Chaos, coexisting attractors, and circuit design of the generalized sprott C system with only two stable equilibria

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This paper carries out the coexistence of chaotic attractors and stable equilibria in a generalized Sprott C system with only two stable equilibria. The discovery of this result is striking, because one typically would anticipate non-chaotic and even asymptotically converging behaviors. The simulation results are verified with a circuit implementation.

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1. Introduction

Since Lorenz found the first classical chaotic attractor in 1963 [1], chaos, as a very interesting nonlinear phenomenon, has been intensively studied in the last decades [2-6]. It is found to be either useful or has great potential in many fields, such as in engineering, biology and economics.

For a generic three-dimensional smooth quadratic autonomous system, Sprott found by exhaustive computer searching about 19 simple chaotic systems with no more than three equilibria[1-3]. It is very important to note that some 3D autonomous chaotic systems have three particular fixed points: one saddle and two unstable saddle-foci(for example, Lorenz system[1], Chen system [4], Lü system [5], the conjugate Lorenz-type systemet. et al [6]). The other 3D chaotic system, such as the original Rölsser system [2], DLS [7] and Burke-Show system [8], have two unstable saddle-foci. Yang and Chen found another 3D chaotic system with three fixed points: one saddle and two stable fixed points [9]. Recently, Yang, Wei and Chen [10] introduced and analyzed a new 3-D chaotic system in a form very similar to the Lorenz, Chen, Lü and Yang-Chen system [9], but it has only two stable node-foci. In 2011, Wang and Chen discovered a simple three-dimensional autonomous quadratic system that has only one stable equilibrium [11], revealing some new mysterious features of chaos. That is to say, the analytic criterion that the system has at least an unstable equilibrium for emergence of chaos is certainly not necessary. Moreover, many theoretical analysis and numerical simulation results about these systems were obtained [12-16].

It should be noted that one commonly used analytic criterion for generating and proving chaos in autonomous systems is based on the fundamental work of Sil'nikov [17, 18] and its subsequent embellishment and slight extension [19]. However, Shi'linikov criteria is sufficient but certainly not necessary for emergence of chaos. Another form of complexity arises when two or more asymptotically stable equilibria and other attracting sets coexist as the system parameters are being varied. The trajectories of the kinds of system selectively converges on either of the attracting sets depending on the initial state of the system. Another form of complexity arises when two or more asymptotically stable equilibria or attracting sets co-exist as the system parameters are being varied. There has been increasing interest in exploiting chaotic dynamics in engineering applications, where some attention has been focused on effectively creating chaos via simple physical systems, such as some electronic circuits [20-26].

In this paper, by using linear feedback, we introduce a generalized Sprott C system with six terms. When all of equilibria of generalized Sprott C system are stable, the system generates a double-scroll chaotic attractor, which can coexist with period attractors and stable equilibria.This is usually referred to as co-existing attractors and when this occurs, the trajectories of the system selectively converges on either of the attracting sets depending on the initial state of the system. When co-existing attractors occur in a system, engineers and scientists are usually interested in obtaining the basins of attraction of the different attracting sets, defined as the set of initial points whose trajectories converge on the given attractor. Trajectories selectively converge on either of the attracting sets depending on the initial condition of the system. It might render future prediction of the system's steady state behavior almost impossible.

2. The generalized Sprott C system dynamic properties

The generalized Sprott C system is described by the following equations:

$$\begin{aligned} \dot{x} &= a(y - x) \\ \dot{y} &= -cy - xz \\ \dot{z} &= y^2 - b, \end{aligned} \tag{2.1}$$

where a, b, c are real parameters. When $b > 0$ and $a \neq 0$, the model (2.1) has the following two fixed points:

$$E_1(\sqrt{b}, \sqrt{b}, -c), \quad E_2(-\sqrt{b}, -\sqrt{b}, -c).$$

In addition, the divergence of the system is $-a-c$ which implies that the system is dissipative for $a+c>0$, since the volume of the system contracts according to the Liouville formula. It is easy to show that the system (2.1) is topologically equivalent to the original Sprott C system when $c=0$ and $a=b^2>0$ [3]. In the parametric space $\{(a,b,c) \mid a>0, b>0, c=0\}$, the system (2.1) has two non-hyperbolic equilibria whose characteristic values both are: $\lambda_1 = -a$, $\lambda_{2,3} = \pm\sqrt{2bi}$.

Generally, it is difficult to analytically specify parametric regions of a chaotic system. Therefore, certain numerical indices for identifying chaotic properties of system orbits are verified. According to the aforementioned analysis, there are three kinds of cases about the type of equilibria in system (2.1): (i) two saddle-foci. (ii) two stable node-foci. (iii) two non-hyperbolic equilibria. Consequently, a question naturally arises: the generalized Sprott C system can generate chaotic attractors when $c>0$?

To answer this question, now we investigate the influence of initial condition on the dynamics of system with the parameters space $\{(a,b,c) \mid a>0, b>0, c>0\}$. In particular, when we fix $a=10, b=100, c=0.4$ and change initial values slightly, dynamical behaviors of the system may produce large variations in the long term. Besides the two stable equilibrium points, chaotic attractors of system (2.1) also are obtained, which implies that chaos coexists with period attractors and the two stable fixed points:

(a) A chaotic attractor with initial values (11.2, 4.81.-0.2) is obtained. The Lyapunov exponents of the system (2.1) are found to be $L_1 = 1.2804, L_2 = 0$ and $L_3 = -11.6805$;

(b) For initial values (11.2, 4.85.-0.2), trajectories converge to stable equilibrium E_1. The Lyapunov exponents of the system (2.1) are found to be $L_1 = -0.1272, L_2 = -0.1308$ and $L_3 = -10.1421$;

(c) A chaotic attractor with initial values (11, 4.85.-0.2) is obtained again. The Lyapunov exponents of the system (2.1) are found to be $L_1 = 1.2714, L_2 = -0.0002$ and $L_3 = -11.6713$;

(d) For initial values (11, 4.88, -0.2), trajectories converge to stable equilibrium E_1. The Lyapunov exponents of the system (2.1) are found to be $L_1 = -0.1233, L_2 = -0.1301$ and $L_3 = -10.1466$.

Notice that the behavior of system (2.1) is very sensitive to the parameter c and initial values. It can be seen from the above results that a small change in the initial condition of the system (2.1) will probably create totally different dynamic behavior. For different initial conditions, trajectory of system (2.1) with only stable equilibria can converges to two types of attractors (chaos or stable equilibrium). These attractors are shown in Fig. 1. From this viewpoint, one is confident that there are still abundant complex properties and phenomena to be further investigated, towards some unified theories on the celebrated Lorenz system and other chaotic systems.

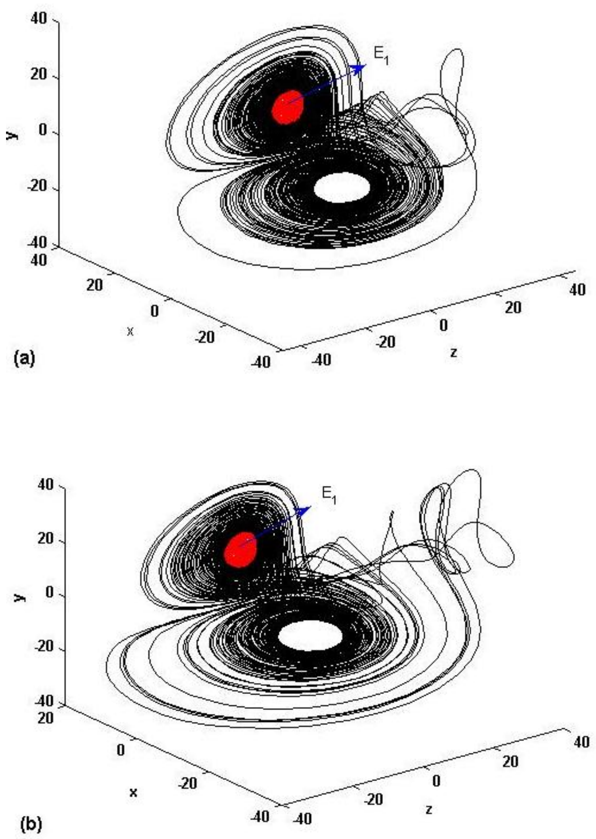

Fig. 1. Coexistence of chaotic attractors and stable equilibrium of the system (2.1) for the case $a=10, b=100, c=0.4$ *: (a) initial conditions (11.2, 4.85.-0.2) (in black) and (11.2, 4.85.-0.2) (in red) ; (b) initial conditions (11, 4.85.-0.2) (in black) and (11, 4.88, -0.2) (in red).*

3. Circuit realization of the new attractor

In fact, the strong random property is also demonstrated by the following circuit implementation [3]. When system (2.1) has only stable equilibria, the simple electronic circuit is designed that can be used to study chaotic phenomena. The circuit employs simple electronic elements such as resistors, and operational amplifiers, and is easy to construct. Fig. 2 shows the circuit schematic for implementing the new chaotic system Eq. (2.1). There are 3 capacitors, 8 resistors, 4 opamps and 2 multipliers in the circuit. Orcad-Pspice simulation of the new chaotic system is realized for parameters a = 10; b = 10, and c = 1.25 (R11=320 K), and initial conditions x_0=0.29, y_0=0.19, z_0=-0.04. TL081 opamps, and the Analog Devices AD633/AD multipliers are used and R1=40K, R_2=40K, R_5=40K, R_6 = 400K, R_7=40K, R_9= R_{10}=100K, R_{11}= 320K, C1 = C_2 = C_3 = 1nF, V_N = -15V, V_P =15V are chosen. All of the electronic components are easily available. Acceptable inputs to the AD633 multiplier IC are –10 to +10 V. The output voltage is the product of the inputs divided by 10 V. Orcad-PSpice simulations of the new chaotic system are also attained in Fig. 3, 4, and 5 (xy, xz, yz attractors respectively). In order to practical realization of the new chaotic circuit, a microcontroller based circuit have to be designed to applying the initial condition voltages to the capacitors. The real oscilloscope outputs and the initial condition applying circuit will be present in the next publication.

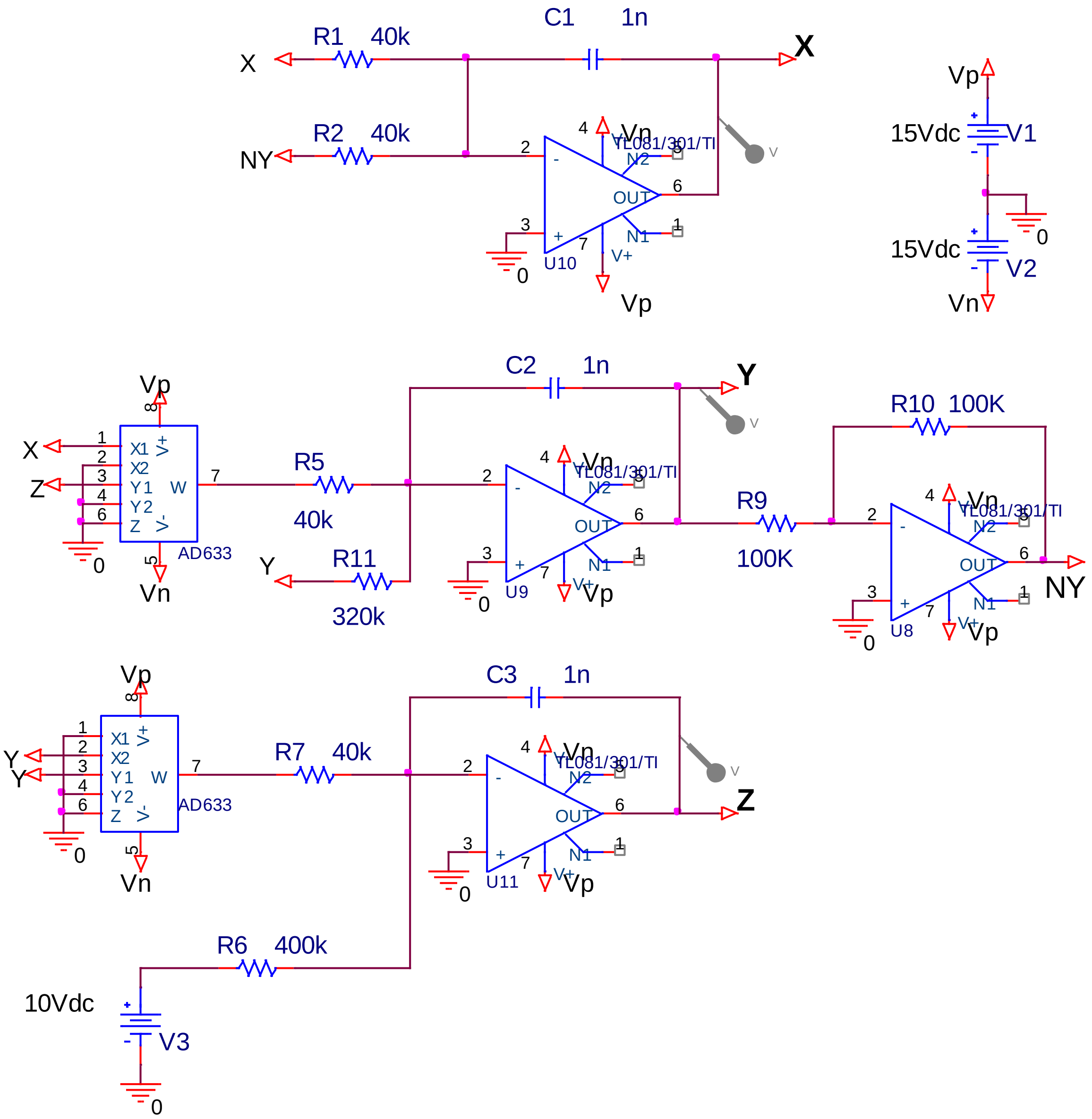

Fig. 2. The electronic circuit schematic of the new chaotic system.

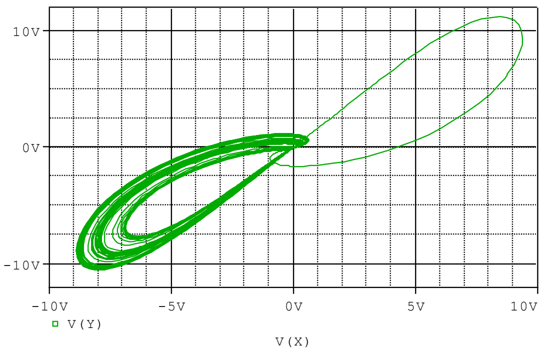

Fig. 3. Pspice simulation result of the new chaotic system's electronic oscillator, (xy strange attractor).

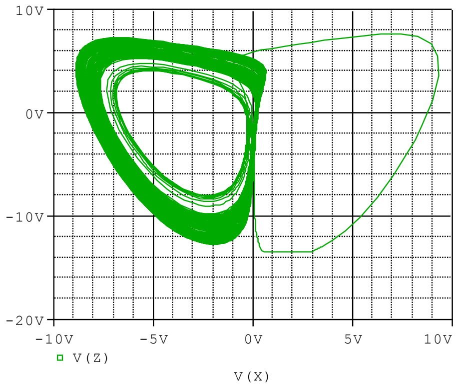

Fig. 4. Pspice simulation result of the new chaotic system's electronic oscillator, (xz strange attractor).

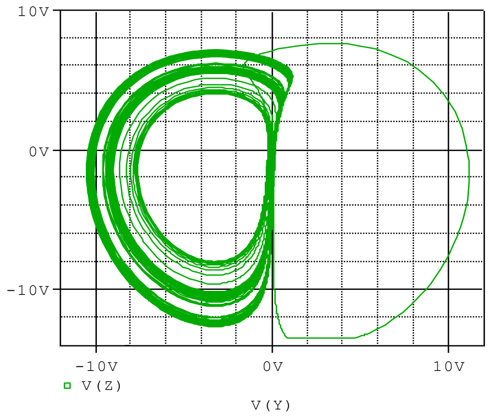

Fig. 5. Pspice simulation result of the new chaotic system's electronic oscillator, (yz strange attractor).

4. Conclusion

In summary, we have given a brief review of the chaotic behavior of the generalized Sprott C system and illustrated that the system may possess topologically different chaotic attractors. It is to indicate that some chaotic attractors may appear quite differently from the other kinds of equilibria, from a topological point of view. The co-existence of chaotic attractors and stable equilibria is very desirable for some engineering applications, as electronic circuit shows.

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