Band gaps in phononic crystal beam-foundation systems

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Phononic crystal (PC) beam-foundation systems are introduced to study the flexural vibration band gaps (BGs). Two general beam types, Euler beam and Timoshenko beam, are both considered. The finite periodic structures are constructed to describe the PCs in practice. The finite element analysis of the finite periodic structures shows that the stiffness of foundation influences the distribution of BGs. Moreover, the PC Timoshenko beam-foundation system has wider applicability in the engineering than the system with Euler beam. The periodic numbers of the finite structures affect the attenuation in BGs; the more periodic numbers give more distinct BGs.

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1. Introduction

In the last twenty years, the apparent existence of elastic/acoustic band gaps (BGs) in periodic composite materials has caused much attention [1-4]. Since Kushwaha [2] firstly proposed the concept of phononic crystals (PCs) to define these kinds of artificial periodically arrayed materials, some relevant potential applications about PCs, such as acoustic insulations [1, 3], vibration control [5], transducers [2], and wireless communications [6] have been designed. Furthermore, the researchers have performed a series of study mainly laid on the experimental tests of 1D, 2D and 3D problems and theoretical researches [7]. The common theoretical methods include the transfer matrix (TM) method [5, 8] the plane wave expansion method [2, 9], the finite difference time domain method [10], the multiple scattering theory [11], the lumped-mass method [12] and the finite element method [13].

At present, PC attracts researchers mainly because of its BG properties. However, in the actual applications, a PC component might be just a part of a complex system. Therefore, the influences of other components can not be neglected for these combined systems. Moreover, the PC component used in practice can not have the ideal infinite structure. The vibration properties of the finite periodic structures corresponding to the ideal PCs also should be emphasized. Considering the PC could adjust and control the elastic waves in its BGs [5, 14] while the general beam-foundation system is widely used in engineering [15-17], we try to propose a PC beam-foundation system and study its BG behaviors for the flexural vibrations based on the finite periodic structures [18-20]. Winkler model [21-23] is used here to describe the influence of the foundation for the preliminary study.

In this paper, we first present the general PC beam-foundation systems and construct the finite periodic structures to describe them. Then, the corresponding frequency responses are calculated by ABAQUS, and the influence of the stiffness of foundation, beam type and periodic number are discussed. Finally, we draw the conclusion.

2. PC beam-foundation system

2.1 General description

Fig. 1 shows a general PC beam on a Winkler foundation. In the x direction, the beam is composed of an infinite repetition of alternating segment i with material i, where i=1, 2, ... , n. The length, width and height of the segment i are a_i, b_i and h_i. In the Winkler model, the parameter c represents the stiffness of foundation.

According to the length height ratio of a beam, Timoshenko and Euler beam models could be used. Generally, the shear deformation and the rotary inertia with respect to the central axis exist naturally. Considering these effects, Timoshenko beam model could be given. The governing equation of the free flexural vibration of a segment in the PC Timoshenko beam on a Winkler foundation can be expressed as [21]

$$\left(EI\frac{\partial^4 w(x,t)}{\partial x^4}\right)+\rho A\frac{\partial^2 w(x,t)}{\partial t^2}-\rho I\left(1+\frac{E}{\kappa G}\right)\frac{\partial^4 w(x,t)}{\partial x^2\partial t^2}+\frac{\rho^2 I}{\kappa G}\frac{\partial^4 w(x,t)}{\partial t^4}+cbw(x,t)=0 \qquad (1)$$

where E and G are the Young's modulus and shear modulus. I is the second axial moment of area. A is the cross section area. $w(x,t)$ is the transverse displacement. κ is the shape factor of cross section, for the rectangular section, κ=5/6.

For a segment, when the length is larger enough than the height, the effects of the shear deformation and rotary inertia become slightly and the Euler beam model could give the sufficiently precise results. Neglecting the shear deformation and the rotary inertia, the governing equation of the free flexural vibration of a segment in the PC Euler beam on a Winkler foundation could be written as [21]

$$EI\frac{\partial^4 w(x,t)}{\partial x^4}+\rho A\frac{\partial^2 w(x,t)}{\partial t^2}+cbw(x,t)=0 \qquad (2)$$

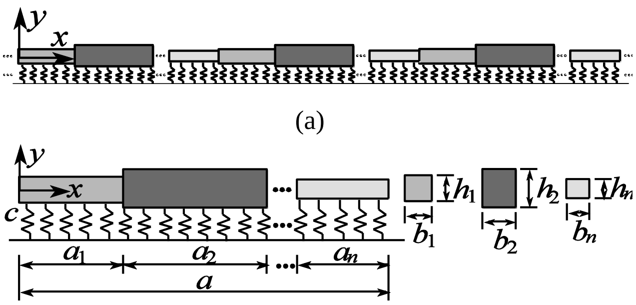

Fig. 1. (a) A PC beam on a Winkler foundation composed of an infinite repetition of cells. The lengths and cross-section sizes of different segments in a cell are shown in (b).

Considering the solution, $w(x,t)=v(x)\exp(\mathrm{i}\omega t)$, due to the periodicity of the infinite structure along the x direction, the band structure of the PC beam-foundation system can be solved as an eigenvalue problem by using the Bloch's theorem [24]. One can see the detailed method in Ref. [8, 18].

2.2 Finite periodic description

The vibration properties of the finite structure with the same periodicity as the PC surely could reflect the BGs of the PC to a certain degree [25, 26]. In this study, the finite periodic structures are constructed to describe the systems of the PC Euler/Timoshenko beam on the Winkler foundation. The software ABAQUS is used to analyze their frequency responses of the flexural vibrations. A two component PC beam is considered. The process can be summarized as follows. First, two materials, aluminum and epoxy, are chosen. The parameters are respectively ρ_A=2730 kg/m^3, E_A=77.56 GPa, G_A=28.87 GPa, ρ_E=1180 kg/m^3, E_E=4.35 GPa and G_E=1.59 GPa. Second, the geometric parameters are given as $a_A=a_E$=0.035 m, $b_A=b_E$=0.02 m, $h_A=h_E$=0.01 m. Third, the finite structure with a given periodic number is constructed. Next, the stiffness of the Winkler foundation should be added to the model. Finally, the frequency response analysis could be done at one end of the beam from applying the harmonic displacement impulses from 0 Hz to 15000 Hz to the other end. In this process, two beam types, "cubic formulation" and "shear-flexible", could be selected to represent the Euler and Timoshenko beam models respectively. The distribution of BGs can be easily found and analyzed from the distinct attenuation frequency ranges.

The frequency response of the 8-cell above mentioned finite system with the Euler beam model and the stiffness of foundation as 2.5×10^7 N/m^3 is calculated and shown in Fig. 2. Three BGs exist in 0-15000 Hz. For comparison and validation, we also calculate the BGs of the corresponding PC Euler beam-foundation system by the TM method [18]. The comparison of the BGs ranges are shown in table 1. Surely, the results have agreements in the main and the frequency response analysis of the finite periodic structure could reflect the BG properties of the ideal PC, but the differences can not be neglected directly and the frequency response analysis gives more realistic vibration properties. The comparison of the PC Timoshenko beam-foundation system is not shown here, because the case has the similar situation as that with the Euler beam model.

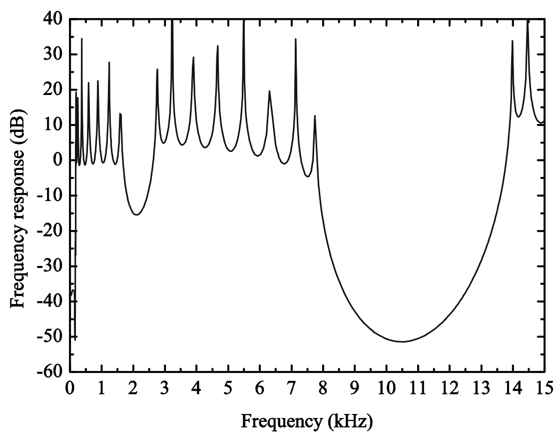

Fig. 2. Frequency response of the 8-cell aluminum-epoxy PC Euler beam on the Winkler foundation with the stiffness of 2.5×10^7 N/m^3.

Table 1. Comparison of ranges of BGs (Hz).

BG No.	TM method		Frequency response	
	Beginning	End	Beginning	End
1	0.0	180.0	0.0	185.0
2	1789.9	2655.8	1673.6	2621.8
3	7999.7	13850.4	7828.8	13764.0

3. Results and discussion

3.1 Influence of the stiffness of foundation

Using the 8-cell structure with the Euler beam model, we study the influence of the stiffness of foundation. We choose several values of the stiffness of foundation from 1.0×10^4 N/m^3 to 1.0×10^8 N/m^3. The frequency responses have the similar shape as that shown in Fig. 2. The detailed ranges of the first two BGs are shown in table 2. Along with the increase of the stiffness, the first BG which starts at 0 Hz becomes wider, while the boundaries of the other BGs all rise with a lesser degree for the higher frequencies.

Table 2. Ranges of BGs with different foundations (Hz).

c (N/m^3)	1st BG		2nd BG	
	Beginning	End	Beginning	End
1.0×10^4	0.0	3.6	1657.3	2621.8
1.0×10^5	0.0	11.5	1657.3	2621.8
1.0×10^6	0.0	36.4	1664.3	2621.8
2.5×10^7	0.0	185.0	1673.6	2621.8
5.0×10^7	0.0	262.2	1692.7	2621.8
8.0×10^7	0.0	331.0	1705.8	2640.7

3.2 Influence of the beam type

Then we apply the Euler and Timoshenko beam models respectively to the 8-cell structure with the stiffness of 2.5×10^7 N/m^3. Fig. 3 shows the frequency responses. The structure with the Timoshenko beam model gives lower frequency BGs compared with that with the Euler beam model. The difference becomes lager along with the increase of the frequency. Considering the geometric features of the beam segments in the model, actually, neglecting the shear deformation and the rotary inertia is not proper. Thus the structure with the Timoshenko beam model should give the more precise results. However, the first BG is caused by the existence of the foundation and its end frequency is determined by the fundamental mode of flexural vibration, the transverse translation motion. Therefore the first BGs obtained from the two systems with different beam models are the same. In addition, the influences of the shear deformation and the rotary inertia are bigger to the relatively high frequency vibrations than the relatively low frequency vibrations. Thus the frequency responses of the two systems with different beam models are in better agreement at lower frequency than that at higher frequency.

3.3 Influence of the periodic number

Finally, we study the influence of the periodic number of the finite periodic structure. Fig. 4 shows the frequency responses of the 4-cell, 8-cell and 12-cell structures with the Euler beam model and the stiffness of foundation as 2.5×10^7 N/m^3. Clearly, the periodic number mainly affects the attenuation in BGs. The more periodic numbers give stronger attenuation and more distinct BGs. Thus, enough periodic number should be designed in the applications, in order to guarantee that no vibrations exist in the ranges of BGs.

4. Conclusion

In conclusion, we study the flexural vibration BGs of the PC beam-foundation systems by the frequency response analysis based on the finite element method. Using the finite periodic structure corresponding to the ideal PC, the results close to the cases in applications could be given. The influence of the stiffness of foundation, beam type and periodic number to the BGs are discussed. The boundaries of BGs rise, following the increase of the stiffness of foundation, especially for the first BG. The PC Timoshenko beam-foundation system has wider applicability in the engineering than the system with Euler beam model. The periodic numbers of the finite structures affect the attenuation in BGs; the more periodic numbers give more distinct BGs. This study helps to design and manufacture PC beam-foundation systems.

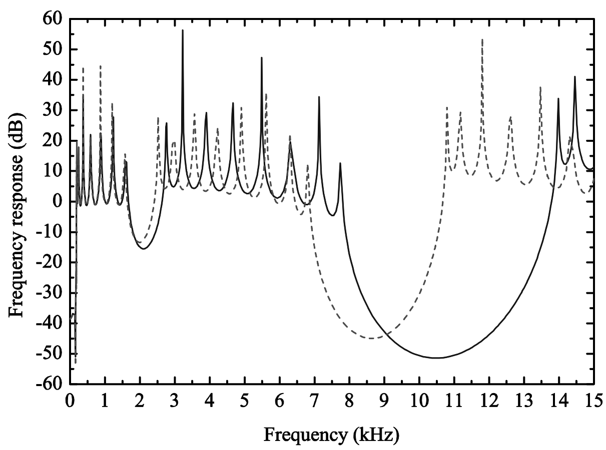

Fig. 3. Frequency responses of the 8-cell aluminum-epoxy PC beam on the Winkler foundation with the stiffness of 2.5×10^7 N/m^3 in the cases of Timoshenko (dash line) and Euler (continuous line) beam models.

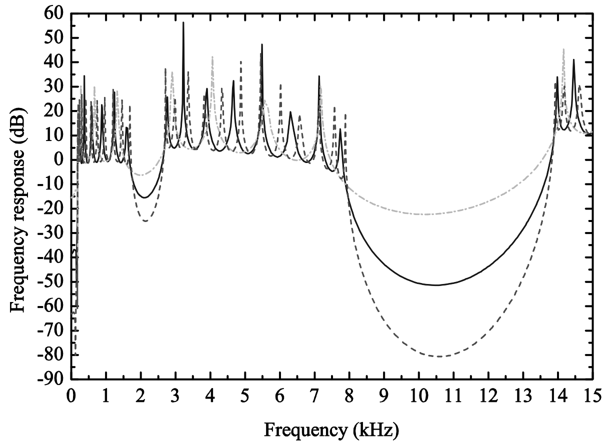

Fig. 4. Frequency responses of the aluminum-epoxy PC Euler beam on the Winkler foundation with the stiffness of 2.5×10^7 N/m^3 in the cases of 4-cell (dash dot line), 8-cell (continuous line) and 12-cell (dash line) structures.

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