

Compound quartz depolarizer

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A compound quartz depolarizer was analysed in detail by using matrix method; the expression of degree of polarization of emergent light was also given. These results would be useful in design of the monochromatic quartz depolarizer.

(Received January 12, 2008; accepted March 12, 2008)

Keywords: Quartz, Rotation, Depolarizer

1. Introduction

Depolarizers can convert an arbitrary incident polarization state into a collection of states that have temporally and spatially random polarization. Polarized light is normally needed in optical modulation, information transmission, storage and so on, however, unpolarized light is also needed in some cases. Almost any optical instrument especially spectrometer has polarization sensitivity. In order to reduce the undesired polarization sensitivity of optical instruments and improve the measurement precision, depolarizers are necessarily used in optical system. Many depolarizers [1-7] have been developed since the Lyot depolarizer was invented.

Light beam will split and the transmission quality of light beam will be impaired when light passes through a quartz wedge. However, the compound quartz depolarizer can overcome these disadvantages. These features will be analyzed the later in detail.

2. Theory

The compound quartz depolarizer consists of two quartz wedges with the same angle β . Two quartz wedges are left-handed and right-handed, respectively. Fig. 1 shows the structure of a quartz depolarizer: the left is the side view of the depolarizer, the right is the end view of it; Z axis is the optical direction of propagation; the optical axis of quartz crystal is along the Z-direction; R is the radius of light beam. Length of polarizer is noted d_0 , $OA=d_1$ and $AC=d_2$.

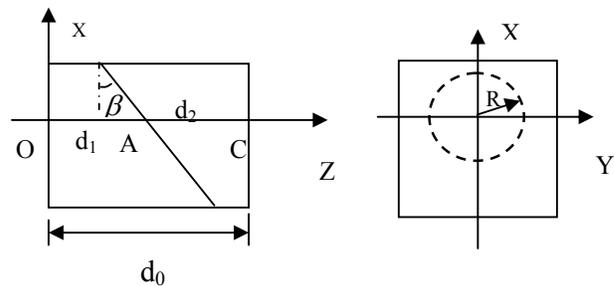


Fig. 1. The structure of quartz depolarizer.

For monochromatic light, the polarization scrambling is performed over the space domain. For simplicity, (1) we assume that the incident polarized light is homogeneous, perpendicular cylinder light beam. (2) we ignore the absorption and scattering in the crystal.

The Mueller matrix for depolarizer is

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\Phi & \sin 2\Phi & 0 \\ 0 & -\sin 2\Phi & \cos 2\Phi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (1)$$

where $2\Phi = 2a(d_1-d_2)-4\alpha \operatorname{Rtg} \beta$ $x=fx+b$; α is the specific rotation of quartz crystal; X is normalized : $-1 \leq x \leq 1$, so $-R \leq Rx \leq R$

The integral of $\cos 2\Phi$ and $\sin 2\Phi$ over space domain is

$$(\cos 2\Phi)_{\Sigma} =$$

$$\frac{1}{\pi} \int_0^1 \rho d\rho \int_0^{2\pi} \cos(f\rho \cos \theta) \cos b d\theta = 2 \cos b \frac{J_1(f)}{f} \quad (2)$$

$$(\sin 2\Phi)_{\Sigma} = \int_{\Sigma} \sin 2\Phi dx dy = 2 \sin b \frac{J_1(f)}{f}$$

where J_1 is the first order Bessel function. Thus, the average Mueller matrix for depolarizer is given by:

$$\bar{M} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 \cos b \frac{J_1(f)}{f} & 2 \sin b \frac{J_1(f)}{f} & 0 \\ 0 & -2 \sin b \frac{J_1(f)}{f} & 2 \cos b \frac{J_1(f)}{f} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

If the value of f is sufficiently large, $J_1(f)/f$ approach zero.

In this case, the average Mueller matrix is

$$\bar{M} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \text{ This is different from the Mueller}$$

$$\text{matrix of ideal depolarizer} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

So, it is not for all kinds of polarization states that the compound quartz depolarizer is effective.

Discussion:

i) if incident light is linearly polarized light

Let θ be the azimuth of incident linearly polarized light. So, for an incident light beam of Stokes vector S , the Stokes vector S' of the emergent light beam is given by

$$S' = \bar{M} S = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 \cos b \frac{J_1(f)}{f} & 2 \sin b \frac{J_1(f)}{f} & 0 \\ 0 & -2 \sin b \frac{J_1(f)}{f} & 2 \cos b \frac{J_1(f)}{f} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ \cos 2\theta \\ \sin 2\theta \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 2 \cos b \cos 2\theta \frac{J_1(f)}{f} + 2 \sin b \sin 2\theta \frac{J_1(f)}{f} \\ -\sin b \cos 2\theta \frac{J_1(f)}{f} + 2 \cos b \sin 2\theta \frac{J_1(f)}{f} \\ 0 \end{pmatrix} = \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} \quad (3)$$

Hence, the degree of depolarization of emergent light is

$$\text{given by } P = \frac{(S_1^2 + S_2^2 + S_3^2)^{1/2}}{S_0} = 2 \left| \frac{J_1(f)}{f} \right| \quad (4)$$

Using equation (4), the curves of degree of polarization of emergent light P versus λ , β and R were obtained as follow

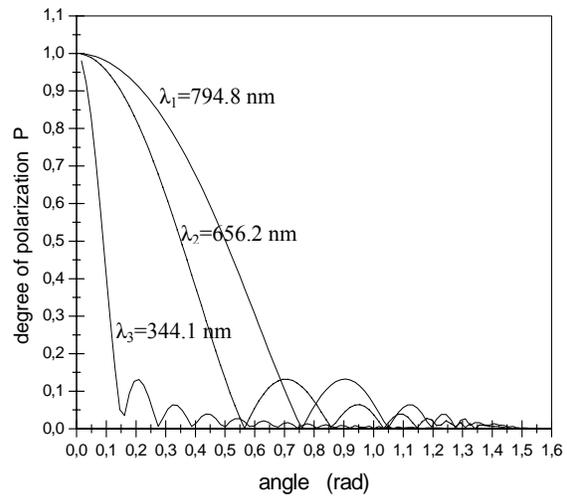


Fig. 2. Relationship between P and β ,

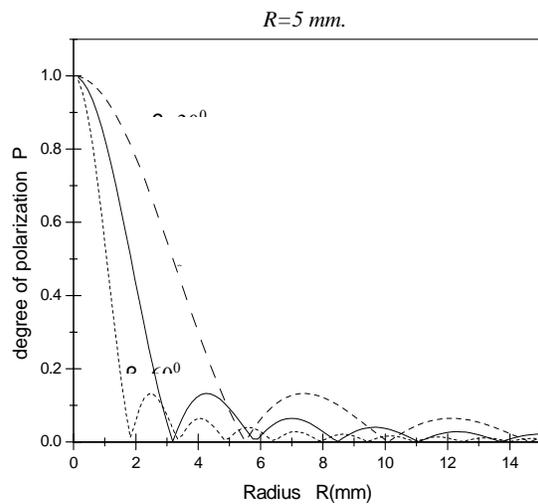


Fig. 3. Relationship between P and R ,

$\lambda = 656.2 \text{ nm}$.

Analysis:

- (1) Fig. 2 shows that the degree of polarization of emergent light surge decreasing with angle β increasing. This change gets slower as the wavelength increases.
- (2) Fig. 3 and Fig. 4 show that the degree of polarization of emergent light surge decreasingly with the radius of incident light beam increasing.
- (3) Fig. 5 shows that the radius required is different when the incident linearly polarized light can be depolarized effectively. The shorter the wavelength is, the smaller the radius of light beam is.

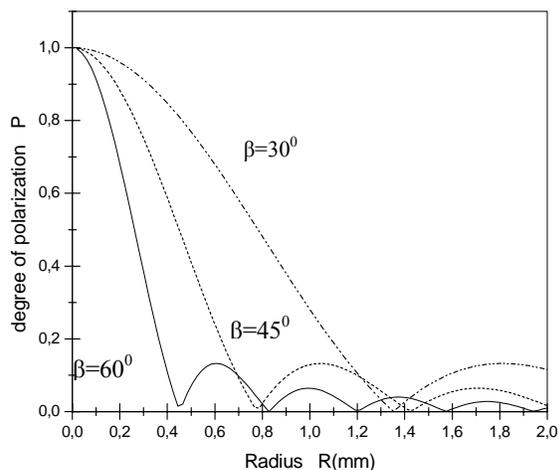
ii) if incident light is circularly polarized light

Fig. 4. Relationship between P and R ,
 $\lambda=344.1 \text{ nm}$.

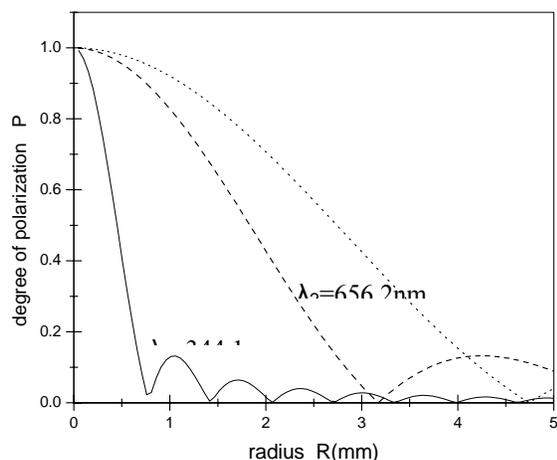


Fig. 5. Relationship between P and R , $\beta=45^\circ$.

The Stokes vector S' of emergent light can also be obtained as

$$S' = \overline{M} \begin{pmatrix} 1 \\ 0 \\ 0 \\ \pm 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 \cos b \frac{J_1(f)}{f} & 2 \sin b \frac{J_1(f)}{f} & 0 \\ 0 & -2 \sin b \frac{J_1(f)}{f} & 2 \cos b \frac{J_1(f)}{f} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ \pm 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \pm 1 \end{pmatrix} \quad (5)$$

The equation (5) shows the compound quartz depolarizer is not effective for circularly polarized light.

3. Conclusion

The compound quartz depolarizer is independent of the azimuth of the incident linearly polarized light and the thickness of quartz wedge. The polarized light for short wavelength is depolarized more easily than the polarized light for long wavelength; the depolarization effect is good when radius R and angle β is large if incident light is linearly polarized light. Thus, the linearly polarized light can be effectively depolarized if only we increase R and angle β properly. In addition, compound quartz depolarizer is not effective for circularly polarized light. Because circularly polarized light represents an homogeneous property similar to unpolarized light in optical system, it will not affect the optical measurement, and then the quartz depolarizer discussed in this paper is not effective for circularly polarized light.

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