Analysis of Hermite-Gaussian and Laguerre–Gaussian modes in mode division multiplexing based FSO system

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Free space optics (FSO) is a non-line of sight (NLoS) technique for providing ubiquitous digital services, especially in sectors where the frequency allocation is pretty tight and not practically possible to accommodate all the users. In this work, we have designed a model that transmits four independent modes (HG_{00}, HG_{01}, LG_{00}, and LG_{10}) carrying pseudo-random bit sequence multiplexed into a single free space channel and studied under the subject of various attenuation and link length values. We found the HG family outperforming the LG family by having ~7.7% lower bit error rate (BER), and 4% higher Q-factor for 18 dB/km attenuation at 600 m link range.

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1. Introduction

Optical wireless communication (OWC) uses visible light to transmit data through the atmosphere. With high bandwidth and low cost for implementation, free space optical (FSO) communications are attracting a lot of attention and could potentially tackle the issue of last-mile bottlenecks in local area access networks. The leading cause of attenuation in free space is atmospheric attenuation, which might affect the performance and availability of the same. Lasers can be leveraged to quickly deliver larger bandwidth to customers [1].

FSO communication uses a laser beam to send very high bandwidth data based on LOS technology through atmospheric. Fig. 1 depicts the FSO communication system. The electrical signal is converted to an optical signal by transmitter and sends the data via the atmospheric channel. FSO communication takes place at the bottom of the troposphere layer. The receiver then converts the optical signal back to an electrical signal. The PIN photodiode and avalanche photodiode (APD) are mostly used due to their good quantum efficiency and easy availability of materials [2].

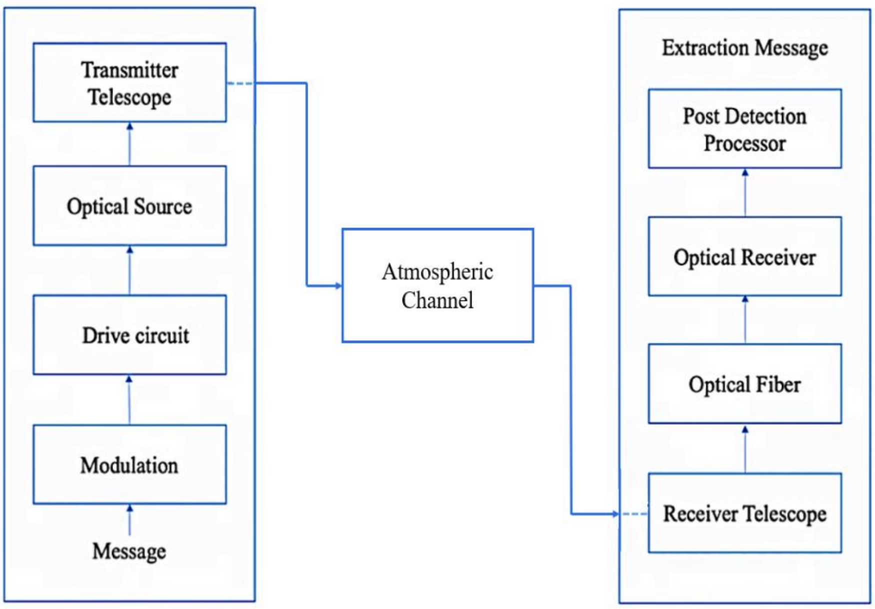


Fig. 1. The main components of terrestrial FSO system

Mode division multiplexing (MDM) system uses eigenmodes to transmit independent data by multiplexing modes over a single wavelength. Evidently, MDM systems are spectral efficient and cost-effective due to no requirement of large, tunable broadband light sources [3]. It was demonstrated that the MDM-WDM scheme can supply four channels, each of which has a 10 Gbps capacity that can be upconverted to 40 GHz using an FSO link, making it appropriate for providing broadband and communication services inside the hospital premises [4]. It offers better utilisation of individual optical channels and high data rates. In a hybrid system, combining MDM and code division multiplexing can boost capacity while providing greater flexibility and bandwidth granularity beyond space, time, and frequency [5].

Copper connections have a maximum data throughput of 2Mb/s per pair, much slower than Gigabit Ethernet. Base stations may send radio signals to other base stations through a faster optical carrier, which is subsequently transformed back to radio signals at the central station, thanks to an optical fibre backbone. Combining free-space and optical networks is a practical technique to use optical fibre's large capacity and the inherent mobility of wireless networks. Although optical fibre provides faster Gigabit Ethernet communication rates, placing cables in cities is becoming increasingly unpopular, and local governments are putting restrictions in place to reduce interruption [6].

Our primary aim is to investigate the performance analysis of MDM-based OWC system, to satiate the ever-increasing need for high-speed data transmission. Since we are using spatial laser as a source, understanding data transmission through Gaussian modes and comparing the same via several factors is the idea.

The paper is organised as follows. Section 1 describes the free space optics system and MDM scheme. Section 2 discusses literature and their analysis on these topics. Mode theory, particularly Gaussian modes, which are used in implementation are covered in Section 3. The proposed model is discussed in Section 4. Results are analysed in detail in Section 5. Section 6 concludes our findings.

2. Literature survey

The different weather conditions such as rain, fog, haze, physical obstruction etc. are discussed, and employed the techniques to mitigate their effects to add to the numerous advantages over existing approaches, which can be optical, radio, or microwave in nature [7].

A free-space link of 800 m is investigated for 2.5 Gbps data transfer over two 10 GHz radio subcarriers, each modulated across two 850 nm optical carriers on Hermite Gaussian modes (HG_{03} and HG_{01}). It concludes that the channel propagating HG_{01} is more robust than the channel bearing HG_{03} in terms of SNR (signal to noise ratio), received power, BER (bit error rate), and modal decomposition data. The technique might connect microcells to the central station through 5G backhaul cables in high-density areas [8]. A PSK-MDM Ro-FSO transmission system.is studied for millimeter applications having capability of transmitting 20 Gbps–40 GHz data utilising the LG_{00} (Laguerre Gaussian) and HG_{01} modes and the PSK-MDM (Phase Shift Keying-MDM) system. Two radio channels were broadcasted across a 50 km FSO connection with an acceptable SNR of 20 dB and received power [9].

A high-speed, inexpensive FSO system was proposed for delivering broadband services in healthcare. The suggested MDM-FSO system uses a straightforward On off Key (OOK) encoding strategy in contrast to the MDM scheme, which requires 00 and 01 modes of LG. According to the results, 20 Gbps data may be transmitted successfully across a 27 km FSO link when the weather is clear. Additionally, the planned MDM-FSO link is assessed in light of the effects of low, medium, and heavy fog situations [10]. The performance of simultaneous optical mode division multiplexing of a donut mode, a LG mode, and two HG modes in a Ro-FSO system under clear weather conditions is investigated. Power coupling coefficients, received power, SNR, and constellations are all investigated [11]. Photonic radar based advanced transport systems is proposed using MDM, and FSO transmission is developed to enhance the information carrying capacity by incorporating scheme of MDM with polarization shift keying (PolSK) in [12] and [13], respectively.

3. Mode theory

In waveguides, optical resonators, and free-space, modes are self-consistent electric field distributions. The fibre shape and composition dictate the discrete set of electromagnetic fields or fibre modes that may propagate in the fibre.

Theoretically simplest modes in free space are plane waves, described with infinite transvers and lateral spread. Despite this simplicity, plane waves do not resemble any natural occurring wave due to the assumption of unlimited transverse extent. With the idea of confined transverse spatial dimension and laser as the source, Gaussian beam or modes are considered into the picture. During propagation, the beam expands or contracts, gets scaled only in the transverse dimension and has a constant Gaussian form.

Most common utilised mode families are HG and LG modes. Each higher order mode’s transverse extent changes in proportion to the fundamental mode’s during propagation.

$$\begin{aligned} \mathrm{I(r)} &= \mathrm{I_0 e}^{\left(-\frac{2\mathrm{r}^2}{\omega(\mathrm{z})^2}\right)} \\ &= \frac{2\mathrm{P}}{\pi\omega(\mathrm{z})^2}\exp\left(\frac{-2r^2}{\omega(z)^2}\right) \end{aligned} \tag{1}$$

In Eq. (1) [14], I_0 stands the peak irradiance at the beam's centre, r represents the radial distance away from the axis, $\omega(z)$ represents the radius of the laser beam where the irradiance is $1/e^2$ (13.5 % of I_0), z represents the distance propagated from the plane where the wavefront is flat, and P represents the total power of the beam.

(a) Hermite Gaussian Modes:

These are approximate wave equation solutions that work for weak focusing. The phase term effectively defined by the combination of a Hermite polynomial and a Gaussian function. The Hermite polynomial with the non-negative integer index n is H_n (x). The indices n and m determine the profile's form in the x and y directions according to Eq. (2) [14]. The values w and R evolve in the z direction.

$$E_{nm}(x,y,z) = \frac{E_0\omega_0}{\omega(z)} H_n\left(\frac{\sqrt{2}x}{\omega(z)}\right) e^{-\frac{x^2}{\omega(z)^2}} H_m\left(\frac{\sqrt{2}y}{\omega(z)}\right) e^{-\frac{y^2}{\omega(z)^2}} \times e^{\left[-i\left(kz-(1+n+m)\arctan\left(\frac{z}{z_r}\right)+\frac{k(x^2+y^2)}{2R(z)}\right)\right]} \quad (2)$$

(b) Laguerre Gaussian Modes:

Based on polar coordinates, better suited to situations with rotational symmetry as given in Eq. (3) [14].

$$U_{(p,l)}(r,\theta,z) = \sqrt{\frac{2p!}{\pi(p+|l|!)}} \times \frac{1}{\omega(z)} \left(\frac{r\sqrt{2}}{\omega(z)}\right)^{|l|} \times Lp^{|l|}\left(2\frac{r^2}{\omega(z)^2}\right) \times e^{\left[-\frac{r^2}{\omega(z)^2}\right]} e^{\left[-\frac{ikr^2}{2R(z)}\right]} e^{[il\theta]} \quad (3)$$

where, ω_0 = Minimum spot size, $r = \sqrt{x^2+y^2}$, R (z) = Radius of curvature of beam at z, and Z_R = Rayleigh length

Although, in practice components are placed at an angle to the laser beam thus breaking rotational symmetry, hence preferring HG modes.

4. System design

A schematic diagram of proposed model of MDM based optical communication system is shown in Fig. 2.

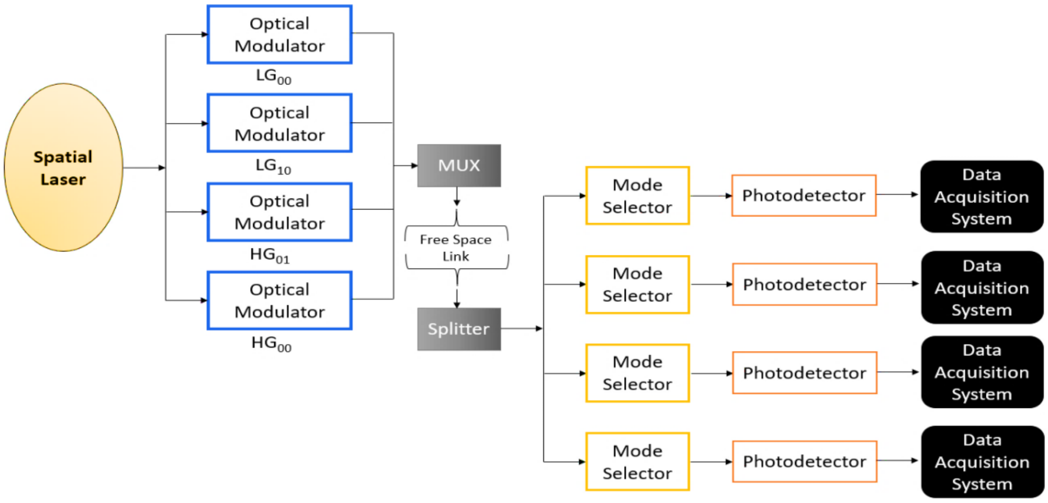


Fig. 2. Schematic layout of MDM based FSO system (color online)

In the proposed model, two Laguerre Gaussian modes: LG_{00} and LG_{10}, and two Hermite Gaussian modes: HG_{01} and HG_{00} were multiplexed through free space.

Firstly, the modes were modelled in MATLAB. The intensity distributions of all the modes was observed using MATLAB through the steps given in a Fig. 3. The modes were then generated in OptiSystem to validate the selected modes. Only after successful validation we moved ahead with the designing of the entire transmission model.

The results of MATLAB and OptiSystem were found to be identical. The results were validated as shown in Fig. 4 and Fig. 5, respectively.

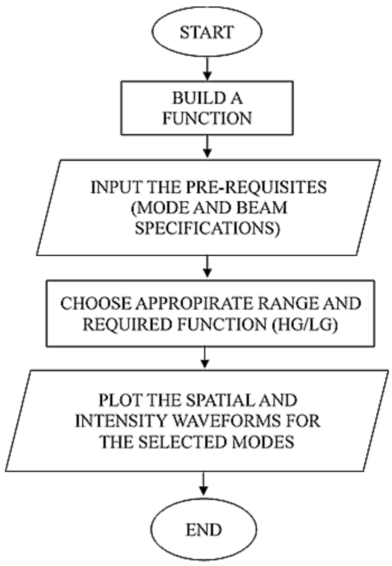
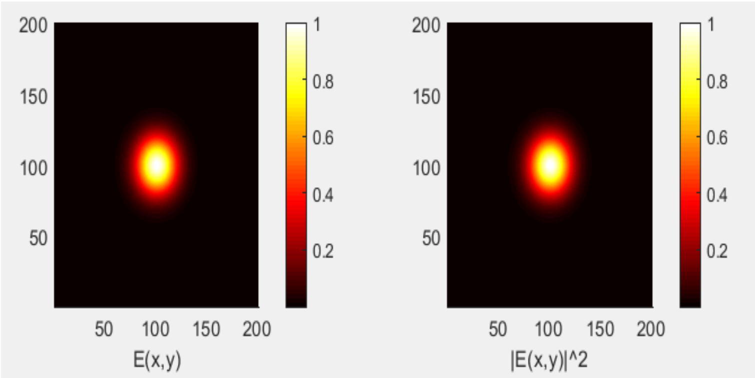
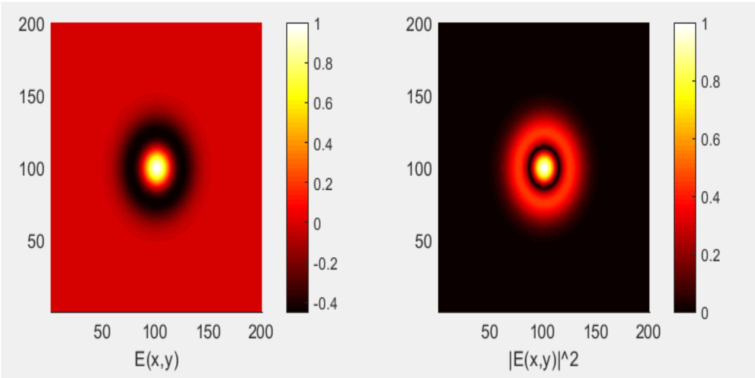


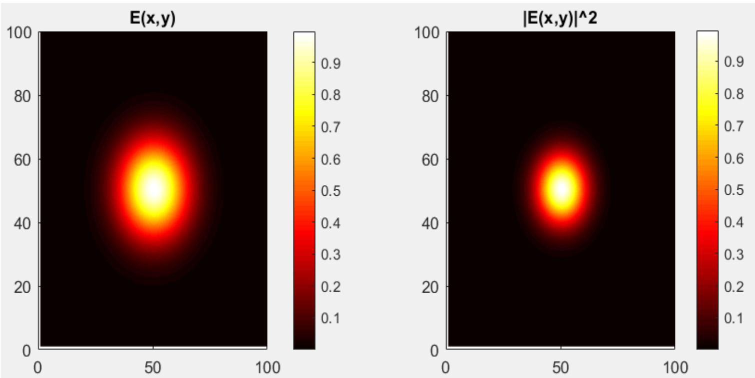
Fig. 3. Flowchart for mode generation



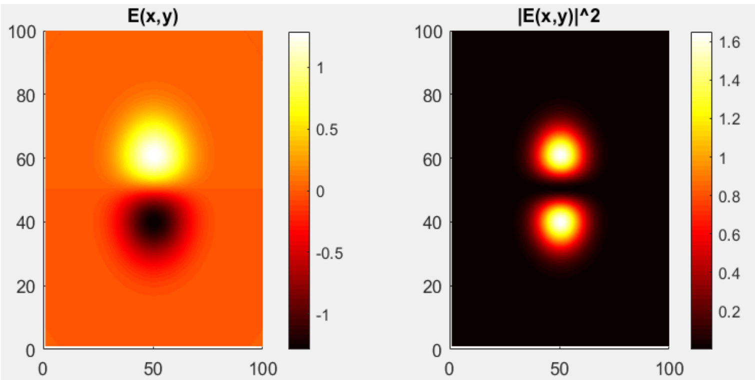
(a)



(b)



(c)



(d)

Fig. 4. Intensity Fields of Generated Modes of (a) LG_{00} (b) LG_{10} (c) HG_{00} (d) HG_{01} (color online)

Firstly, four pseudo random bit sequences (PRBS) were generated using optical transmitters and these were transmitted using four different modes using the multimode generator. Then, multiplexed signal of all the four modes was transmitted through FSO channel.

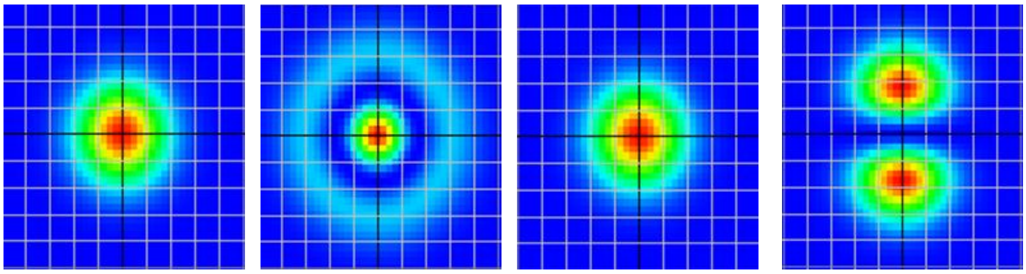


Fig. 5. Simulative Trans-received modes of (a) LG_{00}, (b) LG_{10}, (c) HG_{00} and (d) HG_{01} (color online)

The received signal was then de-multiplexed and using mode selector the appropriate modes were received. An optical receiver is connected with one of the mode selectors to observe the Eye diagram and get the values of BER, Q-factor. With the other mode selector, spatial PIN photodiode is connected for O/E conversion and thus a low pass Bessel filter is used to get a good enough practical output which can thus be observed with an oscilloscope visualizer. System parameters with associated values are summarized in Table 1.

The free space link equation is used as follows [15]:

$$P_T = \frac{d_R^2}{(d_T+\theta R)} 10^{-\alpha R/10} \tag{4}$$

where P_T is transmitted power, d_R is receiver aperture, d_T is transmitter aperture, θ is beam divergence, R is link range and α is atmospheric attenuation.

Table 1. Specifications and corresponding values of the simulative system

Parameters	Parameters/Values
LG_{00} in MATLAB	P = 0, L = 0, A = 1, ω = 1 Grid = -0.5:0.05:0.5
HG_{00} in MATLAB	n = 0, m = 0, $\omega_0 = 1.5$ $k = 2\pi/1550 \times 10^{-9}$m
General parameters	Bit rate = 10 Gbps Sequence length = 128 bits Samples per bit = 64 Reference wavelength = 1550 nm
Optical transmitter	Wavelength =1550 nm Power = 5 mW Modulation type = NRZ Aperture diameter = 5 cm Beam divergence = 1 mrad
Multimode generator	Mode polarization = X Power ratio = 1 1 1 1 Mode types = LG_{00}, LG_{10}, HG_{01} and HG_{00}
FSO channel	Link range = 0.6 km Atmospheric Attenuation = 15 dB/km
Spatial PIN photodiode	Aperture diameter = 30 cm Responsivity = 1 A/W Dark current = 10 nA

5. Results and discussion

The results of BER values and Q factor with varying attenuation and link length were observed. The electrical SNR can mathematically be expressed as given below:

$$SNR = \frac{S_E}{N_E}. \tag{5}$$

where S_E and N_E represents electrical signal power and noise power, respectively. Q-Factor can be calculated using the formula given in Eq. (6) [16]:

$$Q = \frac{10^{\frac{SNR}{20}}}{2} \qquad (6)$$

The BER is defined as $BER = \frac{1}{2}erfc\left(\frac{Q}{\sqrt{2}}\right)$. Where $erfc$ is error function [17].

In Fig. 6, max Q factor variation with attenuation was observed for all the four modes. The Q factor values decrease as the attenuation is increased. We observed that HG modes outperform the LG modes. Also, HG_{00} mode gives better Q factor than HG_{01}. Whereas LG_{00} mode gives better Q factor than LG_{10}.

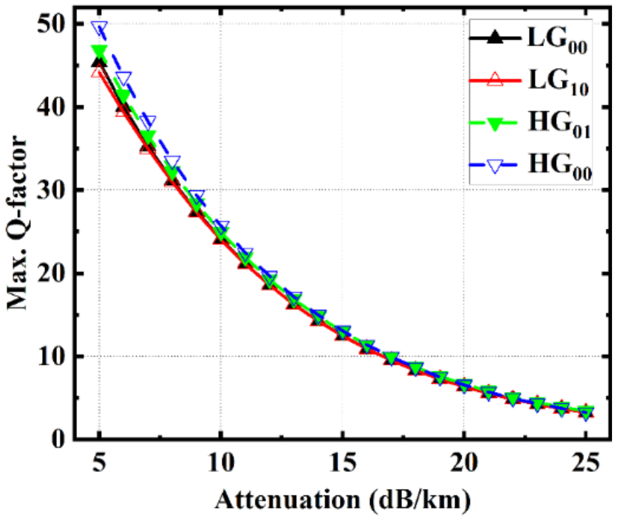


Fig. 6. Maximum Q-factor as function of attenuation at constant link length of 600 m (color online)

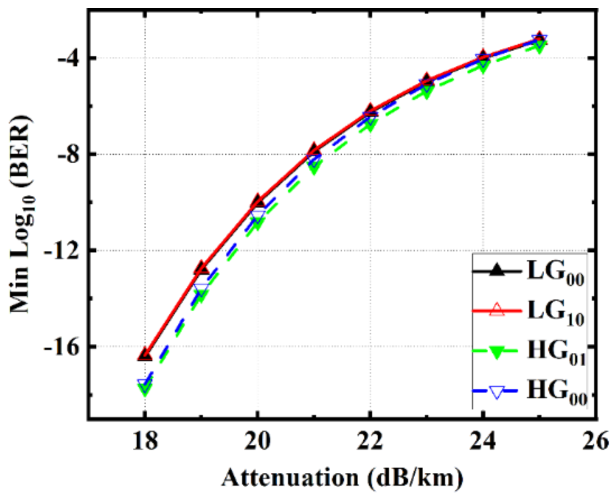


Fig. 7. Minimum log of BER against attenuation at constant link length of 600 m (color online)

Fig. 7 gives the minimum log of BER versus attenuation for all the four modes. The BER values increase as the attenuation is increased. We observed that HG modes outperform the LG modes. The rotational symmetry is broken by components used in lasers because they are positioned at an angle to the beam. Therefore, HG modes give better outcomes for laser beam Fig. 8 represents maximum Q factor variation with link length. As the link length is increased, the max Q factor decreases. For link length variation also, HGs modes outperform the LGs modes.

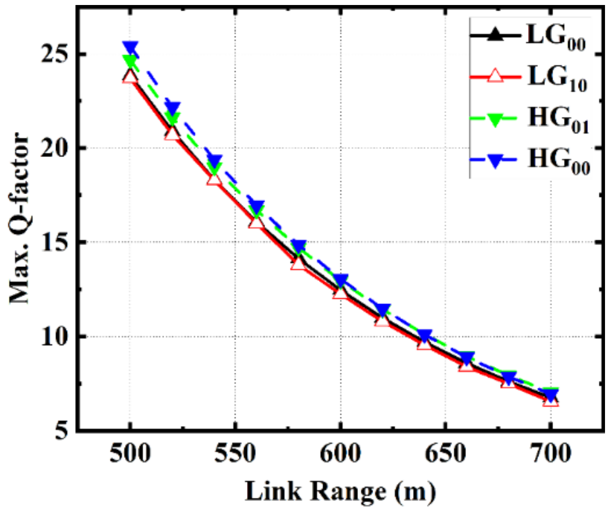


Fig. 8. Plot of Q-Factor as function of increasing range at attenuation of 15 dB/km (color online)

Fig. 9 gives the variation of min log of BER as function of link length. The BER values increase as the link length is increased. Fig. 10 shows the variation in min log of BER with variation in data rate. As the data rate increases, the BER values increase which shows that lesser the data rate, the better the signal received. These results also show that HG modes outperform the LG modes in MDM transmission.

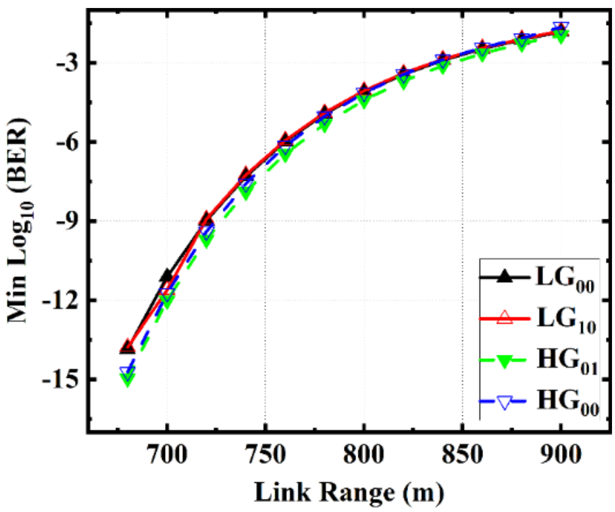


Fig. 9. Minimum log of BER with varying link range at attenuation of 15 dB/km (color online)

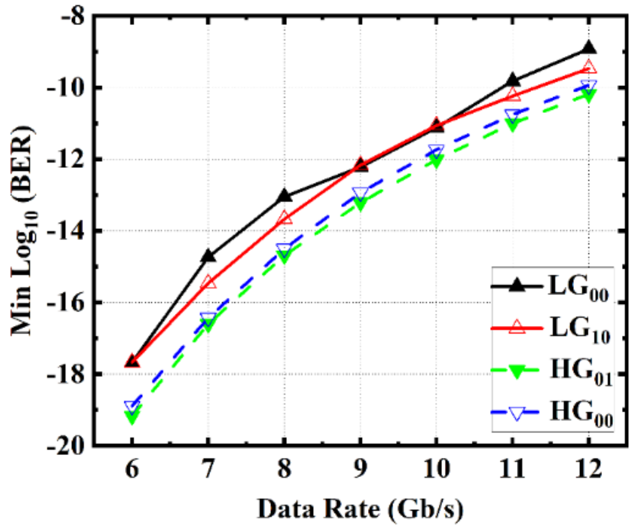


Fig. 10. Minimum log of BER versus bit rate at link length of 700 m and attenuation of 15 dB/km (color online)

6. Conclusion

Four optical modes, LG_{00}, LG_{10}, HG_{01} and HG_{00} were multiplexed for transmitting PRBS data through free-space for a long-haul communication. Eigen modes are used to transmit many independent channels simultaneously, it increases the spectral efficiency and data rates of transmission. At the receiver, HG_{00} performs better than HG_{01}, while LG_{00} performs better than LG_{10}. The overall HG family outperforms LG family. The suggested system's findings imply that fundamental modes provide greater response than higher modes of particular families. The HG family has 7.7% lesser BER and 4% higher Q-factor than the LG family at 18 dB/km for 600 m link length.

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