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Comments on SVD related image compression

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This paper analyses the distribution of singular vectors entries of an image to guarantee the quality of the compressed images. Distribution of singular vectors entries of all images follows a multi Gaussian probability density function. The proposed idea is employed for determination of compression rate of images. It is shown that while the Gaussian distribution of singular vectors entries is valid for all type of images, it depends to the complexity level of the image.

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1. Introduction

Determination of the probability density function of an image has been always a challenging research in statistical analysis of DCT coefficients [1-3]. It helps to find the suitable compression method for images. On the other hand, the SVD theorem has been recently applied in image quality assessment [4-5]. It has been shown that some useful properties in image quality relates to SVD theorem. Based on the linear algebra Equation 1 shows the SVD theorem:

$$A_{m\times n} = U_{m\times m}.S_{m\times n}.V^T_{n\times n} \qquad (1)$$

where $U^TU=I$ and $V^TV=I$ (I is identity matrix). Also the columns of $U=\{U_1, U_2, \ldots, U_m\}$ and $V=\{V_1, V_2, \ldots, V_n\}$ are orthonormal eigenvectors of AA^T and A^TA, respectively. S is a diagonal matrix containing singular values, s_i, which are the square roots of eigenvalues of U or V in descending order and in the case of m<n, only m nonzero singular values are computed. The methods [4-5] were tested on the Live database [6] and showed that the structural properties of an image can be explained by singular vectors. By using the features of SVD, other interesting property of matrix A could be determined. In fact, matrix A can be accurately estimated by using the first r column of matrix U, (U_r) and the top-left r×r sub-matrix S, S_r, and the first r rows of matrix V^T, V_r^T, as following:

$$\tilde{A}_r = U_r.S_r.V_r^T \qquad (2)$$

The most energy of Matrix A can be found in $\tilde{A}_r$. Furthermore, when A is m×n matrix, $\tilde{A}_r$ could be saved by (m×r)+r+(r×n)=(m+n+1)×r matrix. For example, when m=n=256; and r=100, the total entries to be saved will decrease $100\% - \frac{(256+256+1)100}{256^2}\times 100\% = 22\%$. In fact with small values of r, the approximated $\tilde{A}_r$ keeps most of the energy of matrix A and this characteristic indicates to the attractiveness of SVD theorem. However, determination of optimum value for r which grants adequate approximation of $\tilde{A}_r$ is a critical step in this approach. The present paper offers a new criterion for determination of such value for r.

2. Method presentation

The statistical property of the values in the singular vector matrices has been considered in this work. Based on the definition these vectors are linearly independent from together. Since any image could be reconstructed from a few column of singular vectors, this leads that how is the rate of the compact energy in the image If the image can be reconstructed by appropriate numbers of singular vectors, its compression could be done in low bit rate while the image quality is preserved. We observed that histogram of singular vector matrices entries, (U or V in Eq. (1)) of all type images benefits from Multi Gaussian distribution. Fig. 1 shows three different types of images, i.e. Buildings as high complex, Lenna as medium complex and Block as low complex images. Their histograms are normalized to form the probability distribution function (P.D.F.).

Matrix U_r which contains only r first columns of U is considered. With increasing r, its distribution becomes closer to gaussian distribution. For example, in Fig. 2, histogram of U_1, first column (most significant column) of reference image "Buildings", U_{10}, U_{50}, U_{100} and the resulted images A_1, A_{10}, A_{50} and A_{100} are shown in columns from left to right. In last row, histogram of matrix U and the original image are shown to compare. Also, r_square fitting histograms with Gaussian distribution are typed under each histogram. In addition root mean square error (rmse) each resulted image and original image are shown.

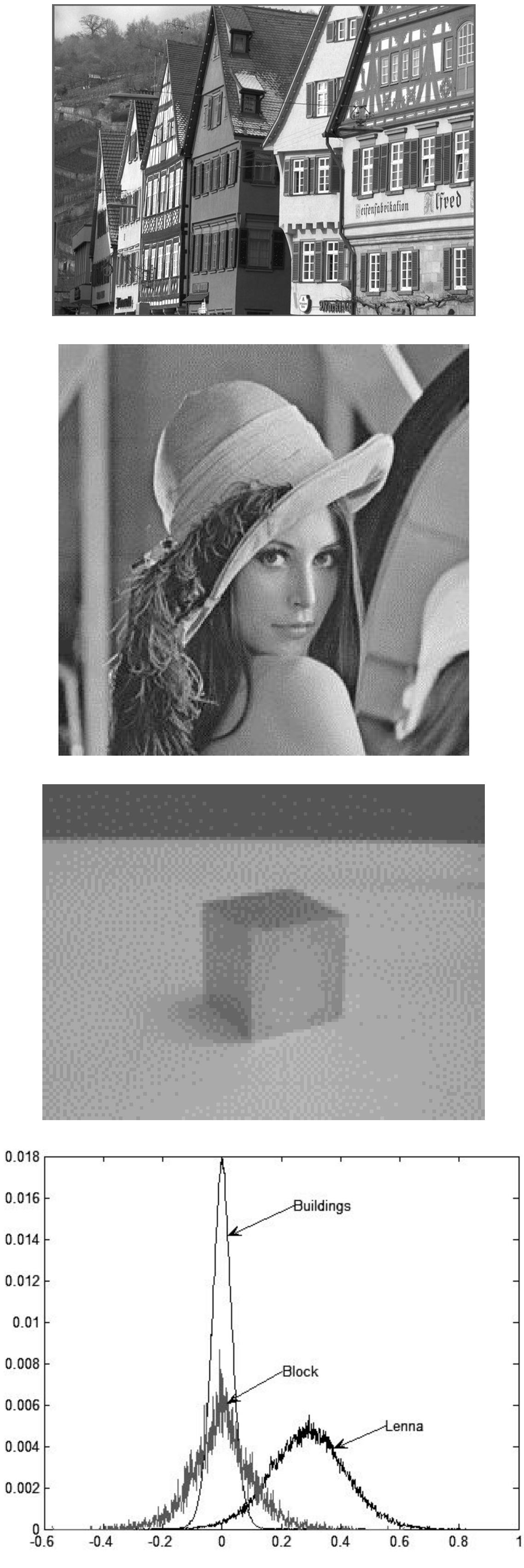


Fig. 1. Three different types of images and their histograms P.D.F. of left singular vector matrix.

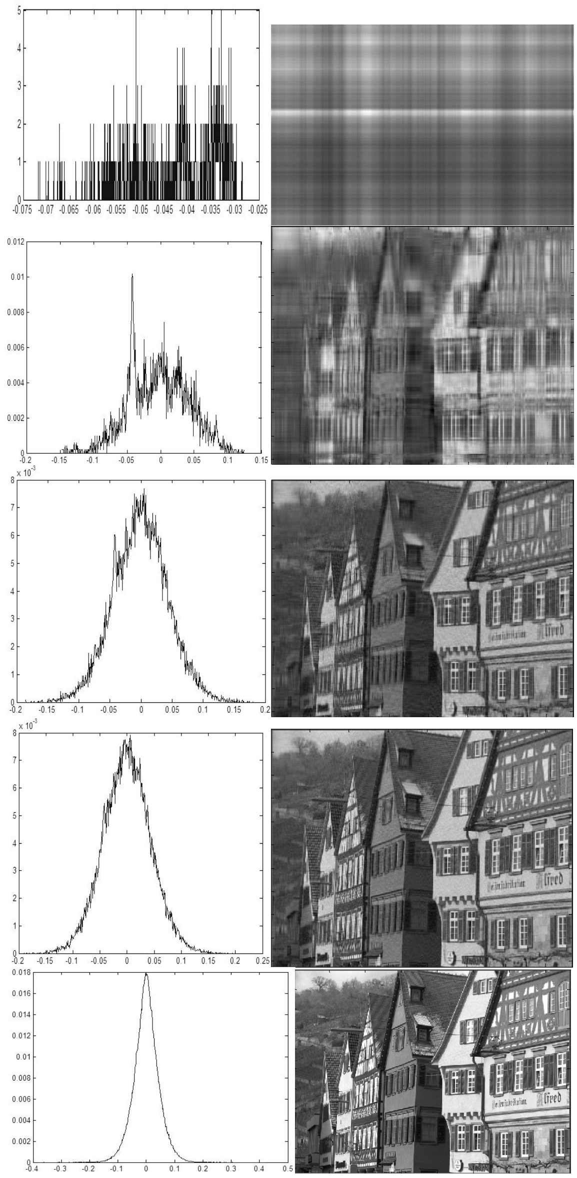


Fig. 2. rows, up to buttom: U_1 and A_1, U_{10} and A_{10}, U_{50} and A_{50}, U_{100} and A_{100}, U and A.

Hence, r could be determined based on the fitting of Gaussian curve to the histogram of singular vector matrix of the reference image with a threshold level for R_Square which grants the quality of reconstructed image. As shown before, if m<n, it is quite often to have m non singular values. In such case, matrix U or V with size of m×m is considered and the ratio of r to the whole number of columns, m, could be considered as the available compression rate. This could simultaneously guaranties insufficient quality of compressed image. Table 1 shows the values of (r/m)%, r and final R-Square of images of the LIVE database [6] while multi Gaussian probability density function has been fitted to r first columns of matrix U with threshold level of 0.99 for R_Square. In simulations, we consider sum of three Gaussin functions as curve to fit.

name	(r/m) %	r	Final R-Squre
Bikes	54.688	280	0.99853
Building2	21.289	109	0.99899
Buildings	37.695	193	0.99892
Caps	19.336	99	0.99885
Carnivaldolls	45.082	220	0.99733
Cemetery	17.635	85	0.99761
Churchandcapitol	16.436	83	0.99655
Coinsinfoutain	54.492	279	0.99871
Dancers	37.307	169	0.99786
Flowersonih35	21.484	110	0.99618
House	23.828	122	0.99815
Lighthouse	22.917	110	0.99538
Lighthouse2	15.82	81	0.99635
Manfishing	17.808	78	0.9963
Ocean	8.0078	41	0.99884
Paintedhouse	21.875	112	0.99733
Parrots	14.063	72	0.99762
Plane	42.969	220	0.99779
Rapids	19.531	100	0.9986
Sailing1	26.367	135	0.99797
Sailing2	24.167	116	0.99858
Sailing3	21.25	102	0.99862
Sailing4	9.7656	50	0.99827
Statue	19.792	95	0.9984
Stream	11.914	61	0.99851
Studentsculptur	67.723	342	0.99798
Woman	22.708	109	0.99484
womanhat	21.458	103	0.99883

Fig. 3 shows the resulted Buildings image using r=193 as computed in Table 1. Its rmse equals by 0.0105.

Fig. 3. The resulted Buildings image, r=193.

Fig. 4 shows the original and the resulted ocean image image using r=41 as it is in Table 1.

(a)

(b)

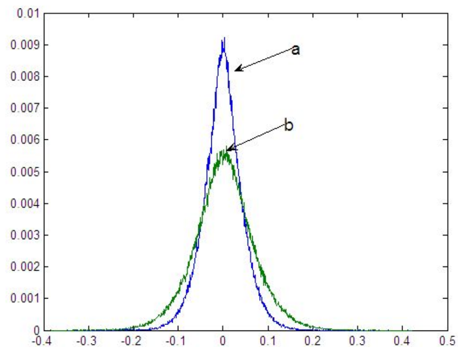
Fig. 4. (a) The original ocean image, (b) the resulted ocean image, r=41.

As, it is seen in Fig. 4, Ocean image is too simple and could be constructed with only 41 columns which is only 8% of all columns. However, in the case of the Buildings image, reconstruction needs 37% of all columns.

Usually the entropy value is used to discuss the image information. However, the entropy does not show the distribution characteristics of pixel values. In other words, images with same histograms have same entropy values. However, singular vector matrices of different images are independent from histogram and are based on image structural. Figure 5 shows the original house image, its singular vector matrix and its histogram. To investigate the weakness of entropy, the image is blocked and four parts of original image are changed. The entropies of two images are same while their singular vectors and their singular vectors entries histograms are completely different as shown in Fig. 5. Besides, the values of r are different in original and disturbed images. They are respectively r=122 and r=101 for original and disturbed images.

(a)

(b)



(c)

Fig. 5. (a) Original image; (b) Disturbed image; (c) Histogram of singular vectors of a and b.

3. Conclusions

The statistical properties of singular vector matrices entries showed that their histogram follow a multi guassianl curve. Since original matrix can be reconstructed by using a few numbers of singular vectors, determination of the optimum number of singular vectors which grants the quality of image is a critical issue. In this paper, a method based on fitting of normal curve to the histogram of the first columns of the singular vectors with a threshold level of rmse has been proposed to determine the proper value of such vectors. As shown in the paper, the number of columns varies with the complexity level of the images.

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