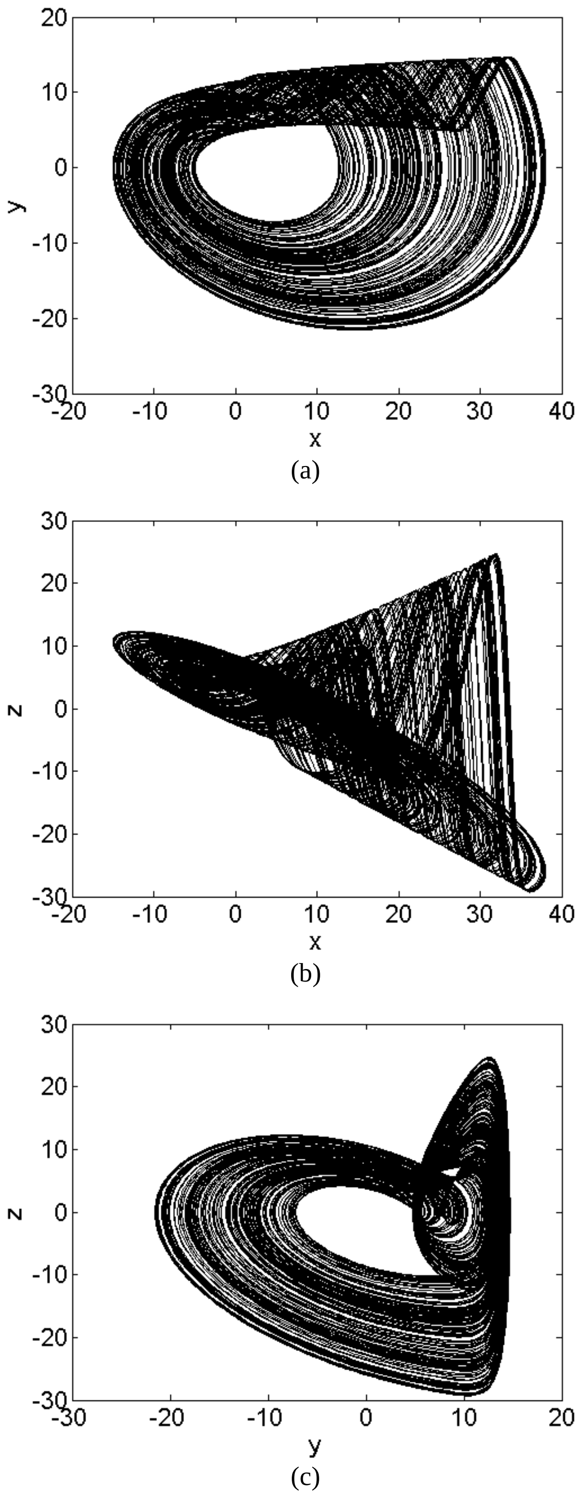


Fig. 2. Chaotic attractor of system (4) for a = 1.9231×10⁻⁴, and b = 1 in (a) x–y plane, (b) x–z plane, and (c) y–z plane

As have been known, the Kaplan-Yorke fractional dimension [51], which presents the complexity of attractor, is defined by

$$\mathrm{D_{KY}} = j + \frac{1}{\left|L_{j+1}\right|}\sum_{i=1}^{j} L_i,$$

where j is the largest integer satisfying $\sum_{i=1}^{j} L_i \geq 0$ and $\sum_{i=1}^{j+1} L_i < 0.$ The Kaplan-Yorke dimension of system (4) for $a = 1.9231\times10^{-4}$ and $b = 1$ is $\mathrm{D_{KY}} = 2.0894 > 2,$ which indicates a strange attractor.

For a clear view of the nonlinear dynamics of system (4), its bifurcation diagram is reported in Fig. 3 by plotting the local maxima of the state variable x when changing the value of the bifurcation parameter b. In addition, the maximal Lyapunov exponents (MLEs) of system (4) have been calculated using the algorithm in [52] and are presented in Fig. 4.

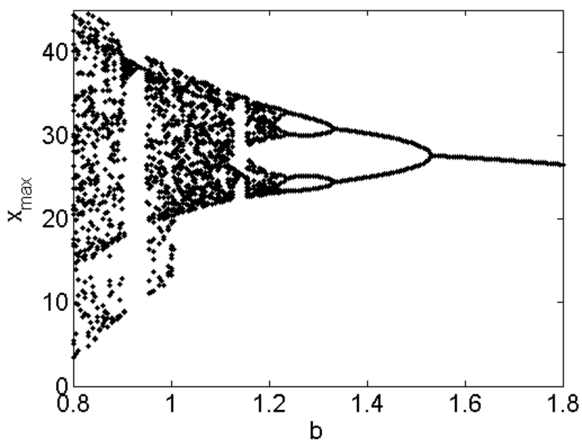


Fig. 3. Bifurcation diagram of system (4) when varying b.

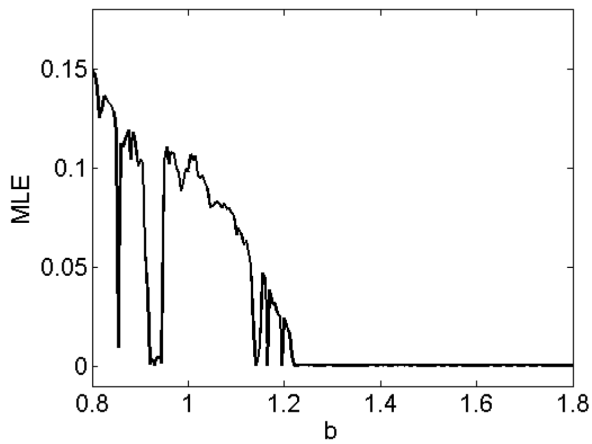
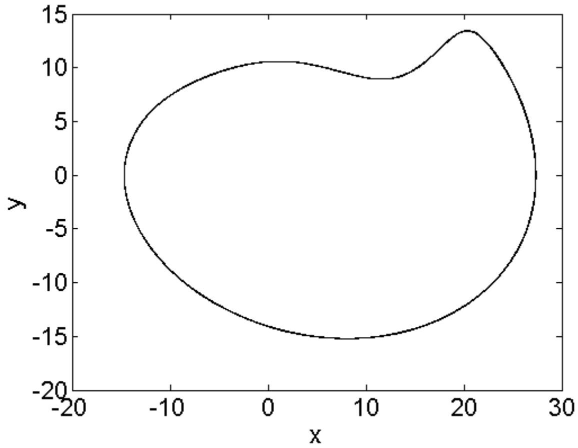
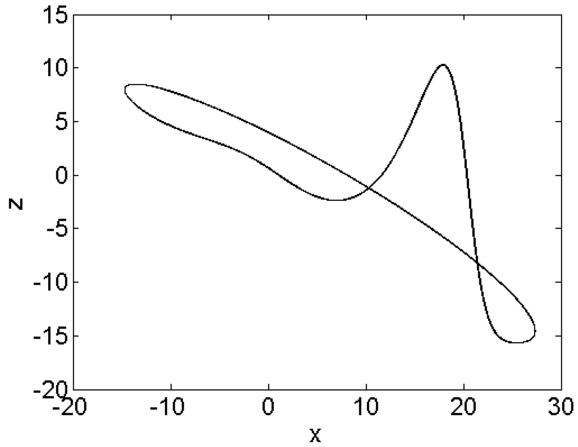


Fig. 4. Maximal Lyapunov exponents of system (4) when varying b.

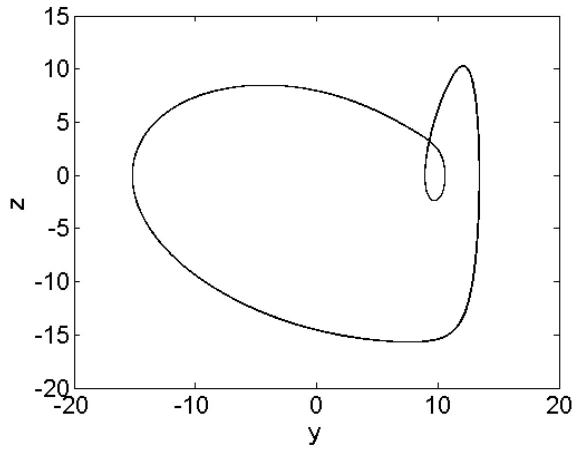
The bifurcation diagram is a valuable tool which gives the change of system's dynamic behavior [21]. Maximal Lyapunov exponent determines a notion of predictability of system (4). A positive MLE is an indication that the system is chaotic [21]. Moreover, the system displays a periodic state when MLE is equal to zero.



(a)



(b)



(c)

Fig. 5. The periodic orbit of system (4) when b = 1.6 in (a) x–y plane, (b) x–z plane, and (c) y–z plane

As seen in Figs. 3, 4 there are some windows of limit cycles, and chaotic behavior. In more detail, in the range $1.135 < b < 1.215$, the system exhibits not only periodic behavior but also chaotic behavior. For the value of the parameter $b < 1.135$, a more complex behavior is merged. In contrast, for the value of $b > 1.215$, system (4) remains always in periodic state. For example, Fig. 5 illustrates the periodic orbit of system (4) for the parameter $b = 1.6$. In addition, as reported in Fig. 4, it easy to see that the oscillator presents a reverse period-doubling route to chaos.

Moreover, dynamical behavior of system (4) has been studied by varying the parameter *a*. The bifurcation diagram and the maximal Lyapunov exponents are displayed when changing the value of the bifurcation parameter *a* as reported in Fig. 6 and Fig. 7, respectively. As shown in Fig. 6 and Fig. 7, the system can exhibit periodical and chaotic behaviors with different values of the parameter *a*.

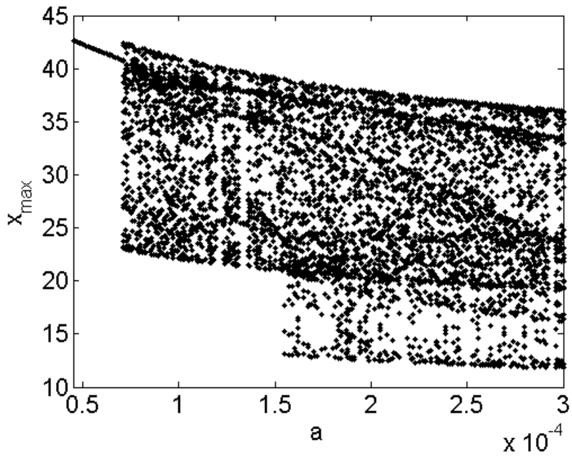


Fig. 6. Bifurcation diagram of system (4) when changing the parameter a

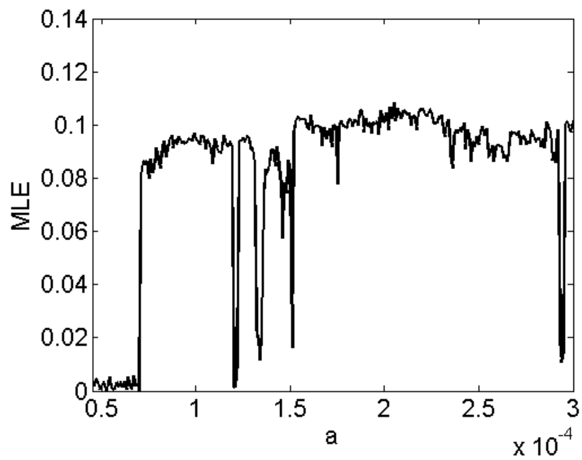


Fig. 7. Maximal Lyapunov exponents of system (4) when changing the parameter a

In order to illustrate the coexistence of attractors in system (4), we have been plotted two bifurcation diagrams of system (4) with two different initiation conditions in Fig. 8. Interestingly, the coexisting attractors can be observed in the range of the parameter b from 1.12 to 1.195.

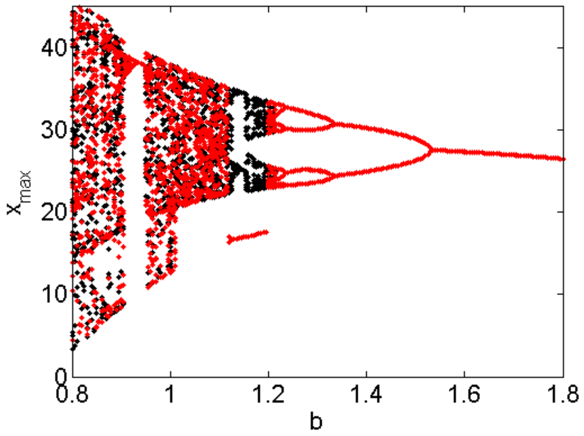


Fig. 8. Bifurcation diagrams of system (4) when varying b for different initial conditions: x(0),y(0),z(0))=(0,0.1,0) (black) and (x(0),y(0),z(0))=(0.1,0,0.1) (red)

4. Experimental results

The proposed system has been implemented in breadboard by using off-the-shelf discrete electronic components, as shown in Fig. 9.

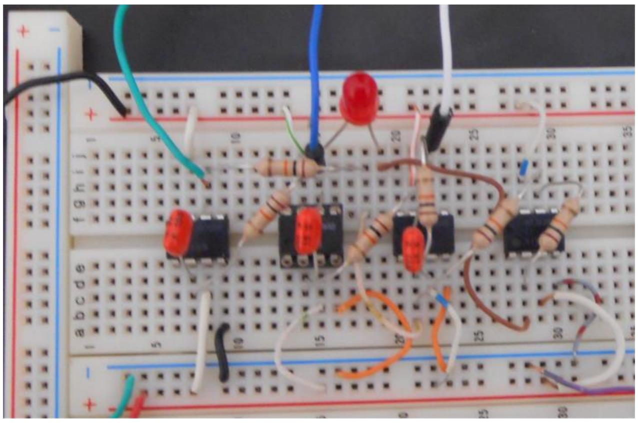
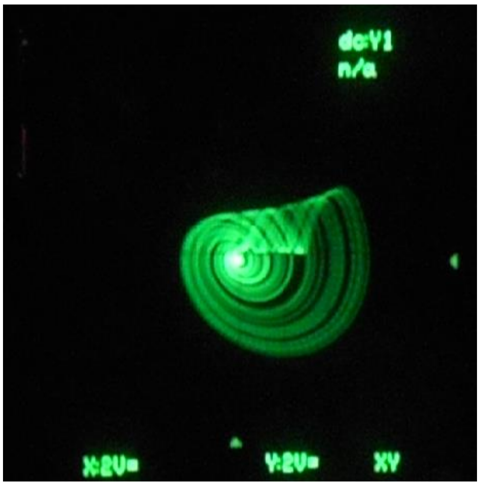
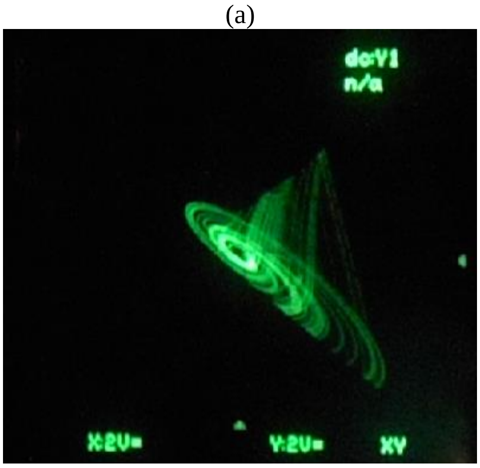


Fig. 9. Implemented circuit in breadboard in which the red light-emitting diode can be used to display the chaotic flicker of the light

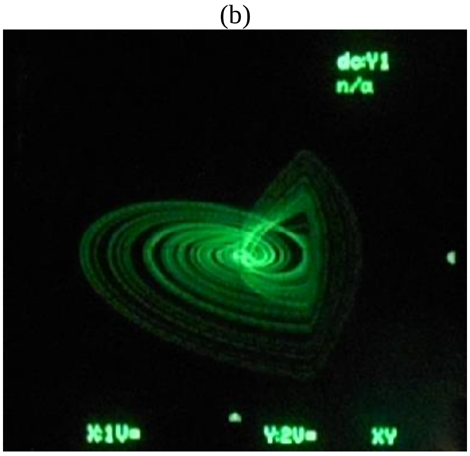
The circuit is easy to be built with commercially cheap components such as resistors, capacitors, operational amplifiers and a red light-emitting diode. In a simple experimental prototype, the values of electronic components in Fig. 9 have been selected as follows: $R = R_a = R_b = 10$ kΩ, and $C_1 = C_2 = C_3 = C = 10$ nF.



(a)



(b)



(c)

Fig. 10. Experimental attractors of the electronic circuit displayed by oscilloscope in (a) x–y plane, (b) x–z plane, and (c) y–z plane

The single light-emitting diode provides not only the nonlinearity but also a visual tool for observing chaos directly. In addition, an advantage of the circuit is the absence of inductors, thus it allows integrating such circuit in view of on-chip implementation. Experimental attractors are measured from the oscilloscope and shown in Fig. 10. There is an agreement between numerical simulations (Fig. 2) and experimental results (Fig. 10).

Different kinds of LEDs, for example the green LED and the yellow LED, have been tested to confirm the robust of our circuit. There are slightly differences in the values of parameter *b*, in which the circuit's dynamical behavior changed. This is due to the slightly different characteristics of each LED, such as the threshold voltage. However, the circuit exhibits chaos in all cases.

The main advantage of the proposed circuit is its system simplicity. It has only six terms and only one nonlinear (the exp(y) function). Also, the system is implemented with a LED, that makes it different from reported works. As have been known, the inductors always exist in simple chaotic circuits [24,25,33]. However, the inductor is a less desirable circuit element because of its inevitable parasitic resistance and its relatively large space compared to the size of the entire circuit. Our circuit includes only resistors, capacitors, operational amplifiers and a LED. Thus, the circuit is suitable for the manufacture of integrated circuits. In addition, there is no presence of multipliers in our proposed circuit; therefore we avoid the nonidealities of analog multipliers [53]. Moreover, the state variable x appears only once in the third equation of (4), therefore, the state variable x can be controlled conveniently by introducing a control parameter k in the third equation of (4). As a result, chaotic signal x can be moved from a bipolar signal to a unipolar signal easily, which is useful in practical applications.

5. Conclusions

In this work, a simple chaotic circuit has been introduced and investigated through numerical simulations and experimental implementation. Interestingly, the presence of a single light-emitting diode provides not only the nonlinearity term in the circuit but also a direct approach for observing chaotic behavior. The proposed optoelectronic circuit is able to show complex behavior, like chaos in spite of its simple structure. From a practical standpoint, the circuit is constructed by common components and is a robust standalone chaotic circuit [22]. Therefore, the circuit is suitable for chaos-based engineering applications. However further studies must be done when using it for secure communications. For example, we have to consider the synchronization of systems, the maximum operation frequency or actual communication protocols. It is noted that we should be aware of the limitations of commercially available amplifiers [54,55].

In addition, simple chaotic circuits with hidden attractors [56-60] should be studied in future works.

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