Bright solitons in an optical lattice induced in nonlocal media with infinite range of nonlocality

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We investigate the propagation of a spatial soliton in an optical lattice imprinted in a nonlocal thermal media with infinite range of nonlocality. A variational approach is used to obtain dynamical equation for beam width, amplitude and curvature. Furthermore an approximate formula to form a quasi-soliton is obtained. What implying that Gaussian function is a good approximation to a nonlocal spatial soliton in such thermal media with lattices when the lattice period is larger. Numerical simulations show the dynamic propagation of the quasi-solitons.

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1. Introduction

Optical spatial solitons in optical lattices are widely investigated in both local and nonlocal nonlinear media in recent years [1-19], ranger from photorefractive crystals, local and nonlocal kerr media, to liquid crystal, lead glass, etc. Recent progress in creation of reconfigurable optical lattices in photorefractive crystals [2-6], mobility of solitons in nonlocal kerr media [7-10], nematic liquid crystals [11-14] and lead glass [15-19] opened the direction to explore the properties of solitons by varying the lattice period, depth and nonlocality of media. All the investigations prove that the optical lattices and the nonlocality play an important role in the formation and steering of such spatial solitons, which could make the solitons stable. In generality, the nonlocality of media can be classified as three kinds: local, finite range of nonlocality, infinite range of nonlocality, and the difference of which are discussed in Ref [15]. At the same time the nonlocal system with infinite range of nonlocality have attract much interest recently. For example, Efremidis investigated the properties of nonlocal lattice solitons in thermal media (with infinite range of nonlocality) [16], nonlocal high-order surface soliton in such nonlocal thermal media [17]. Localization of light in a parabolically bending waveguide array in thermal nonlinear media [18]. Propagation of solitons in thermal media with periodic nonlinearity [19], etc.

Inspired by their work, we investigated the propagation of a spatial soliton in an optical lattice imprinted in such nonlocal thermal media with infinite range of nonlocality, and obtained the dynamical equation for beam width, amplitude and curvature by using a variational approach. Furthermore how the input power, modulation period and depth of optical lattices act on the formation and propagation of such solitons are discussed. Results show that a stable propagation of the spatial soliton in this medium is possible, and an approximate formula to form a quasi-soliton is obtained. What implying that Gaussian function is a good approximation to a nonlocal spatial soliton in such thermal media with optical lattice when the lattice period is larger. Pertinent numerical numerical examples are presented to show their propagation properties.

2. Theoretical model and numerical simulations

Propagation of one dimensional optical beam in nonlocal thermal media with optical lattice is given by the following equations [15-21]:

$$i\frac{\partial A}{\partial Z}+\frac{1}{2}\frac{\partial^2 A}{\partial X^2}+\left[\Delta N(X)+V(X)\right]A=0 \quad (1)$$

$$\frac{\partial^2 \Delta N(X)}{\partial X^2}=-\left|A\right|^2 \quad (2)$$

Where $V(X)$ is a period potential, here we consider the cos-function potential $V(X)=h\cos(2\pi X/T)$ with h, T lattice length and period, respectively. $\Delta N(X)$ beam induced refractive index. In particular, we assume the sample of length be $2d$, $-d\le X\le d$. Under the assumption of $\Delta N(X=\pm d,Z)=0,\partial\Delta N(X=0,Z)/\partial X=0$, the exact solution of possion equation be:

$$\Delta N(X,Z) = -\frac{1}{2}|a|^2\sigma\left[\sqrt{\pi}X\mathbf{erf}(\frac{X}{\sigma}) + \sigma\exp(-\frac{X^2}{\sigma^2}) - \sqrt{\pi}d\mathbf{erf}(\frac{d}{\sigma}) - \sigma\exp(-\frac{d^2}{\sigma^2})\right]$$
$$= -\frac{1}{2}|a|^2\sigma\left[\sqrt{\pi}X\mathbf{erf}(\frac{X}{\sigma}) + \sigma\exp(-\frac{X^2}{\sigma^2}) - \sqrt{\pi}d\right] \tag{3}$$

Where $\mathbf{erf}(d/\sigma)\approx 1,\ \exp(-d^2/\sigma^2)\approx 0$ when $d \gg \sigma$.

Equations (1-2) could be rewritten as a variation problem [22], and the Lagrange density be

$$L = \frac{i}{2}\left[AA_Z^* - A^*A_Z\right] + \frac{1}{2}|A_X|^2 - \frac{1}{2}|A|^2\Delta N(X) \tag{4}$$

Since we consider the bright solitons in this media, a Gaussian tril function is introduced:

$$A(X,Z) = a(Z)\exp(-\frac{1}{2}\frac{X^2}{\sigma^2(Z)})\exp(i\beta(Z)X^2) \tag{5}$$

Where $a(Z),\ \sigma(Z),\ \beta(Z)$ are complex amplitude, beam width and curvature of the beam, respectively.

Then the effective Lagrange is

$$\langle L\rangle = \int_{-\infty}^{\infty} LdX = \int_{-\infty}^{\infty}\left\{\frac{i}{2}\left[aa_Z^* - a^*a_Z\right] + \frac{1}{2}|a_X|^2 - \frac{1}{2}|A|^2\Delta N(X)\right\}dX$$
$$= \frac{\sqrt{\pi}}{2}\left\{\begin{array}{l} i\sigma\left[aa_Z^* - a^*a_Z\right] + |a|^2\sigma^3\beta_Z + \frac{|a|^2}{2\sigma}(1+4\beta^2\sigma^4) + \\ \frac{\sqrt{2}}{2}|a|^4\sigma^3 - \frac{\sqrt{\pi}}{2}da^4\sigma^2 - 2h\sigma|a|^2\exp(-\frac{\pi^2\sigma^2}{T^2}) \end{array}\right\} \tag{6}$$

the following differential equations are obtained:

$$\frac{\delta\langle L\rangle}{\delta a^*} = 0 \Rightarrow$$
$$\frac{d}{dZ}(i\sigma a) = -i\sigma a_Z + a\sigma^3\beta_Z + \frac{a}{2\sigma}(1+4\beta^2\sigma^4) \tag{7}$$
$$+\sqrt{2}|a|^2 a\sigma^3 - \sqrt{\pi}d|a|^2 a\sigma^2 - 2h\sigma a\exp(-\frac{\pi^2\sigma^2}{T^2})$$

$$\frac{\delta\langle L\rangle}{\delta a} = 0 \Rightarrow$$
$$\frac{d}{dZ}(-i\sigma a^*) = i\sigma a_Z^* + a^*\sigma^3\beta_Z + \frac{a^*}{2\sigma}(1+4\beta^2\sigma^4) \tag{8}$$
$$+\sqrt{2}|a|^2 a^*\sigma^3 - \sqrt{\pi}d|a|^2 a^*\sigma^2 - 2h\sigma a^*\exp(-\frac{\pi^2\sigma^2}{T^2})$$

$$\frac{\delta\langle L\rangle}{\delta\sigma} = 0 \Rightarrow$$
$$i\left[aa_Z^* - a^*a_Z\right] + 3|a|^2\sigma^2\beta_Z + \frac{|a|^2(12\beta^2\sigma^4 - 1)}{2\sigma^2} + \frac{3\sqrt{2}|a|^4\sigma^2}{2}$$
$$-\sqrt{\pi}d|a|^4\sigma - 2h|a|^2\exp(-\frac{\pi^2\sigma^2}{T^2})(1 - \frac{2\pi^2\sigma^2}{T^2}) = 0 \tag{9}$$

$$\frac{\delta\langle L\rangle}{\delta\beta} = 0 \Rightarrow \frac{d}{dZ}(|a|^2\sigma^3) = 4|a|^2\beta\sigma^3 \tag{10}$$

After some calculations the following relations are true:

$$|a|^2\sigma = |a_0|^2\sigma_0 = \text{costant} \tag{11}$$

$$\sigma_Z = 2\beta\sigma \tag{12}$$

$$\beta_Z = \frac{1}{2\sigma^4} - \frac{\sqrt{2}}{4}|a|^2 - 2\beta^2 - \frac{2\pi^2}{T^2}h\exp(-\frac{\pi^2\sigma^2}{T^2}) \tag{13}$$

Then the ordinary differential equation for the beam width can be obtained:

$$\sigma_{ZZ} = \frac{1}{\sigma^3} - \frac{\sqrt{2}}{2}|a|^2\sigma - \frac{4\pi^2}{T^2}h\sigma\exp(-\frac{\pi^2\sigma^2}{T^2}) \tag{14}$$

Equation (14) is equivalent to Newtonian second law in classical mechanics for the motion of a one-dimensional particle acted by an equivalent force

$$F = 1/\sigma^3 - |a|^2\sigma/\sqrt{\pi} - \frac{4\pi^2 h\sigma}{T^2}\exp(-\pi^2\sigma^2/T^2).$$

In this paper we consider only the case when $\pi^2\sigma^2/T^2 << 1$, which mean that the lattice period is larger. Then under this assumptions Eq. (14) can be rewritten as

$$\sigma_{ZZ} \approx \frac{1}{\sigma^3} - \frac{\sqrt{2}}{2}|a|^2\sigma - \frac{4\pi^2 h\sigma}{T^2} \tag{15}$$

When $F = \sigma_{ZZ} = 0,\ \sigma(0) = 1$, the input power to form a soliton is

$$P_{Soliton} = \sqrt{2\pi}\left(1 - \frac{4\pi^2 h}{T^2}\right) \tag{16}$$

Easy to find $P_{Soliton} < \sqrt{2\pi}$, which means the soliton power is less than that in uniform nonlinear media. The potential has the following form under the condition $\sigma(0)=1, \sigma_Z(0)=0$,

$$\mathbf{V(\sigma)}=-\frac{1}{2}(\sigma_Z)^2=\frac{1}{2}\left(\frac{1}{\sigma^2}-1\right)+\frac{P_0}{\sqrt{2\pi}}(\sigma-1)+\frac{2h\pi^2}{T^2}\left(\sigma^2-1\right) \quad (17)$$

Where $P_0=\sqrt{\pi}a^2\sigma=\sqrt{\pi}a_0^2\sigma_0$.

Typical potential with relation to input power, modulation period and modulation length are presented in Figs.1-3, respectively.

Fig. 1 shows the potential of solitons with different input power when given $T=4\pi$ and $h=0.2$. From Fig. 1 we see that the potential of solitons are very sensitive to the input power around the normalized soliton width, i.e., $\sigma=1$. So in the formation of such solitons, the solitons are very sensitive to the input power when the the normalized soliton width $\sigma=1$.

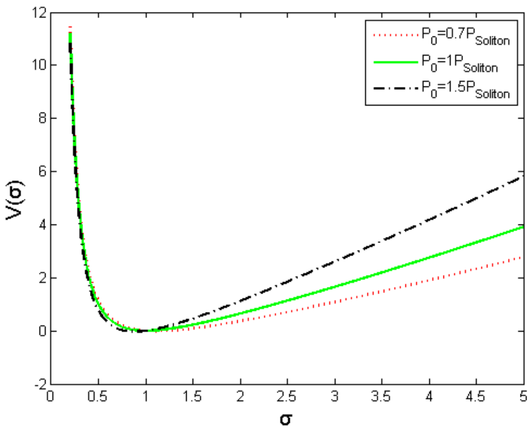

Fig. 1. Potential of solitons with different input power when given the modulation period $T=4\pi$ and modulation length $h=0.2$.

Figs. 2-3 show the potential of solitons with different modulation period (length) when given $h=0.2$ ($T=4\pi$), respectively. Where $P_0=P_{Soliton}$. From Fig. 2 we see that the potential of solitons is not sensitive to the modulation period when $\sigma=1$. The potential of solitons is not sensitive to the modulation length when $\sigma=1$ as shown in Fig. 3.

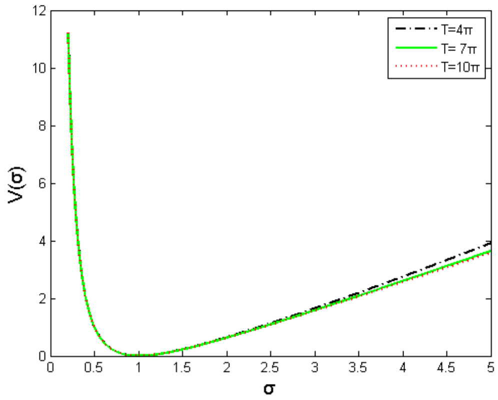

Fig. 2. Potential of solitons with different modulation period when given $P_0=P_{Soliton}$ and modulation length $h=0.2$.

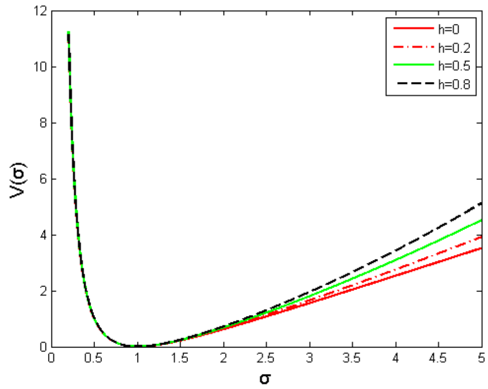

Fig. 3. Potential of solitons with different modulation length when given $P_0=P_{Soliton}$ and modulation period $T=4\pi$.

Dynamic propagation of the solitons in this media with an optical lattice is shown in Fig. 4. As is discussed above that $P_{Soliton}=\sqrt{2\pi}\left(1-\frac{4\pi^2h}{T^2}\right)$, so larger lattice depth support only lower power solitons when given modulation period, then the soliton power decrease with the increase of modulation length h. Dynamic propagation of solitons in such media when $P_0=P_{Soliton}$ and $T=4\pi$ for different modulation length is shown in Fig. 4. From Fig. 4 we see that the quasi-soliton propagate stably even for longer propagation distance. What implying that Gaussian function is a good approximation to a nonlocal spatial soliton in such thermal media with lattices when the lattice period is larger. Additionally, we find that the input beam will vibrate when the input power does not equal to the soliton power.

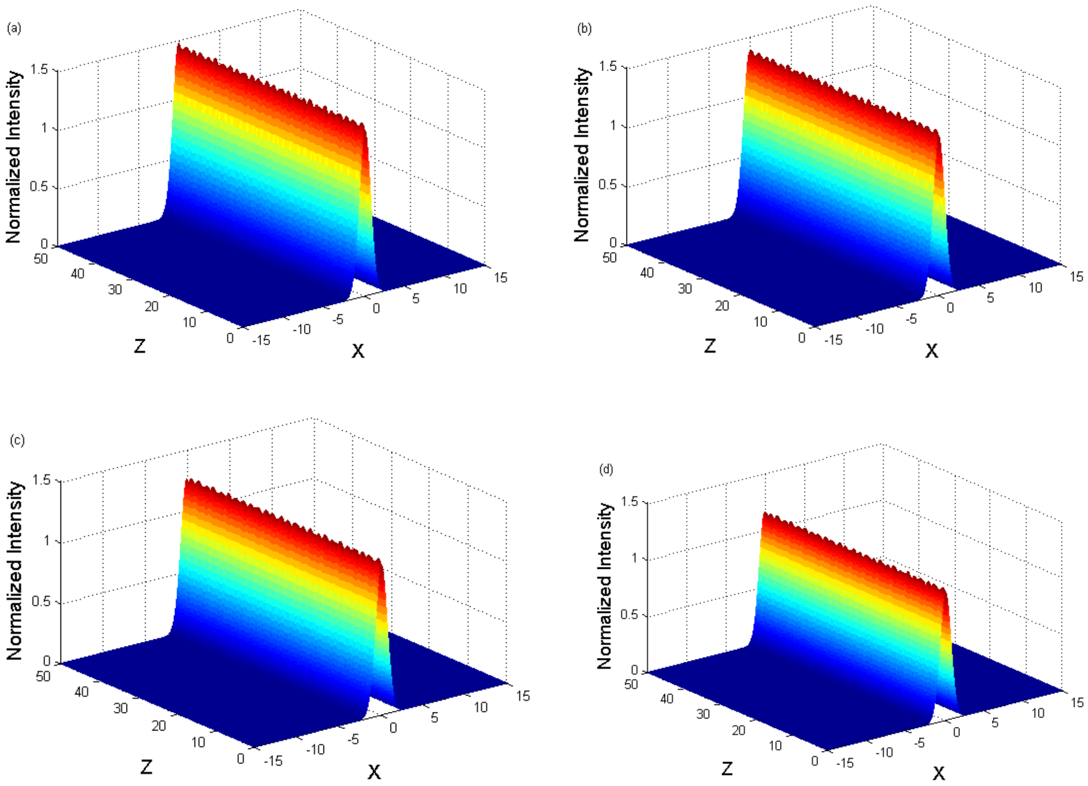

Fig. 4. Dynamic propagation of solitons in nonlocal media when $P_0 = P_{Soliton}$ *and* $T = 4\pi$ *for different modulation length (a).h=0; (b).h=0.2; (c).h=0.5; (d).h=0.8.*

Several issuers deserve discussion. Firstly, all the investigations confirm that Gaussian function is a good approximation to a nonlocal spatial soliton in nonlocal thermal media with optical lattice when the lattice period is larger. For the Gaussian beam is not the exact solution in such nonlocal system, there is some little vibration around the Gaussian solitons. Secondly, in this paper, we consider only the spatial solitons propagation in such nonlocal thermal media with an optical lattice, so the modulation periods is not very small compared to the soliton width.

3. Conclusions

We study the propagation of a spatial soliton in an optical lattice imprinted in a nonlocal thermal media with infinite range of nonlocality using a variational approach, and obtain the dynamical equations for beam width, amplitude and curvature. Results show that a stable propagation of the spatial soliton in this medium is observed when the input power is equal to the soliton power. All the investigations confirm that Gaussian function is a good approximation to a spatial soliton in nonlocal thermal media with optical lattice when the modulation period is larger.

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