Combined optical solitons with nonlinear dispersion and spatio-temporal dispersion

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The nonlinear dynamics of optical solitons with spatio-temporal dispersion, nonlinear dispersion and inter-modal dispersion have been investigated. The well-posed nonlinear Schrödinger equation is solved analytically suing the complex envelope function ansatz. The combined solitons are obtained. Finally, the effects of nonlinear dispersion as well as spatio-temporal dispersion, on the combined solitons, are analyzed.

(Received August 24, 2014; accepted January 21, 2015)

Keywords: Solitons, Nonlinear Schrödinger equation, Nonlinear dispersion, Spatio-temporal dispersion, Inter-modal dispersion

1. Introduction

Soliton theory was developed rapidly during the past few decades. In nonlinear optics, the study of optical solitons in birefringent fibers, metamaterials, hollow-core photonic crystal fibers (HC-PCFs) and other forms of optical waveguides has recently attracted widespread interest [1-8].

The dynamics for the propagation of optical solitons through optical fibers for trans-continental and trans-oceanic distances is modeled by the well-known nonlinear Schrödinger equation (NLSE) [1-18]. However, recent studies show that this model is ill-posed [19, 20]. To Make the NLSE well-posed, one should to revisit the model, and then an additional term that is the spatio-temporal dispersion (STD) should be taken into consideration [19]. Therefore, under investigation in this paper is the NLSE with STD. In the presence of group velocity dispersion (GVD), self-phase modulation (SPM), detuning, nonlinear dispersion (ND), inter-modal dispersion (IMD) and third order dispersion (TOD), the governing equation is given by

$$iq_t + aq_{xx} + bq_{xt} + c|q|^2 q + dq + i\theta |q|^2 q_x + i\mu q_x + i\gamma q_{xxx} = 0 \tag{1}$$

In Eq. (1), $q(x,t)$ represents the normalized electric-field envelope that is the function of longitudinal coordinate x and time coordinate t. Here a, c, d, θ, μ, and γ are related to the GVD, SPM, detuning, ND, IMD and TOD coefficients respectively, while b represents the STD coefficient.

In order to investigate the nonlinear dynamics of optical solitons with STD, the first and most important task for us is to construct analytical solutions to Eq. (1), particularly the solitons. Optical solitons including the dark and bright solitons, which are non-diffracting localized electromagnetic waves that maintain their shapes in the long distance transmission, have important applications in tele-communication and ultrafast signal routing systems [21, 22].

2. Combined solitons to Eq. (1)

First we introduce the following complex envelope function ansatz that is the combination of single dark and single bright solitons [21, 23-25]:

$$q(x,t) = \lambda \tanh[B(x - vt)] + ip\, \mathrm{sech}[B(x - vt)] + i\beta \tag{2}$$

where λ, p, and β can be the real or complex constants that are the dark soliton amplitude, bright soliton amplitude and soliton greyness respectively, while B represents the soliton width, and v represents the soliton speed. It is needed to note that, when one takes $\lambda = 0$ ($p = 0$), then Eq. (2) gives the single dark (single bright) soliton with a grayness.

Then the amplitude $|q(x,t)|$ is given by

$$|q(x,t)|^2 = \lambda^2 + \beta^2 + 2\beta p\, \mathrm{sech}[B(x - vt)] + (p^2 - \lambda^2)\, \mathrm{sech}^2[B(x - vt)] \tag{3}$$

Now inserting Eq. (2) into Eq. (1), then utilizing the homogeneous balance method, i.e. equating the coefficients of each independent term of $\operatorname{sech}^{\alpha}\eta\tanh^{\delta}\eta$ ($\alpha = 0,1,2,3,4,5$, $\delta = 0,1$) equal to zero, one get

$$\alpha = 0, \delta = 0 : [c(\lambda^2 + \beta^2) + d]\beta = 0 \tag{4}$$

$$\begin{aligned} &\alpha = 1, \delta = 0 : \\ &[aB^2 - bB^2 v + c(\lambda^2 + 3\beta^2) + d]p = 0 \end{aligned} \tag{5}$$

$$\begin{aligned} &\alpha = 2, \delta = 0 : [\theta(\lambda^2 + \beta^2) + \mu + 4\gamma B^2 - v]\lambda B \\ &+ c\beta(3p^2 - \lambda^2) = 0 \end{aligned} \tag{6}$$

$$\begin{aligned} &\alpha = 3, \delta = 0 : 2B(-apB + bpBv + \theta\beta\lambda p) \\ &+ c(p^2 - \lambda^2)p = 0 \end{aligned} \tag{7}$$

$$\alpha = 4, \delta = 0 : [\theta(p^2 - \lambda^2) - 6\gamma B^2]\lambda B = 0 \tag{8}$$

$$\alpha = 0, \delta = 1 : [c(\lambda^2 + \beta^2) + d]\lambda = 0 \tag{9}$$

$$\begin{aligned} &\alpha = 1, \delta = 1 : \\ &[\theta(\lambda^2 + \beta^2) + \mu + \gamma B^2 - v]pB + 2c\lambda\beta p = 0 \end{aligned} \tag{10}$$

$$\begin{aligned} &\alpha = 2, \delta = 1 : \\ &2B[-a\lambda B + b\lambda Bv + \theta\beta p^2] + c(p^2 - \lambda^2)\lambda = 0 \end{aligned} \tag{11}$$

$$\alpha = 3, \delta = 1 : [\theta(p^2 - \lambda^2) - 6\gamma B^2]pB = 0 \tag{12}$$

Comparing Eq. (7) with Eq. (11), one obtains

$$p = \pm\lambda \tag{13}$$

Using Eqs. (8) and (12), or using Eqs. (6) and (10), one must have

$$\gamma = 0 \tag{14}$$

which means there is no TOD. Then Eqs. (4)-(12) reduce to

$$c(\lambda^2 + \beta^2) + d = 0 \tag{15}$$

$$aB^2 - bB^2 v + c(\lambda^2 + 3\beta^2) + d = 0 \tag{16}$$

$$[\theta(\lambda^2 + \beta^2) + \mu - v]B + 2c\lambda\beta = 0 \tag{17}$$

$$-aB + bBv + \theta\beta\lambda = 0 \tag{18}$$

Solving those equations above, one get

$$B = -\frac{2c\beta}{\theta\lambda} \tag{19}$$

$$v = \frac{a}{b} + \frac{\theta^2\lambda^2}{2bc} \tag{20}$$

$$v = \theta(\lambda^2 + \beta^2) + \mu - \theta\lambda^2 \tag{21}$$

Upon equating the two values of the soliton velocity v from Eqs. (20) and (21), one obtains the constraint condition

$$2ac + \theta^2\lambda^2 = 2bc[\theta(\lambda^2 + \beta^2) + \mu - \theta\lambda^2] \tag{22}$$

Hence, finally the combined solitons to Eq. (1) are obtained that are given by Eq. (2). The relation for dark soliton amplitude and bright soliton amplitude is given by Eq. (13), while the soliton width is given by Eq. (19), and the soliton speed is given by Eq. (20) or Eq. (21). Additionally the corresponding existence conditions are given by Eqs. (15) and (22).

3. Results and discussion

In the previous section, the complex envelope function ansatz is applied to extract the combined solitons to Eq. (1). This section is going to investigate the influence of ND and STD on the properties of the combined solitons. The presented results are useful for describing the propagation of optical solitons through optical fibers with ND and STD.

In the rest of this section, without loss of generality, we take the $p = \lambda$ as an example to illustrate. Then Form Eq. (3), one get the amplitude

$$|q(x,t)|^2 = \lambda^2 + \beta^2 + 2\beta\lambda\operatorname{sech}[B(x - vt)] \tag{23}$$

It should be noted that the Eq. (30) represents a bright soliton if $\beta\lambda > 0$, and a dark soliton if $\beta\lambda < 0$. This feature shows that the combined bright-dark solitons can propagate simultaneously.

Now, we are going to analyze the effect of ND and STD on the solitons. By choosing the appropriate parameters in Eq. (30), the bright and dark solitons are formed as shown in Fig. 1. Changing the values of θ and b can just adjust the soliton width as shown in Figs. 2 and 3. This means that we can control the soliton width by choosing the parameter values of the ND and STD.

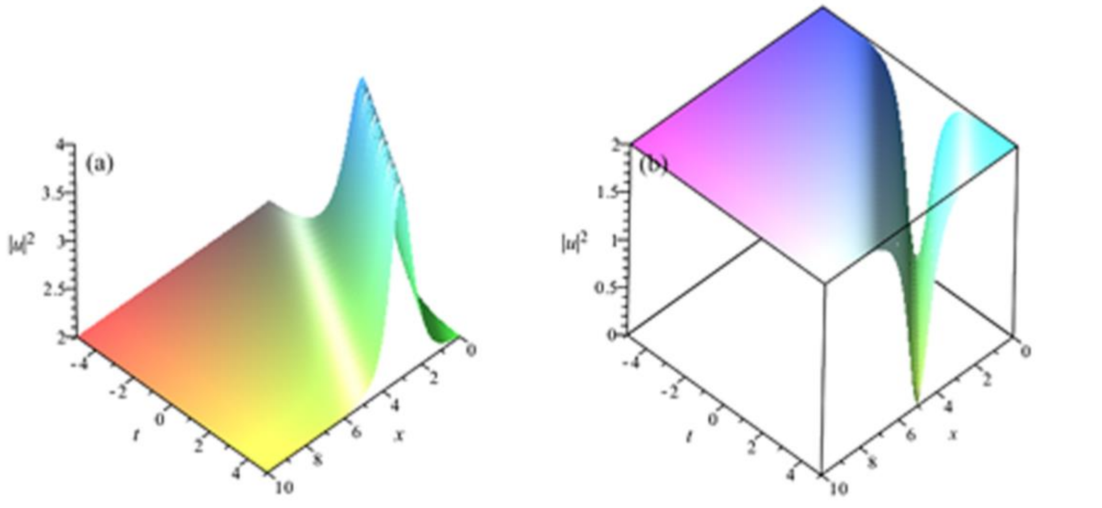

Fig. 1. (a) Evolution of the bright soliton. The parameter values are $\lambda = 1$, $\beta = 1$, $c = -0.5$, $a = 1$, $\theta = 0.5$, *and* $b = 1.25$. *(b) Evolution of the dark soliton. The parameter values are same as in (a) except for* $\beta = -1$ *and* $c = 0.5$.

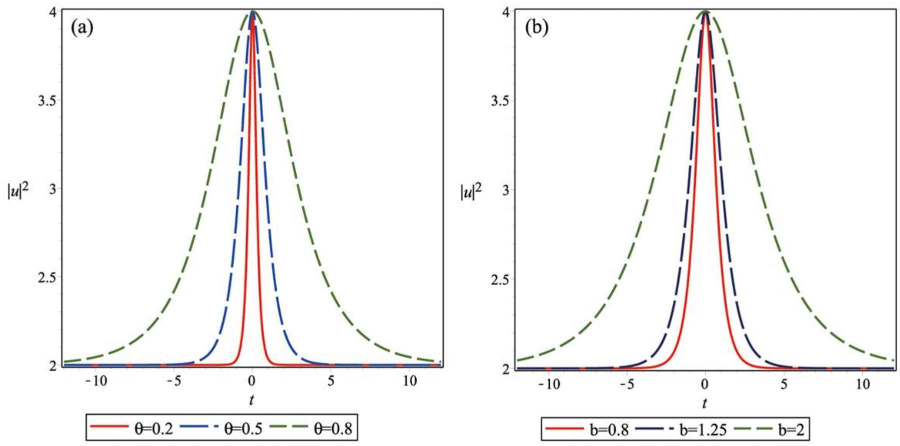

Fig. 2. Evolution of the bright soliton at x=0 on (a) different nonlinear dispersion with the same values as those given Fig. 1(a), but with $b = 1$. *(b) different spatio-temporal dispersion with the same values as those given Fig. 1(a).*

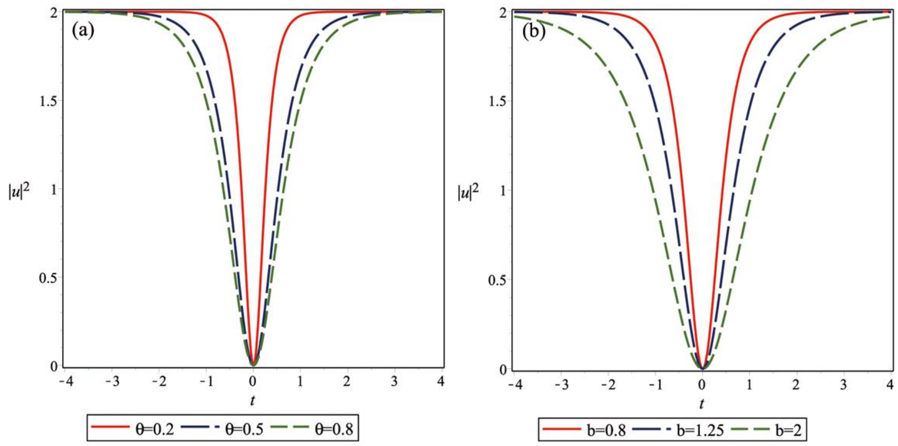

Fig. 3. Evolution of the dark soliton at x=0 on (a) different nonlinear dispersion with the same values as those given Fig. 1(b), but with $b = 1$. *(b) different spatio-temporal dispersion with the same values as those given Fig. 1(b).*

4. Conclusions

In this work the combined solitons in an optical fiber with detuning, ND, IMD as well as TOD are obtained along with necessary constraint conditions. The well-posed dynamical model that is studied is the NLSE with STD, i.e. Eq. (1). The tool of integrability that is used is the complex envelope function ansatz. We investigate the influence of ND and STD on the properties of the combined solitons. The results show that we can control the soliton width by choosing the parameter values of the ND and STD. Later, we will continue to study the combined solitons in other models arising in nonlinear optics and Bose–Einstein condensates.

Acknowledgement

The work of the second author was supported by the Scientific Research Fund of Hubei Provincial Education Department [grant number B2013193].

References

[1] M. Mirzazadeh, M. Eslami, B. Vajargah, A. Biswas, Optik **125**, 4246 (2014).
[2] Q. Zhou, D. Z. Yao, F. Chen, W.W. Li, J. Mod. Opt. **60**, 854 (2013).
[3] S. C. Wen, Y. W. Wang, W. H. Su, Y. Xiang, X. Fu, D. Fan, Phys. Rev. E **73**, 036617 (2006).
[4] R. G. Li, R. C. Yang, Z. Y. Xu, Phys. Rev. E **82**, 046603 (2010).
[5] A. Biswas, K. R. Khan, A. Rahman, A. Yildirim, T. Hayat, O. M. Aldossary, J. Optoelectron. Adv. Mater. **14**, 571 (2012).
[6] W. J. Liu, H. N. Han, L. Zhang, R. Wang, Z. Y. Wei, M. Lei, Laser Phys. Lett. **11**, 045402 (2014).
[7] Q. Zhou, J. Mod. Opt. **61**, 500 (2014).
[8] Q. Zhou, D. Z. Yao, S. J. Ding, Y. F. Zhang, F. Chen, F. Chen, X. N. Liu, Optik **124**, 5683 (2013).
[9] B. Ahmed, A. Biswas, Proc. Romanian Acad. A **14**, 111 (2013).
[10] Q. Zhou, Optik **125**, 3142 (2014).
[11] Q. Zhou, Optik **125**, 5432 (2014).
[12] A. Biswas, A. Yildirim, T. Hayat, O. M. Aldossary, R. Sassaman, Proc. Romanian Acad. A **13**, 32 (2012).
[13] W. J. Liu, B. Tian, M. Lei, Appl. Math. Lett. **30**, 28 (2014).
[14] Q. Zhou, D. Z. Yao, F. Chen, J. Mod. Opt. **60**, 1652 (2013).
[15] Q. Zhou, Q. Zhu, A. H. Bhrawy, L. Moraru, A. Biswas, Optoelectron. Adv. Mater. – Rapid Comm. **8**, 800 (2014).
[16] H. Kumar, F. Chandb, Opt. Laser Technol. **54**, 265 (2013).
[17] Q. Zhou, D. Z. Yao, Q. Q. Xu, X. N. Liu, Optik **124**, 2368 (2013).
[18] Q. Zhou, D. Z. Yao, Z. H. Cui, J. Mod. Opt. **59**, 57 (2012).
[19] M. Savescu, S. Johnson, A. H. Kara, S. H. Crutcher, R. Kohl, A. Biswas, J. Electromagn. Waves Appl. **28**, 242 (2014).
[20] M. Savescu, E. M. Hilal, A. A. Alshaery, A. H. Bhrawy, L. Moraru, A. Biswas, J. Optoelectron. Adv. Mater. **16**, 619 (2014).
[21] Z. H. Li, L. Li, H. P. Tian, G. S. Zhou, Phys. Rev. Lett. **84**, 4096 (2000).
[22] Q. Zhou, D. Z. Yao, X. N. Liu, F. Chen, S. J. Ding, Y. F. Zhang, F. Chen, Opt. Laser Technol. **51**, 32 (2013).
[23] A. K. Sarma, Commun. Nonlinear Sci. Numer. Simulat. **14**, 3215 (2009).
[24] H. Triki, F. Azzouzi, P. Grelu, Opt. Commun. **309**, 71 (2013).
[25] F. Azzouzi, H. Triki, K. Mezghiche, A. El Akrmi, Chaos, Solitons and Fractals **39**, 1304 (2009).

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