Conservation laws for cubic–quartic optical solitons in birefringent fibers with Sasa–Satsuma equation

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This paper presents the conservation laws for optical cubic–quartic solitons in birefringent fibers where the governing model is Sasa–Satsuma equation. The multiplier approach reveals the conserved densities to the model. The conserved quantities are finally computed from these soliton solutions that were reported earlier.

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1. Introduction

One of the key features in soliton transmission is its conservation laws. It says a whole lot about the physical features that stays invariant during its transmission across intercontinental distances unless perturbative effects creep in. These conservation laws are derivable in a number if ways. The usage of Lie symmetry or through the computation of the Lagrangian for the system or even the multiplier approach are some of the commonly studied methods to recover these laws.

The current paper secures the conservation laws for cubic–quartic (CQ) solitons in birefringent fibers that are modeled by Sasa–Satsuma equation (SSE) [1–5]. The concept of CQ solitons was introduced about half a decade ago when low count chromatic dispersion is replaced with third–order dispersion (3OD) and fourth–order dispersion (4OD). The three conservation laws that emerge from the model are computed and exhibited in the paper. SSE was studied by many authors and several solutions were recovered including quiescent solitons for nonlinear chromatic dispersion.

1.1. Governing model

The CQ-SSE in birefringent fibers with Kerr law nonlinearity is written, for the first time, as:

$$iq_t + ia_1 q_{xxx} + b_1 q_{xxxx} + (c_1|q|^2 + d_1|r|^2)q + i\left\{\begin{matrix}(e_1|q|^2 + f_1|r|^2)q_x \\ +[g_1(|q|^2)_x + h_1(|r|^2)_x]q\end{matrix}\right\} = 0, \quad (1)$$

and

$$ir_t + ia_2 r_{xxx} + b_2 r_{xxxx} + (c_2|r|^2 + d_2|q|^2)r + i\left\{\begin{matrix}(e_2|r|^2 + f_2|q|^2)r_x \\ +[g_2(|r|^2)_x + h_2(|q|^2)_x]r\end{matrix}\right\} = 0, \quad (2)$$

where a_j, b_j, c_j, d_j, e_j, e_j, g_j and h_j $(j = 1,2)$ are real valued constants, while $u(x,t)$ and $v(x,t)$ are complex-valued functions representing the wave profile. In Eqs. (1) and (2), the first term is the linear temporal evolution term, a_j and b_j $(j = 1,2)$ are the coefficients of 3OD and 4OD, respectively. Next, c_j $(j = 1,2)$ are the coefficients of Kerr law nonlinearity, d_j $(j = 1,2)$ are the coefficients of cross-phase modulation, e_j and g_j are the self-steepening and the stimulated Raman scattering, while f_j and h_j are the nonlinear terms parameters.

2. Conserved quantities

The soliton pulses in birefringent fibers are given in the form:

$$q(x,t) = A_1 \mathrm{sech}^2[B(x - vt)]e^{i(-\kappa x + \omega t + \theta_0)}, \quad (3)$$

and

$$r(x,t) = A_2 \text{sech}^2[B(x-vt)]e^{i(-\kappa x+\omega t+\theta_0)}. \quad (4)$$

In the system above, we let $q = U + iW$ and $r = V + iZ$ so that the system splits into a system of four pdes whose conserved flows (T^t, T^x) are constructed using the multiplier approach. It turns out that if $f_1 = f_2$ and $h_1 = h_2$, we have a single multiplier $Q = (-u, -v, w, z)$ giving rise to the 'power' conserved density

$$T_P^t = \frac{1}{2}(U^2 + W^2 + V^2 + Z^2), \quad (5)$$

so that a corresponding conserved density of the complex system is

$$\Phi_P^t = |q|^2 + |r|^2. \quad (6)$$

Also, if $g_1 = g_2 = h_1 - h_2 = 0$ and $d_1 = d_2$ we have linear momentum conservation

$$T_m^t = -\left[-\frac{1}{2}V_x Z - \frac{1}{2}U_x W + \frac{1}{2}Z_x V + \frac{1}{2}W_x U\right], \quad (7)$$

and the momentum density is

$$\Phi_M^t = I(q^* q_x) + I(r^* r_x). \quad (8)$$

The conserved density corresponding to energy is obtained only if, in addition to the above restrictions on the parameters, $f_2 = f_1 = 0$, viz.,

$$\Phi_H^t = -\frac{1}{2}\{a_1 I(q^* q_{xxx}) + a_2 I(q^* q_{xxx})\}$$

$$+\frac{1}{2}\{b_1 R(q q_{xxxx}^*) + b_2 R(q q_{xxxx}^*)\}$$

$$+4(c_1|q|^2 + c_2|r|^2) + \frac{1}{2}d_2|q|^2|r|^2$$

$$-\frac{1}{4}\{e_1|q|^2 I(q^* q_x) + e_2|r|^2 I(r^* r_x)\}. \quad (9)$$

Therefore, the conserved quantities are:

$$P = \int_{-\infty}^{\infty} \Phi_P^t dx$$

$$= \int_{-\infty}^{\infty} (|q|^2 + |r|^2)dx = \frac{4}{3B}(A_1^2 + A_2^2), \quad (10)$$

$$M = \int_{-\infty}^{\infty} \Phi_M^t dx$$

$$= \int_{-\infty}^{\infty} \{I(q^* q_x) + I(r^* r_x)\}dx = \frac{4\kappa}{3B}(A_1^2 + A_2^2), \quad (11)$$

and

$$H = \int_{-\infty}^{\infty} \Phi_H^t dx$$

$$= -\frac{2\kappa}{105B}(132B^2 + 35\kappa^2)(a_1 A_1^2 + a_2 A_2^2)$$

$$+\frac{2}{105B}(80B^4 + 264\kappa^2 B^2 + 35\kappa^4)(b_1 A_1^2 + b_2 A_2^2)$$

$$+\frac{16}{3B}(c_1 A_1^2 + c_1 A_2^2) + \frac{16 d_2 A_1^2 A_2^2}{35B} + \frac{8\kappa}{35B}(e_1 A_1^4 + e_2 A_2^4). \quad (12)$$

3. Conclusions

This paper recovers conservation laws for CQ solitons in birefringent fibers that are modeled by SSE. The multiplier method recovers the conserved densities while the soliton solutions, reported earlier, give the conserved quantities from these densities [5]. The results serve as a way to move along with additional avenues to pursue. The conservation laws lead to the dynamics of quasi–stationary solitons that would give way to its adiabatic dynamics due to quasi–monochromaticity. Additional features would be to compute collision–induced timing jitter for the soliton pair in birefringent fibers. These upcoming results will be aligned with the recently reported works [6-15]. Such results are forthcoming.

Disclosure

The authors claim there is no conflict of interest.

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