OPTOELECTRONICS AND ADVANCED MATERIALS – RAPID COMMUNICATIONS Vol. 4, No. 11, November 2010, p. 1811 - 1814

A theoretical analysis and development of a quartz strip mass-sensitive resonator AT-cut

Y. LAZAROV*, B. DULMET[a], Z. RAICHEVA, L. SPASSOV, V. GEORGIEVA, V. GADJANOVA, M. ATANASSOV, Ts. YORDANOV

"Georgi Nadjakov" Institute of Solid State Physics, Bulgarian Academy of Sciences, 72 Tzarigradsko Chaussee Blvd., 1784 Sofia, Bulgaria

[a] *FEMTO-ST, CNRS, Université of Franche Comté, Ecole Nationale Supérieure de Mécanique et des Microtechniques, 26 Chemin de l'Epitaphe 25000 Besançon, France*

A theoretical analysis and development of an AT-cut quartz strip mass-sensitive resonator operating at 20 MHz fundamental thickness-shear mode is presented in this paper. The frequency spectra of acoustic waves propagating in a quartz strip plate is analyzed. Quartz plate dimensions ratio is determined on the basis of the theoretical analysis results. This assures a dominant contribution of the main thickness-shear mode in the complex wave, with a minimal modification as a result of the coupling with unwanted modes. A new miniature quartz mass-sensitive resonator is designed. An experimental series of resonators is produced and resonators motional parameters are measured. Samples of mass-sensitive resonators can be investigated as high sensitive sensors for registration of noxious gases in the atmosphere, like CO_2, nitric oxides etc.

(Received August 2, 2010; accepted November 10, 2010)

Keywords: Theoretical analysis, Quartz, Mass-sensitive resonator, Sensor

1. Introduction

The theoretical analysis is based on a previously [1] developed two-dimensional, semi-analytical model for analysis of coupled acoustic waves in quartz plates. Now the model is used to analyse the frequency spectra of acoustic waves propagating in a quartz strip plat At-cut. The coupling between thickness shear and flexural waves propagating along the plate's width is of great importance in forming the complex nature of the main mode. It causes a modulation of the mechanical displacement along the width depending on the width to thickness dimensions ratio. According to the theory the resonator will have high mass-sensitivity because of its high resonance frequency and the small electrodes area. If there will be such a possibility in the future, this miniature quartz mass-sensitive resonator can be investigated as a sensor for registration of noxious gases in the atmosphere.

2. Theoretical analysis

The analysis consists of several steps: solving the acoustic waves propagation equations and boundary conditions on the free major surfaces of the plate, located at $x_2 = \pm h$, where h is a half thickness; obtaining the dispersion curves giving the dependence of the frequency of acoustic modes propagating in strip plate cross-section versus the lateral wavenumber; solving the edge boundary conditions; obtaining the dispersion curves giving the dependence of the frequency versus width to thickness dimensions ratio; obtaining the mechanical displacement amplitude distribution along plate's width for values of the dimensions ratio varying between 24.2 and 25.7; determination of the best dimensions ratio in the investigated interval.

The principle of a two-dimensional semi-analytical model for a theoretical analysis of the frequency spectra in quartz strips was presented in our previous papers.

A quartz strip cross-section is shown on Fig. 1.

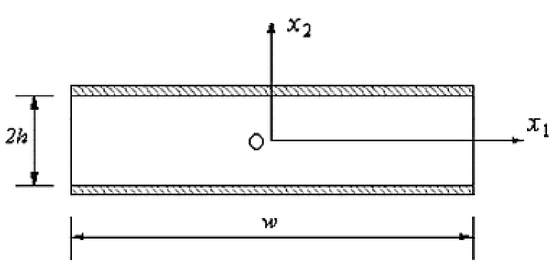

Fig. 1. Quartz strip cross-section.

The first step is the analysis of guided waves propagating along x_1 axes, i.e. the direction of the strip width, and having a piezoelectrically excitable component u_1 of mechanical displacement which is simultaneously antisymmetric along the thickness (x_2 axis) and symmetric w.r.t. the middle of the strip.

In singly-rotated Y+ θ cuts, the most general form of solution of elasticity having this symmetry and obeying the free boundary conditions on major surfaces of the plate at $x_2 = \pm$ h is the following:

$$u_1 = \cos(\xi x_1) \sum_{n=1}^{3} C_n \beta_1^n \sin(\eta_n x_2)$$
$$u_2 = \sin(\xi x_1) \sum_{n=1}^{3} C_n \beta_2^n \cos(\eta_n x_2) \quad (1)$$
$$u_3 = \sin(\xi x_1) \sum_{n=1}^{3} C_n \beta_3^n \cos(\eta_n x_2)$$

The process of solving the elasticity equations and boundary conditions is realized using the computer program made on Scilab software.

The dispersion curves obtained for AT-cut as a result of the computer program are shown on Fig. 2 where the ordinate and abscissa are the frequency and lateral wavenumber respectively normalized by the values of the thickness-shear fundamental C mode frequency and the vertical wavenumber.

Thus, the dimensionless frequency Ω and wavenumber γ are defined by:

$$\Omega = \frac{\omega}{\omega_F} \qquad \gamma = \frac{2h\xi}{\pi}$$

This first stage of the two-dimensional model is rigorous and well established, and the combinations (1) actually describe branches of the dispersion curves as self consistent solutions of elasticity equations and boundary conditions on the major surfaces of plate located at x_2 ±h. But, in the absence of energy trapping along x_1, a single branch of dispersion curve cannot be a self sufficient solution of the vibration problem for a strip plate in the rectangular cross section in the (x_1, x_2) plane. This is because the form of the solution (1) does not allow to identically satisfy the three boundary conditions on the free edges located at $x_1 = \pm$ w/2, (Fig. 3).

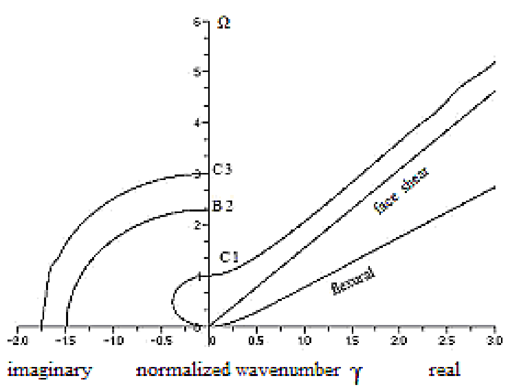

Fig. 2. Dispersion curves.

It means that the behavior of the vibration is actually more complicate than suggested by the solution (1).

The edge boundary conditions are solved using Galerkin variational method [2] in conformity with Hamilton's principle [3]:

$$\int_{t_0}^{t_1} \left\{ \int_V (T_{ij,j} - \rho\ddot{u}_j)\, \delta u_j\, dV - \int_S (f_i - n_j T_{ij})\, \delta u_i dS \right\} dt = 0 \quad (2)$$

The problem is solved by finding a combination of modes from the dispersion curves. Using variational equation (2), the best combination is found to obey the boundary conditions with a minimal error. This mechanism was also presented in our previous work [1,4] concerning the development of quartz temperature sensors NLC-cut.

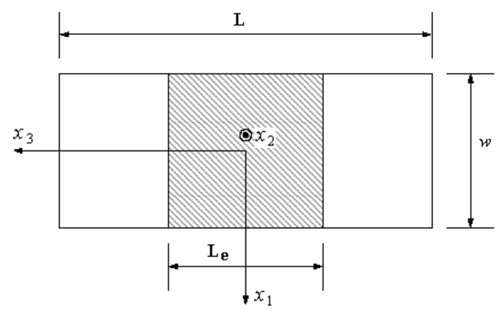

Fig. 3. Typical form of a strip plate.

The dimensions ratio w/2h fixes as a parameter and the computer program finds the corresponding root for the frequency. Thus the dispersion curves – frequency w.r.t. dimensions ratio are obtained. They are shown on Fig. 4. Two anharmonic modes called "mode 1" and "mode2" are found in the investigated interval of dimensions ratio values. In a present case 5 modes are included into the combination – these are the modes shown on Fig. 2: fundamental, flexural, face shear, and the imaginary parts of B2 and C3 modes. The coupling mechanism between modes leads to a modulation of the fundamental mode amplitude distribution as it is shown on Fig. 5.

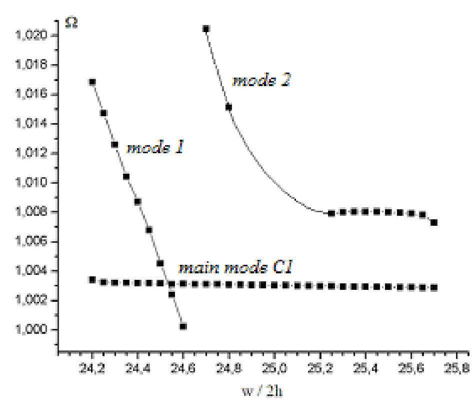

Fig. 4. Dispersion curves Ω = f (w/2h).

The best dimensions ratio was defined using the analysis results. It is in the interval of values 24.9 – 25.1 and assures a dominant contribution of a thickness shear fundamental mode in the complex wave. On the next Fig. 6 we see the opposite case when the anharmonic mode 1 which is essentially flexure, has a dominant contribution at w/2h = 24.55.

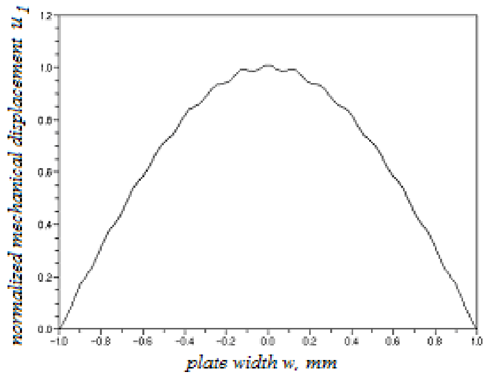

Fig. 5. Theoretical distribution of u1 along the width of a strip with w/2h =25.

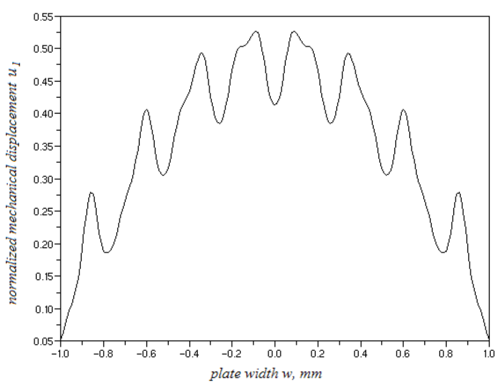

Fig. 6. Theoretical distribution of u1 along the width of a strip with w/2h =24.55

3. Designing of a new quartz strip mass-sensitive resonator and production of experimental series

A new resonator is designed in "Acoustoelectronics " laboratory. The quartz strip design is shown on Fig. 7. Plate dimensions allow the mounting into the standard holder TC 39 used for quartz clock resonators.

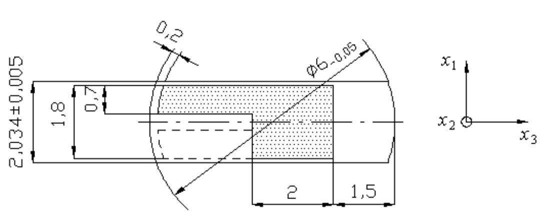

Fig. 7. Quartz strip design.

The nominal resonant frequency is 20.1 MHz fundamental mode. The produced samples have gold as well as silver electrodes. The electrode thickness is 60 nm and 120 nm respectively and the plate thickness is ~ 81 μm.

The resonator mounted in TC 39 holder is shown on the picture.

4. Motional parameters measurements

The resonators motional parameters are presented on the table:

sample number	Co static capacity	Rs dinamic resistance	Fs serial resonance frequency	Fp parallel resonance frequency	Cq dinamic capacity	Lq dynamic inductivity	Q quality factor
No	pF	Ohm	Hz	Hz	F	mH	-
1/1/Ag	2,166	137,3	20187808	20213244	7.12E-15	8.74	8067
2/1/Ag	2,178	56,67	20317867	20344779	7.52E-15	8.17	18395
3/1/Ag	2,198	531	20254662	20285745	8.77E-15	7.05	1688
5/1/Ag	2,143	197	20105754	20120071	3.99E-15	15.71	10071
7/1/Ag	1,996	174,8	20261883	20294418	8.53E-15	7.24	5271
9/1/Au	2,302	188,5	20143262	20168846	7.52E-15	8.31	5574
11/1/Au	2,263	459,7	20223300	20253094	8.61E-15	7.20	1989
12/1/Au	2,759	136,6	20273142	20297906	8.35E-15	7.39	6884
13/1/Ag	2,469	479,2	20220619	20246309	7.95E-15	7.80	2067
1/2/Au	2,239	237,8	20178102	20205381	7.84E-15	7.94	4234
2/2/Au	1,388	106	20278833	20300588	4.39E-15	14.03	16858
6/2/Au	2,186	404	20189864	20219091	8.24E-15	7.55	2369
9/2/Au	2,388	38,4	20213370	20240531	8.19E-15	7.58	25045
10/2/Ag	2,098	171,3	20244158	20269018	6.77E-15	9.13	6779
12/2/Ag	2,186	66,6	19933235	19963420	8.62E-15	7.40	13916
B-1	2,126	151,5	20160200	20187751	7.61E-15	8.19	6847
B-2	2,156	133,5	20081710	20101517	5.55E-15	11.32	10692
B-3	2,297	96,6	20139327	20167732	8.34E-15	7.49	9813
B-4	2,126	189,9	20164822	20187376	6.23E-15	10.01	6672
B-5	2,258	74,2	20180053	20207936	8.06E-15	7.72	13188
B-6	2,034	694	20263460	20287030	6.27E-15	9.85	1807
B-7	2,158	155,7	20287409	20314403	7.50E-15	8.22	6722
B-8	2,146	136,2	20063206	20088728	7.14E-15	8.82	8163
B-11	2,566	161,3	20095918	20123437	8.84E-15	7.11	5560
09.12.13	2,14	119,3	19759425	19802036	1.21E-14	5.38	5594
09.12.11	2,16	205	19699204	19716767	5.03E-15	12.99	7842
09.12.10	2,1	37,2	19681162	19706964	7.24E-15	9.05	30054
09.12.07	2,15	27,3	19743719	19773062	8.35E-15	7.79	35730
09.12.05	2,08	87,3	19646339	19672276	7.23E-15	9.08	12833
09.12.03	2,07	21,2	19685128	19712980	7.73E-15	8.47	49392
09.12.28	2,2	106,3	19660037	19682440	6.52E-15	10.06	11690
09.12.27	2,17	328	19712864	19724948	3.47E-15	18.81	7098

The results of the measurements show that 12.5 % of samples have a motional resistance up to 50 Ω, 15.6 % have a resistance 50 ÷ 100 Ω, and quality factor of these samples is between 10 000 and 50 000.

5. Conclusion

We think that as a first attempt to realize a miniature quartz strip mass-sensitive resonator we have relatively good results. Of course it can be improved in the future and a new resonator can be investigated as a Quartz Crystal Microbalance based sensor having high mass-sensitivity and especially as a new sensor for registration of noxious gases in the atmosphere, like CO_2, nitric oxides etc.

Acknowledgements

The present work is realized with the financial support of the Institute of Solid State Physics - Bulgarian Academy of Sciences under an internal contract with Acoustoelectronics laboratory.

References

[1] B. Dulmet, L. Spassov, R. Bourquin, A. Ivan, Ts. Angelov, J. Lazarov, Proc. of the 19th European Frequency and Time Forum, 2005, Besancon, France, Electron. version, p.627.
[2] G. Dhatt, G. Touzot, Maloine Ed., Paris, 1984.
[3] H. F. Tiersten, J. Math. Phys. **9**, (1968).
[4] B. Dulmet, Yu. Lazarov, L. Spassov, Proc. of the 20th European Frequency and Time Forum, 2006, Braunschweig, Germany, Electron. Version.

*Corresponding author: julian@issp.bas.bg