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Adaptive reduced-order function projective synchronization and circuit design of hyperchaotic DLE with no equilibria

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By introducing a feedback control to a classic DLE system (diffusionless Lorenz equations), an extremely complex hyperchaotic attractor with no equilibria is derived. Based on adaptive control and Lyapunov stability theory, we design a reduced-order projective synchronization scheme for synchronizing the hyperchaotic DLE coexist with no equilibria and the 3-D chaotic Wang-Chen system coexist with one stable equilibrium, which both do not meet the Sil'nikov criteria. Finally, numerical simulations are given to illustrate the effectiveness of the proposed synchronization scheme, and then its circuit implementation is included as real applications.

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Keywords: Hyperchaotic attractors, Reduced-order, Projective synchronization, Lyapunov exponent, Lyapunov stability theorem, Circuit implementation

1. Introduction

It is concerned with the classification and determination of the type of chaos observed experimentally, proved analytically, or tested numerically in theory and practice. However, Sil'nikov criteria is sufficient but certainly not necessary for emergence of chaos.

In 2010, Yang, Wei and Chen [1] introduced and analyzed a new 3-D chaotic system (called generalized DLE system), in a form very similar to the Lorenz, Chen, Lü and Yang-Chen system [2], but it has only two stable node-foci. Recently, Wei and Yang introduced a generalized Sprott C system with six terms by using linear feedback and found that the the control parameter can made the generalized Sprott C system to generate chaotic attractors existing with two stable equilibria [3].Moreover, Research results on related problems can be shown in [4-12].

In 2011, Wang and Chen discovered a simple 3-D autonomous quadratic system that has only one stable equilibrium [13], revealing some new mysterious features of chaos. Later, Wei made the Sprott D system to preserve its chaotic dynamics by a tiny perturbation, and demonstrated that the perturbed Sprott D system with no equilibria has the cascade of period doubling bifurcations and chaotic attractors [14].

On the other hand, hyperchaos characterized with more than one positive Lyapunov exponent, has attracted increasing attention from various scientific and engineering communities, and was first reported by Rossler in 1979 [15]. It might be due to the fact that the hyperchaotic systems are more complex and chaos generation in 4D autonomous systems is more difficult than chaotic systems [16-20]. Generating a hyperchaotic attractor, in particular purposefully designing a hyper-chaotic system from an originally chaotic system with only two stable node-foci by some simple feedback control techniques, is a very attractive and yet technically quite challenging task theoretically. It has wide foreground, important theoretical and practical meanings to carry this research further. In addition, in spite of the natural curiosity and extreme efforts about generation of a three and four autonomous system with different multi-scroll chaotic attractors, there is no report about the result that the coexistence of hyperchaotic attractor and another attractor in the 4-D autonomous system with no equilibria.

In the recent years, many various schemes have been applied for chaos synchronization, for example, complete synchronization [21], anti-synchronization[22], phase synchronization [23], generalized synchronization [24], etc. However, in these synchronization schemes the drive system and the response system always have same order. Now the synchronization of chaotic systems with different order has received less attention [25-27]. In fact, the synchronization phenomena of chaotic systems with different order are the more common form. In the case of thalamic neurons, for instance, such a problem is reasonable if their order is different from the one of the hippocampal neurons [27].

Inspired by the above ideas, a new hyperchaotic system without equilibria form the generalized DLE system is proposed and analyzed in this paper. We also will design asynchronization scheme to realize projective synchronization between the hyperchaotic DLE coexist with no equilibria and the 3-D chaotic Wang-Chen system coexist with one stable equilibrium, which both do not meet the Sil'nikov criteria. Numerical simulations are presented to demonstrate the effectiveness of the proposed adaptive controllers. In order to realize of the new hyperchaotic circuit, a microcontroller based circuit has to be designed to applying the initial condition voltages to the capacitors.

2. The hyperchaotic DLE with no equilibria

The generalized DLE is described by the following equations:

$$\begin{cases} \dot{x}_1 = a(y_1 - x_1) \\ \dot{y}_1 = -x_1 z_1 - c y_1 \\ \dot{z}_1 = -b + x_1 y_1, \end{cases} \tag{2.1}$$

where a, b, c are real parameters. When $b > 0$ and $a \neq 0$, the model (2.1) has the following two fixed points:

$$E_1(\sqrt{b}, \sqrt{b}, -c), \quad E_2(-\sqrt{b}, -\sqrt{b}, -c).$$

In addition, the divergence of the system is $-a-c$ which implies that the system is dissipative for $a+c>0$, since the volume of the system contracts according to the Liouville formula. It is easy to show that the system (2.1) is topologically equivalent to the original DLE[28].For parameter values $(a,b,c) = (10,100,11.2)$, which yields a typical chaotic attractor(see Fig. 1), three characteristic values of the Jacobian of its linearized system evaluated at the equilibria are:

$$\lambda_1 = \text{-}20.9778, \quad \lambda_{2,3} = -0.1111 \pm 9.7635\mathrm{i}.$$

Clearly, system (2.1) has a chaotic attractor coexisting with two stable node-foci. This implies that system (5) with (a, b, c) = (10, 100, 11.2) has neither homoclinic nor heteroclinic orbits, and hence it has a chaotic attractor without any Sil'nikov orbits.

Now, by introducing an additional state w_1 and couple it to the second equation of the chaotic system (2.1), there by obtaining a new 4-D system

$$\begin{cases} \dot{x}_1 = a(y_1 - x_1) \\ \dot{y}_1 = -x_1 z_1 - c y_1 + k w_1 \\ \dot{z}_1 = -b + x_1 y_1 \\ \dot{w}_1 = -m y_1, \end{cases} \tag{2.2}$$

where a, b, c, k, m are constant parameters. It is easy to find the system (2.2) has no equilibrium points. But when a = 5, b = 5, c = 0, k = 1, m = 1, and the initial conditions are (5, -0.1, 1, 2), the system (2.1) is hyperchaotic and its attractor is shown. Then Fig. 2 shows actually hyperchaotic behavior as expected. In this case, the system (2.1) has two positive Lyapunov exponents

$$\lambda_1 = 0.1812, \quad \lambda_2 = 0.0389$$

and the other two are

$$\lambda_3 = -0.0020, \quad \lambda_4 = -5.2181.$$

3. Adaptive reduced-order function projective synchronization between system (2.2) and Wang-Chen system

In the following text, we will design reduced-order projective synchronization scheme between hyperchaotic DLE coexist with no equilibria and the 3-D chaotic Wang-Chen system coexist with one stable equilibrium. The projective subsystem which is constructed by the first three equations of hyperchaotic DLE. The response 3-D chaotic Wang-Chen system is [13]

$$\begin{cases} \dot{x}_2 = y_2 z_2 + \alpha \\ \dot{y}_2 = x_2^2 - y_2 \\ \dot{z}_2 = -1 + 4x_2. \end{cases} \tag{3.1}$$

Note that the system is chaotic when $a = 0.006$. System (3.1) also can display a chaotic attractor with one and only one stable equilibrium (see Fig 3).

Now we give the response system

$$\begin{cases} \dot{x}_2 = y_2 z_2 + \alpha + u_1 \\ \dot{y}_2 = x_2^2 - y_2 + u_2 \\ \dot{z}_2 = -1 + 4x_2 + u_3, \end{cases} \tag{3.2}$$

where u_1, u_2 and u_3 are the controllers.

Define 1. Systems (2.2) and (3.1) are referred to as achieving reduced-order projective synchronization if and only if the three following equalities satisfy simultaneously:

$$\lim_{t\to\infty} e_1(t) = \lim_{t\to\infty} \left| x_2(t) - n \cdot x_1(t) \right| = 0,$$

$$\lim_{t\to\infty} e_2(t) = \lim_{t\to\infty} \left| y_2(t) - n \cdot y_1(t) \right| = 0,$$

$$\lim_{t\to\infty} e_3(t) = \lim_{t\to\infty} \left| z_2(t) - n \cdot z_1(t) \right| = 0,$$

where the error signals

$$e_1(t) = x_2(t) - n \cdot x_1(t),\ e_2(t) = y_2(t) - n \cdot y_1(t),$$
$$e_3(t) = z_2(t) - n \cdot z_1(t).$$

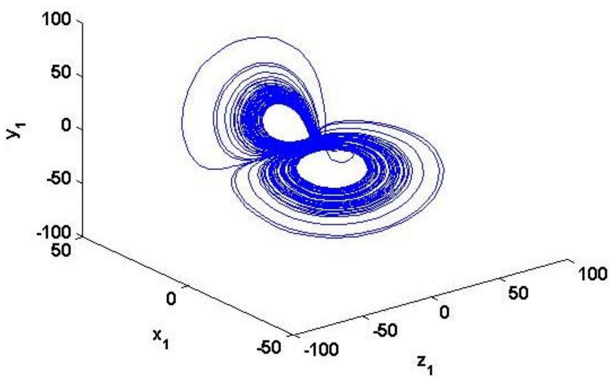

Fig. 1. Coexistence of chaotic attractors and two stable equilibria of the system (2.1) for the case $(a, b, c) = (10, 100, 11.2)$ *with initial conditions (0.001, 0.001, 0.001).*

By using system (3.2) and the error signals, the error dynamical system can be obtained as below:

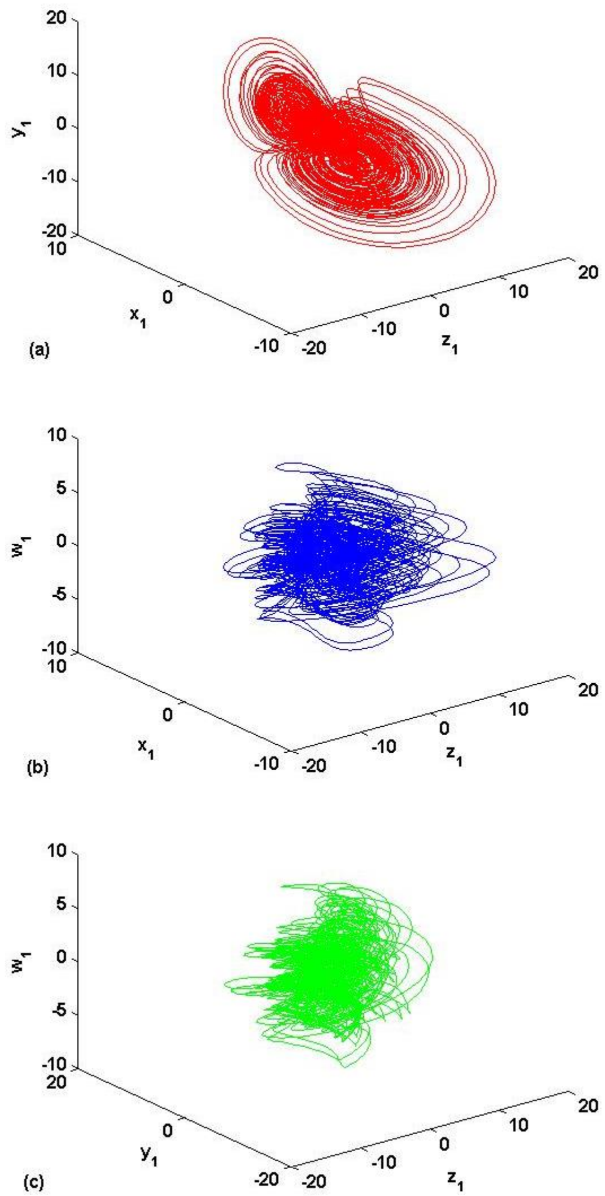

Fig. 2. Hyperchaotic attractors of the system (2.2) with no equilibria for the case $(a, b, c, k, m) = (5, 5, 0, 1, 1)$ *with initial conditions (5, -0.1, 1, 2) : (a) attractor projected in 3-D space* $x_1 - y_1 - z_1$ *; (b) attractor projected in 3-D space* $x_1 - z_1 - w_1$*; (c) attractor projected in 3-D space* $y_1 - z_1 - w_1$.

$$\frac{de_1(t)}{dt} = y_2 z_2 + \alpha - na(y_1 - x_2) + u_1,$$

$$\frac{de_2(t)}{dt} = x_2^{\,2} - y_2 - n(-x_1 z_1 - cy_1 + kw_1) + u_2, \qquad (3.3)$$

$$\frac{de_3(t)}{dt} = -1 + 4x_2 - n(x_1y_1 - b) + u_3.$$

We define the controllers as follows

$$u_1 = -y_2z_2 - \alpha_1 + na_1(y_1 - x_1) - p_1e_1,$$

$$u_2 = -x_2^{\ 2} + y_2 + n(-x_1z_1 - c_1y_1 + k_1w_1) - p_2e_2,$$

$$u_3 = 1 - 4x_2 + n(x_1y_1 - b_1) - p_3e_3.$$

Here p_3 , p_2 and p_3 are positive constants representing control gain. Then the following estimates of parameters update laws are set:

$$\frac{da_1(t)}{dt} = -ne_1(y_1 - x_1),$$

$$\frac{db_1(t)}{dt} = ne_3,\ \frac{dc_1(t)}{dt} = ne_2y_1,$$

$$\frac{dk_1(t)}{dt} = -ne_2w_1,\ \frac{d\alpha_1(t)}{dt} = e_1.$$

Therefore, the reduced-order projective synchronization between the two systems (2. 2) and (3.2) will must satisfy

$$\lim_{t\to\infty}\left|a - a_1(t)\right| = 0,\ \lim_{t\to\infty}\left|b - b_1(t)\right| = 0,\ \lim_{t\to\infty}\left|c - c_1(t)\right| = 0,$$

$$\lim_{t\to\infty}\left|k - k_1(t)\right| = 0,\ \lim_{t\to\infty}\left|\alpha - \alpha_1(t)\right| = 0.$$

Hence, let us define the following Lyapunov function candidate

$$V = \frac{1}{2}(e_1^{\ 2} + e_2^{\ 2} + e_3^{\ 2} + a_1^{\ 2} + b_1^{\ 2} + c_1^{\ 2} + k_1^{\ 2} + \alpha_1^{\ 2}),$$

and we obtain

$$\frac{dV}{dt} = p_1e_1^{\ 2} + p_2e_2^{\ 2} + p_3e_3^{\ 2}.$$

This means that the above designed reduced-order projective synchronization scheme can be achieved. In order to demonstrate the validity of the above designed reduced-order projective synchronization scheme. The parameters of the system (2.2) are set to (a,b,c,k,m) $=(5,5,0,1,1)$, and the parameter of the system (3.1) is set to $a = 0.006$. The initial conditions of the driving and response systems are (5, -0.1, 1, 2) and (1, 12, -2).

Choosing $p_i = 1$ (i=1, 2, 3…, 7) and the initial values of the parameters e_i (i=1, 2, 3), a_1, b_1, c_1, k_1 and α_1 are all set to zero. Fig. 4(a) shows the trajectories of e_i (i=1, 2, 3), and as indicated, the error dynamical system tended to zero after control. Fig. 4(b) shows that the estimates a_1, b_1, c_1, k_1 and α_1 of the unknown parameters converge to a_1, b_1, c_1, k_1 and α_1 as $t \to \infty$.

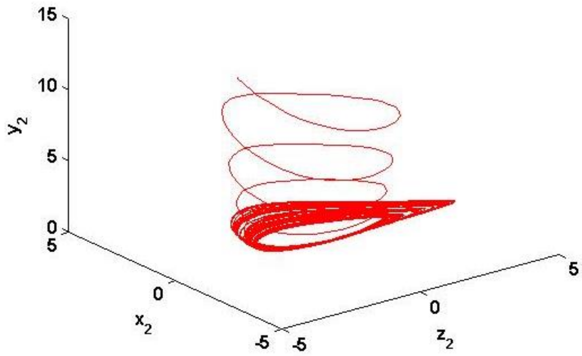

Fig. 3. Coexistence of chaotic attractors and only one stable equilibrium of the system (3.1) for the case a=0.006 with initial conditions (1, 12, -2).

4. Circuit implement of the hyperchaotic attractor

In this section, a physical electronic experimental circuit is designed to realize the new hyperchaotic system. During the design procedure, we choose the value of parameters a = 5, b = 5, c = 0, k = 1, m = 1, and the initial conditions are (5, -0.1, 1, 2) for designing and implementing the system that displays two-scroll hyperchaotic attractor, as shown in Section 2.

The designed circuitry is shown in Fig. 5. In fact, the strong random property is also demonstrated by this circuit implementation. When system (2.2) has no equilibria, the simple electronic circuit is designed that can be used to study hyperchaotic phenomena. The circuit employs simple electronic elements such as resistors, and operational amplifiers, and is easy to construct. The circuit employs simple electronic elements such as resistors, and is easy to construct.

The used circuit equations in terms of the circuit parameters are shown as followings,

$$\begin{cases} \dot{x} = \dfrac{1}{R_2C_1}y - \dfrac{1}{R_1C_1}x \\ \dot{y} = -\dfrac{1}{R_4C_2}xz + \dfrac{1}{R_5C_2}w \\ \dot{z} = -\dfrac{Vp}{R_6C_3} + \dfrac{1}{R_7C_3}xy \\ \dot{w} = -\dfrac{1}{R_{13}C_4}y. \end{cases} \tag{4.1}$$

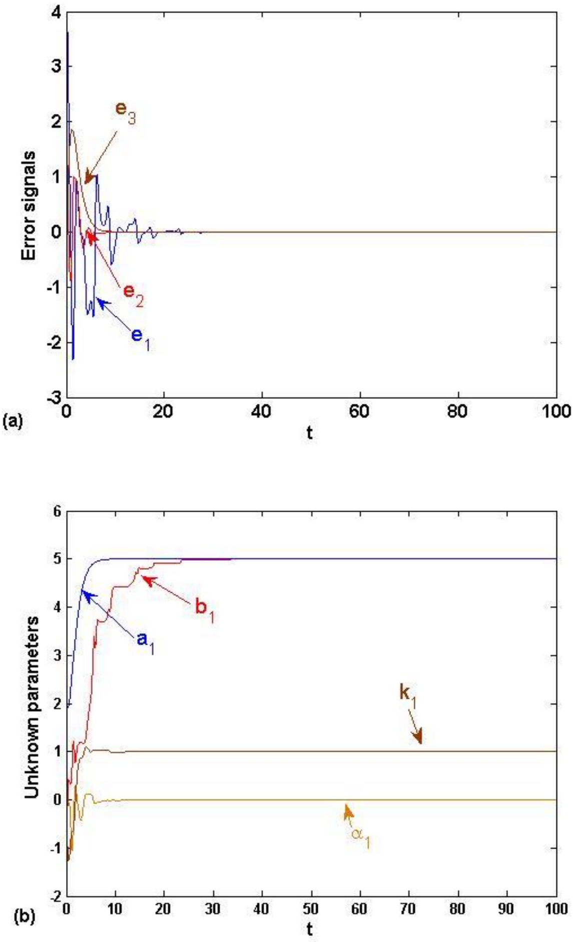

Fig. 4. (a) The behavior of the trajectories e_i (i=1,2,3) of the error system (3.3); (b) the estimates a_1, b_1, c_1, k_1 and α_1 of the unknown parameters converge to 5,5,0,1 and 0.006as $t \to \infty$.

The circuit element values as follow, R1 = R2=8kΩ, R4 = R7 = 4kΩ,R5 =R13 = 40kΩ, R6 = 150kΩ,R9 = R10 = R11 = R12 = R14 = R15 =20kΩ, Vn= −15V, Vp = +15V . To obtain the stable phase portrait in the oscilloscope, we select the capacitor C1 =C2= C3 =C4= 10nF, which only increases the vibration frequency of the chaotic circuit.

The Orcad-PSpice simulation results and oscilloscope outputs of circuitry of the new hyperchaotic system are seen in between Fig. 6 and Fig. 11.

5. Conclusion

In this paper, chaos synchronization between the hyperchaotic system with no equilibria and the chaotic system only with one stable equilibrium by using the adaptive control technique. In addition, the generator of the new hyperchaotic system is confirmed through a novel electronic circuit design. A good qualitative agreement is illustrated between the simulation results and real oscilloscope outputs. It is convenient to use the new system to purposefully generate hyperchaos in chaos applications. We believe that the unknown dynamical behaviors of the strange hyperchaotic attractors deserve further investigation and are very desirable for engineering applications such as secure communications in the near future.

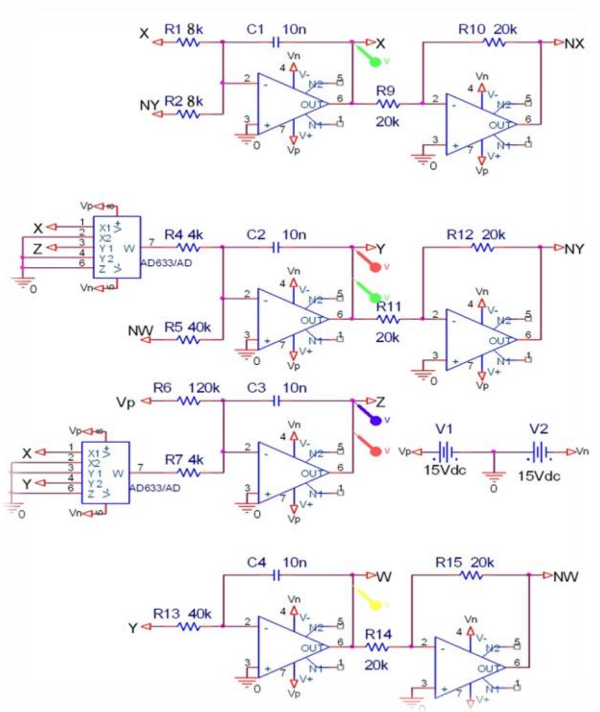

Fig. 5. The designed electronic circuit schematic of the new hyperchaotic system(2.2).

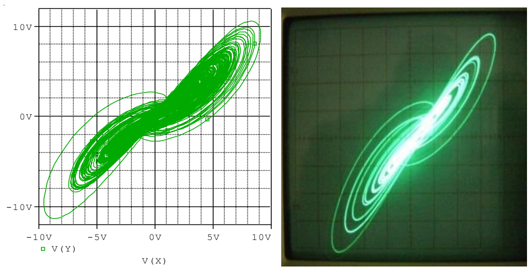

Fig. 6. x-y phase portrait results of the electronic circuit of the new hyperchaotic system obtained from Orcad-PSpice and the oscilloscope screen respectively.

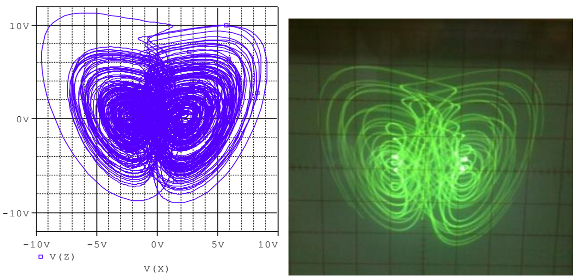

Fig. 7. x-z phase portrait results of the electronic circuit of the new hyperchaotic system obtained from Orcad-PSpice and the oscilloscope screen respectively.

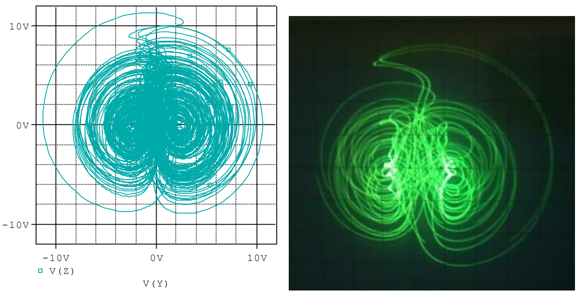

Fig. 8. y-z phase portrait results of the electronic circuit of the new hyperchaotic system obtained from Orcad-PSpice and the oscilloscope screen respectively.

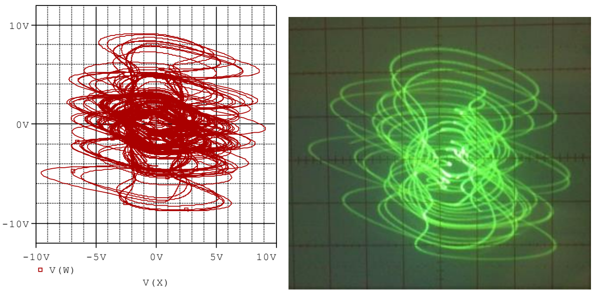

Fig. 9. x-w phase portrait results of the electronic circuit of the new hyperchaotic system obtained from Orcad-PSpice and the oscilloscope screen respectively.

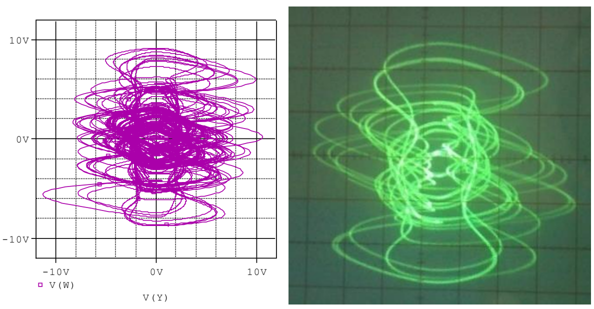

Fig. 10. y-w phase portrait results of the electronic circuit of the new hyperchaotic system obtained from Orcad-PSpice and the oscilloscope screen respectively.

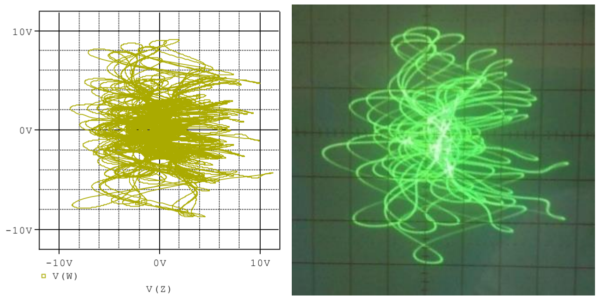

Fig. 11. y-w phase portrait results of the electronic circuit of the new hyperchaotic system obtained from Orcad-PSpice and the oscilloscope screen respectively.

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