Bright solitons in optical metamaterials by traveling wave hypothesis

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This paper obtains bright 1-soliton solutions in optical metamaterials by the aid of traveling wave hypothesis. There are three types of nonlinear media that are considered. They are Kerr law, parabolic law and log law nonlinearity. There are several constraint relations that are obtained for soliton solutions to exist.

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1. Introduction

The dynamics of solitons in optical metamaterials is a very demanding area of research at present times. Several overwhelming results have been reported in the past few decades [1-20]. It is believed that waveguides, made up of metamaterials, will transmit solitons across the globe in future. There are quite a few results that are already reported in this context. Therefore it is necessary to dig a little deeper into this area of research to unearth unprecedented novelty. This paper will therefore extract exact bright solitons by using the most elementary approach, namely the traveling wave hypothesis that gives waves of permanent form.

There are several integration tools applied to extract exact 1-soliton as well as multiple soliton solutions to the model. A few of them are simplest equation approach [4, 14], ansatz method [3,4], F-expansion scheme [6], functional variable method [5], first integral approach [5] and several others. This paper will retrieve bright 1-soliton solutions by the traveling wave hypothesis. The model equation is nonlinear Schrödinger's equation (NLSE) with a few perturbation terms. There are three types of nonlinear media that will be studied in this paper. They are Kerr law, parabolic law and log law nonlinearity. It must be noted that traveling wave hypothesis fails to retrieve soliton solutions to NLSE with power law and dual-power laws.

2. Governing equation

The dimensionless form of NLSE in optical metamaterials is given by [3-6]:

$$\begin{aligned} iq_t + aq_{xx} + F(|q|^2)q &= i\alpha q_x + i\lambda(|q|^2 q)_x \\ &+ i\nu(|q|^2)_x q + \theta_1(|q|^2 q)_{xx} + \theta_2 |q|^2 q_{xx} + \theta_3 q^2 q^*_{xx} \end{aligned} \tag{1}$$

Eq. (1) is the NLSE that is studied in the context of optical metamaterials. The independent variables are x and t that respectively represent the spatial and temporal variables, while the dependent variable is $q(x,t)$ which represents the complex valued wave envelope. Also, a is the coefficient of group velocity dispersion (GVD). The functional F represents the nonlinear term. On the right hand side, α is due to inter-modal dispersion, λ represents the self-steepening term to avoid formation of shock waves, ν is the nonlinear dispersion. The θ_j for $j = 1, 2, 3$ terms appear in the context of metamaterials [3-6].

The functional F represents, in general, non-Kerr law nonlinear media and is a real-valued algebraic function and the smoothness of the complex function $F(|q|^2)q : C \mapsto C$ is needed. Considering the complex plane C as a two-dimensional linear space R^2 , the function $F(|q|^2)q$ is k times continuously differentiable, so that [12, 13]

$$F(|q|^2)q \in \bigcup_{m,n=1}^{\infty} C^k\big((-n,n)\times(-m,m); R^2\big) \tag{2}$$

The traveling wave hypothesis will be introduced in the following section and 1-soliton solution will be obtained for three forms of the nonlinear function F .

3. Traveling wave hypothesis

The starting hypothesis to address (1) is given by [2, 12, 13]

$$q(x,t) = g(x-vt)e^{i(-\kappa x+\omega t+\sigma)} \tag{3}$$

where v represents the speed of the soliton and the function g is the amplitude component of the complex valued function $q(x,t)$. From the phase component, κ is the soliton frequency, ω is the soliton wave number and σ is the phase constant. Substituting (3) into (1) and decomposing into real and imaginary parts lead to

$$\begin{aligned} &ag'' - (\omega+\alpha\kappa+a\kappa^2)g \\ &+\{F(g^2) - \lambda\kappa g^2 + (\theta_1+\theta_2+\theta_3)\kappa^2 g^2\}g \\ &-(3\theta_1+\theta_2+\theta_3)g^2 g'' - 6\theta_1 g(g')^2 = 0 \end{aligned} \tag{4}$$

and

$$\begin{aligned} &(v+\alpha+2a\kappa)g' \\ &+\{3\lambda+2\nu-2(3\theta_1+\theta_2+\theta_3)\kappa\}g^2 g' = 0 \end{aligned} \tag{5}$$

In (4) and (5), the notations $g' = dg/ds$ and $g'' = d^2g/ds^2$ are used where

$$s = x - vt \tag{6}$$

From the real part equation, setting the coe_cients of linearly independent functions to zero gives

$$\theta_1 = 0 \tag{7}$$

and

$$\theta_2 + \theta_3 = 0 \tag{8}$$

Consequently, from the imaginary part equation it follows from the coe_cients of linearly independent functions

$$3\lambda + 2\nu = 0 \tag{9}$$

and the speed of the soliton falls out to be

$$v = -\alpha - 2a\kappa \tag{10}$$

which is valid for all forms of nonlinear media. Therefore with these parameter settings, the governing equation (1) modifies to

$$\begin{aligned} &iq_t + aq_{xx} + F(|q|^2)q = i\alpha q_x + i\lambda(|q|^2 q)_x \\ &+ i\nu(|q|^2)_x q + \theta_2|q|^2 q_{xx} - \theta_2 q^2 q^*_{xx} \end{aligned} \tag{11}$$

and the real part equation simplifies to

$$ag'' - (\omega+\alpha\kappa+a\kappa^2)g + \{F(g^2) - \lambda\kappa g^2\}g = 0 \tag{12}$$

The traveling wave hypothesis of this equation will now be studied. Multiplying both sides of (12) by g' and integrating leads to

$$\begin{aligned} &2a(g')^2 - 2(\omega+\alpha\kappa+a\kappa^2)g^2 \\ &-\lambda\kappa g^4 + 4\int^g F(h^2)hh'dh = 0 \end{aligned} \tag{13}$$

upon simplification after choosing the integration constant to be zero, since the search is for a soliton solution. The next three subsections will focus on the integrability of the ordinary differential equation (ODE) given by (13) for Kerr law, parabolic law and log law where the functional F is known.

3.1 Kerr Law

For Kerr law nonlinearity [1, 3],

$$F(u) = bu \tag{14}$$

for a real constant b so that (11) reduces to

$$\begin{aligned} &iq_t + aq_{xx} + b|q|^2 q = i\alpha q_x + i\lambda(|q|^2 q)_x \\ &+ i\nu(|q|^2)_x q + \theta_2|q|^2 q_{xx} - \theta_2 q^2 q^*_{xx} \end{aligned} \tag{15}$$

and hence the real part ODE gives

$$2a(g')^2 - 2(\omega+\alpha\kappa+a\kappa^2)g^2 + (b-\lambda\kappa)g^4 = 0 \tag{16}$$

After separating variables in (16) and integrating leads to

$$g(s) = g(x-vt) = A\,\mathrm{sech}[B(x-vt)] \tag{17}$$

where

$$A=\sqrt{\frac{2(\omega+\alpha\kappa+a\kappa^2)}{b-\lambda\kappa}} \tag{18}$$

and

$$B=\sqrt{\frac{2(\omega+\alpha\kappa+a\kappa^2)}{a}} \tag{19}$$

Hence, bright 1-soliton solution to (15) is given by

$$q(x,t)=A\,\mathrm{sech}[B(x-vt)]e^{i(-\kappa x+\omega t+\sigma)} \tag{20}$$

where the amplitude (A) of the soliton and the inverse width (B) of the soliton are given by (18) and (19) respectively. It must be noted that these bright solitons exist provided the constraints

$$(b-\lambda\kappa)(\omega+\alpha\kappa+a\kappa^2)>0 \tag{21}$$

and

$$a(\omega+\alpha\kappa+a\kappa^2)>0 \tag{22}$$

4. Parabolic law

In this case,

$$F(u)=b_1u+b_2u^2 \tag{23}$$

where b_1 and b_2 are real-valued constants [2, 8, 12, 13]. Therefore, (11) takes the form

$$\begin{aligned} &iq_t+aq_{xx}+\left(b_1|q|^2+b_2|q|^4\right)q \\ &=i\alpha q_x+i\lambda(|q|^2q)_x+i\nu(|q|^2)_xq+\theta_2|q|^2q_{xx}-\theta_2q^2q^*_{xx} \end{aligned} \tag{24}$$

and hence the real part ODE gives

$$\begin{aligned} &6a(g')^2-6(\omega+\alpha\kappa+a\kappa^2)g^2 \\ &+3(b_1-\lambda\kappa)g^4+2b_2g^6=0 \end{aligned} \tag{25}$$

After separating variables in (25) and integrating leads to

$$g(s)=g(x-vt)=\frac{A}{\sqrt{D+\cosh[B(x-vt)]}} \tag{26}$$

where

$$A=\frac{2\sqrt{6}(\omega+\alpha\kappa+a\kappa^2)}{\sqrt{3(b_1-\lambda\kappa)^2+16b_1(\omega+\alpha\kappa+a\kappa^2)}} \tag{27}$$

and

$$B=2\sqrt{\frac{(\omega+\alpha\kappa+a\kappa^2)}{a}} \tag{28}$$

and

$$D=\frac{\sqrt{3}(b_1-\lambda\kappa)}{\sqrt{3(b_1-\lambda\kappa)^2+16b_1(\omega+\alpha\kappa+a\kappa^2)}} \tag{29}$$

Hence, bright 1-soliton solution to (24) is given by

$$q(x,t)=\frac{A}{\sqrt{D+\cosh[B(x-vt)]}}e^{i(-\kappa x+\omega t+\sigma)} \tag{30}$$

where the amplitude (A) of the soliton and the inverse width (B) of the soliton are given by (27) and (28) respectively. This case introduces a new parameter D that is given by (29). The condition for the existence of the bright soliton is guaranteed for

$$3(b_1-\lambda\kappa)^2+16b_1(\omega+\alpha\kappa+a\kappa^2)>0 \tag{31}$$

and (22) which follows from (27) or (29) and (28).

4.1 Log law

Here,

$$F(s)=b\ln(s) \tag{32}$$

for real valued constant b, so that equation (11) reduces to [2, 12, 13]

$$\begin{aligned} &iq_t+aq_{xx}+bq\ln|q|^2=i\alpha q_x+i\lambda(|q|^2q)_x \\ &+i\nu(|q|^2)_xq+\theta_2|q|^2q_{xx}-\theta_2q^2q^*_{xx} \end{aligned} \tag{33}$$

and hence the real part ODE gives

$$\begin{aligned} &2a(g')^2-(2\omega+2\alpha\kappa+2a\kappa^2+b)g^2 \\ &-2bg^2\ln g=0 \end{aligned} \tag{34}$$

After separating variables in (34) and integrating leads to

$$g(s)=g(x-vt)=Ae^{-B^2(x-vt)^2} \tag{35}$$

where

$$A=\exp\left(\frac{\omega+\alpha\kappa+a\kappa^2+b}{2b}\right) \tag{36}$$

and

$$B=\sqrt{\frac{b}{2a}} \tag{37}$$

Hence, Gausson solution to (33) is given by

$$q(x,t) = Ae^{-B^2(x-vt)^2} e^{i(-\kappa x+\omega t+\sigma)} \tag{38}$$

where the amplitude (A)of the Gausson and the inverse width (B) of the Gausson are given by (36) and (37) respectively. It must be noted that these bright solitons exist provided the constraints

$$b \neq 0 \tag{39}$$

$$ab > 0 \tag{40}$$

hold respectively for (36) and (37). The inequation (40) implies that both GVD and nonlinearity must maintain the same sign for Gaussons to exist.

5. Conclusions

This paper recovered bright 1-soliton solution, in optical metamaterials, by the aid of travelling wave hypothesis. This integration scheme is not applicable to retrieve bright soliton solutions for power law and dual-power law media. Also, it must be noted that there are soliton solutions that are reported earlier by this same integration scheme, namely traveling wave hypothesis applicable to five forms of nonlinearity that includes power law and dual-power law [2, 12, 13]. However, for optical metamaterials, the situation is a little different. The governing equations have parameters that obey constraint relations, as discussed in Section-3, and thus prevent integrability by travelling wave hypothesis for power law and dual-power law.

Another disadvantage of this scheme is that one can retrieve only bright 1-soliton solutions and not dark or singular optical solitons. Later, the focus will be on the application of additional integration techniques to retrieve dark and singular solitons along with bright-dark combo optical solitons. The results of those research will be reported soon. Additionally, soliton perturbation theory as well as quasi-stationary soliton solutions will be obtained. Finally, the quasi-particle theory, for suppression of intra-channel collision, will also be developed and reported.

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