Coupling coefficient of thermally diffused expanded core fiber couplers

G. S. KLIROS *
Department of Aeronautical Sciences, Division of Electronics and Communication Engineering Hellenic Air-Force Academy, Dekeleia Attica GR-1010, Greece

Fiber couplers are widely used as passive optoelectronic devices. The effective power coupling and transmitting from one fiber core to another is mainly described by both coupling coefficient and beat length. Beat length can be calculated directly from coupling coefficient which depends upon the refractive index profile and fiber core separation. In this communication, we theoretically analyze the properties of thermally diffused expanded core (TEC) fiber couplers. The coupling coefficient and beat length have been calculated using coupled-mode theory. Both the refractive index profile change and wavelength dependence of the coupling properties have been investigated. Our results show that the beat length of TEC-couplers can be reduced to millimeter range by an appropriate selection of refractive index difference, core size expansion due to thermal heating and core separation. Moreover, the coupling coefficient shows a less wavelength dependence in the case of TEC-couplers than in conventional SI-SMF couplers. TEC-couplers have a very short beat length even at large fiber-core separations which is very advantageous for integrated optics applications.

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1. Introduction

Recently, compact-size devices and low-cost packaging have become the trend of development in optical communication, MEMs and sensor systems. Thermally diffused expanded core fibers (TEC) [1]-[3] are regarded as one potential technology to meet this trend. Several applications of TEC fibers have been proposed, mainly in the field of low-loss connection where the expanded fiber core reduces the splice losses between different types of fibers [4, 5]. Moreover, TEC fibers have been used for the design of lens-free devices [6], design of band-edge filters with a high cut-off property or tunable comb-filters [7]-[9] as well as for coupling to silicon waveguide gratings [10]. A TEC fiber has an enlarged mode field diameter (MFD) obtained by heating a single-mode fiber (SMF) locally at a high temperature (~1300−1650 ^oC) and diffusing the germanium dopant into the core [1, 2]. The core expansion rate depends on the heating temperature, the heating time and the dopant intensity in the fiber core. The TEC fiber has the feature that although thermal diffusion changes the refractive-index profile, the normalized frequency does not change after the core expansion because the total number of dopants at a cross-section of the fiber remains constant. The refractive index profile $n(r)$ of the TEC fiber can be successfully approximated by [4, 11]:

$$n^2(r) = n_{cl}^2 + \frac{\alpha^2}{A^2}\left(n_{co}^2 - n_{cl}^2\right)e^{-\frac{r^2}{A^2}} \qquad (1)$$

where A is the core radius of the fiber after the heat treatment, n_{co} and n_{cl} are the refractive indexes of the core and the cladding of a step-index fiber, before the heat treatment, respectively. The expanded core radius is given in terms of diffusion coefficient of the dopant D and the heating time t as $A = 2\sqrt{Dt}$.

Optical fiber couplers are passive devices widely used as tunable filters, switches, and power dividers in the field of optical communication as well as in fiber sensor and imaging systems [12-14]. Many useful devices based on a symmetric two touching fiber system can be realized, such as directional couplers, wavelength division multiplexers and polarization beam splitters. In this case, light couples back and forth between the two cores as it propagates along the coupling region. Under suitable conditions, all of the power in one fiber will couple to the other. But once the light has crossed over, the wave couples back into the first fiber so that power is changed continuously as often as coupling region permits. Physically, this is due to the difference between propagation constants of the symmetric and anti-symmetric modes of such a directional coupler, which results in a beating phenomenon, making the electromagnetic wave to oscillate from one fiber to the other. The power transfer can be determined by the coupling coefficient C, which is defined as half the difference between the propagation constants of the symmetric and the anti-symmetric modes. One can also define the beat length L_c as the distance along the fiber-system in which there is total transfer of power from one fiber to the other and back again. In conventional fibers, coupled-mode theory has been systematically analyzed and mode coupling in conventional two-core structures has been studied in detail [15]-[17]. The coupling coefficient C_{lm} for a two single mode fiber system is given by [15]

$$C_{lm} = \frac{k}{4}\sqrt{\frac{\varepsilon_0}{N_l N_m \mu_0}}\int_{A_\infty}\left(n^2 - n_l^2\right)\Psi_l \Psi_m \, dA \quad \text{where} \quad l,m = 1,2 \qquad (2)$$

where k=2π/λ is the free-space propagation constant, $n^2 - n_l^2$ is the difference between the dielectric constants of the fiber core (l) and its cladding, Ψ_l is the transverse electric field profile in fiber core l in the absence of core m and Ψ_m is the transverse electric field profile in fiber core m in the absence of core l. Also, N_l, N_m are the corresponding normalization constants. For single-mode fiber couplers in which the two fibers are identical, the coupling coefficients are equal, i.e., $C_{12} = C_{21}$. As is well known, for two coupled step index single-mode fibers (SI-SMF), Eq. (2) leads to the following expression for the coupling coefficient [17,18]:

$$C = \frac{(2\Delta)^{1/2}}{\alpha} \frac{U^2}{V^3} \frac{K_o(Wd/\alpha)}{K_1^{\,2}(W)} \tag{3}$$

where d is the separation between the two cores, α is the radius of the cores, V is the normalized frequency which is given by: $V = k\alpha n_{co}(2\Delta)^{1/2}$, where $\Delta = (n_{co}^2 - n_{cl}^2)/2n_{co}^2$ is the relative refractive index difference between core and cladding with n_{co} and n_{cl} being the refractive index of the core and the cladding, respectively. The modal parameter U is a solution of the characteristic equation

$$\frac{J_1(Ua)}{UJ_0(Ua)} + \frac{K_1(Wa)}{WK_0(Wa)} = 0 \tag{4}$$

where $W = \sqrt{(ka)^2(n_{co}^2 - n_{cl}^2) - U^2}$ and J_0, J_1 are the zeroth and first order Bessel functions of the first kind, and K_0 and K_1 are the zeroth and first order modified Bessel functions of the second kind, respectively. It is obvious that fiber core-to-core separation d, relative refractive index difference Δ and operational wavelength are three principle parameters to control the coupling coefficient.

In this paper, we present an analytic calculation of the coupling coefficient and beat length for a coupler consisting of two thermally expanded core fibers, both of the same diameters, held in close proximity. The dependence of the coupling coefficient and beat length on the thermal diffusion time, the refractive index difference as well as the operation wavelength is investigated. Our results are compared with the corresponding ones for conventional step-index single mode fiber couplers.

2. Thermally diffused expanded core fiber couplers

We consider the simplest form of a thermally diffused expanded core fiber coupler (TEC-coupler) consisting of two uniform parallel thermally expanded core fibers both of the same diameter, held in close proximity as it is shown in Fig. 1. The two cores contain GeO_2 dopants which are thermally diffused into the cladding changing the refractive index profile of the two-fiber system. Using Eq. (1) and the two-fiber geometry, the refractive index profile of the TEC-coupler can be approximated as follows:

$$\begin{aligned} n^2(r) &= n_{co}^2\left[1 - 2\Delta\left(1 - \frac{a^2}{A^2}e^{-r_1^2/A^2} - \frac{a^2}{A^2}e^{-r_2^2/A^2}\right)\right] \\ &\cong n_{co}^2\left[(1-2\Delta) + \left(2\Delta\frac{a^2}{A^2}\right)e^{-r_2^2/A^2}\right] \end{aligned} \tag{5}$$

where r_1 and r_2 are the radial distances from each fiber core centre, $A = 2\sqrt{Dt}$ is the expanded core of each fiber and D= 3.9 ×10^{-16} m^2 s^{-1} the diffusion constant of GeO_2 [4]. Fig. 2 illustrates the refractive index profile of the TEC-coupler (a) for different heating times and (b) for different core separations d/α when the heating time is 10 h. The parameters of the standard SI-SMF before heat treatment are: relative refractive index difference Δ = 0.3%, refractive index of the cladding n_{cl} = 1.46 and core radius α = 4 μm.

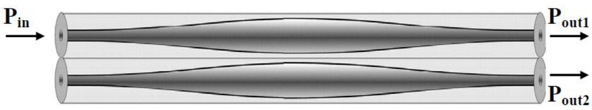
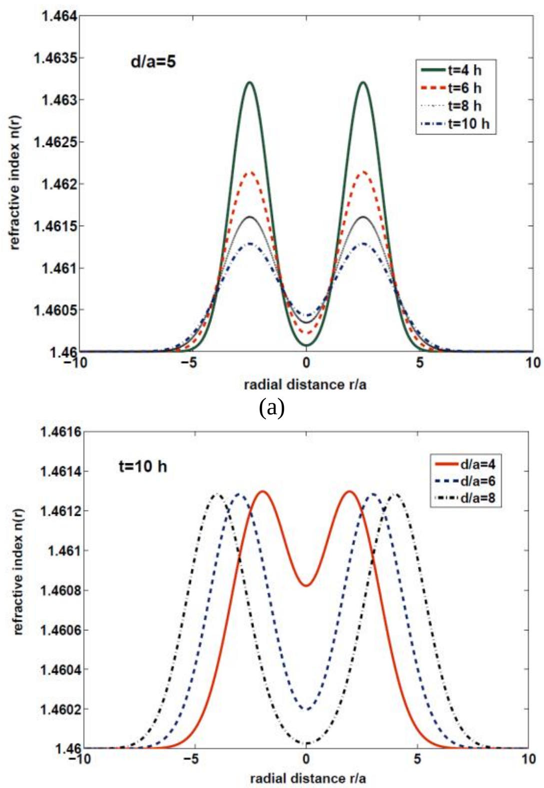


Fig. 1. Schematic illustration of a thermally diffused expanded core fiber coupler.



(a)

(b)

Fig. 2. Refractive index profiles of a TEC-coupler (a) for different heating times (b) for different increasing core separations. The parameters of the standard SI-SMF before heat treatment are: relative refractive index difference Δ = 0.3%, refractive index of the cladding n_{cl} = 1.46 and core radius a = 4 μm.

Application of coupled mode theory leads to the coupling coefficient in terms of the field overlap integral, which is a measure of the interaction between the individual single fiber-core modes (see Eq. 2). In a single mode weakly guiding circular fiber, the electromagnetic fields are transverse to the propagation direction and a radial function Ψ(r) completely describes the spatial variation of the fields in a plane perpendicular to fiber axis. In our case, Eq. (2) can be re-written as

$$C=\frac{k}{4N_1}\sqrt{\frac{\varepsilon_0}{\mu_0}}\int_{A_\infty}\left(n^2(r)-n_{co}^2\right)\Psi_1\Psi_2 dA \qquad (6)$$

where n(r) is given by Eq.(5) and the functions Ψ_1 and Ψ_2 are given by the radial part of the transverse electric field of the single fiber mode centered on the core (1) or core (2) respectively. Using Eqs. (5) and (6) we obtain

$$C=\frac{k}{4N_1}\sqrt{\frac{\varepsilon_0}{\mu_0}}\int_0^{2\pi}\int_0^{\infty}n_{co}^2\left(2\Delta\alpha^2 e^{-R_2^2}-1\right)\Psi_1(R_1)\Psi_2(R_2)R_2dR_2d\phi_2 \qquad (7)$$

where $R_1=r_1/A$, $R_2=r_2/A$, and

$$N_1=\frac{\pi a^2}{2}n_{co}\sqrt{\frac{\varepsilon_0}{\mu_0}}\left(\frac{1}{V-1}\right)$$

Taking as $\Psi_1(R)$ the modal field in the core of the individual TEC fiber and as $\Psi_2(R)$ the evanescent field [4], i.e.,

$$\Psi_1(R)=\exp\left(-\frac{(V-1)}{2A}R^2\right) \qquad (8)$$

$$\Psi_2(R)=\frac{V^2}{V+1}K_0\left[(V-1)R\right]\exp\left(\frac{(V-1)^2}{2(V+1)}\right) \qquad (9)$$

Eq. (7) leads to the following expression for the coupling coefficient:

$$C=2\pi C_0\frac{V^2}{V+1}\exp\left(\frac{(V-1)^2}{2(V+1)}\right)\int_0^{\infty}K_0\left[(V-1)R_2\right]\exp\left\{\left(-\frac{V+1}{2}\right)R_2^2\right\}R_2dR_2d\phi_2 \qquad (10)$$

where $C_0=\frac{k}{4N_1}\sqrt{\frac{\varepsilon_0}{\mu_0}}\left(\alpha^2 2\Delta\right)n_{co}^2$

Using the well-known formula [19]:

$$K_0(\alpha r_1)\cos(m\phi_1)=\sum_{p=-\infty}^{+\infty}\mathrm{sgn}(p)K_{p+m}(\alpha d)I_p(\alpha r_2)\cos p\phi_2 \qquad (11)$$

and, after performing the integration over ϕ_2, Eq. (10) takes the form

$$C=2\pi C_0\frac{V^2}{V+1}\exp\left(\frac{(V-1)^2}{2(V+1)}\right)K_0\left(\frac{(V-1)d}{A}\right)\times$$
$$\times\int_0^{\infty}I_0\left[(V-1)R_2\right]\exp\left\{\left(-\frac{V+1}{2}\right)R_2^2\right\}R_2dR_2 \qquad (12)$$

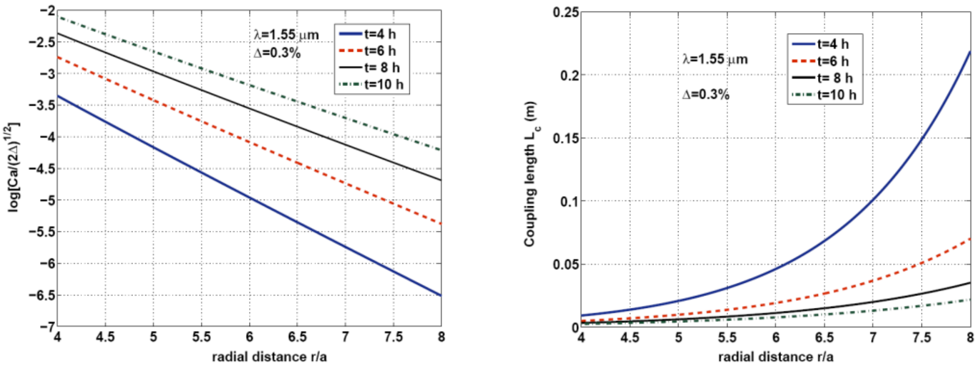
Finally, the integral over R_2 can be calculated in closed form and Eq. (12) leads to the following result

$$C=\frac{\sqrt{2\Delta}}{a}\frac{V^3(V-1)}{(V+1)^2}K_0\left(\frac{(V-1)d}{A}\right)\exp\left(\frac{(V-1)^2}{(V+1)}\right) \qquad (13)$$

which is valid for TEC fibers with normalized frequencies 1<V<2.405 in order to avoid propagation of the next higher-order modes. Consequently, the beat length L_c that is, the distance for complete transfer of the optical power from the input expanded core to the adjacent expanded core and back, can be calculated as $L_c=2\pi/C$. From the above equations it is apparent that both coupling coefficient and beat length are dependent on the core-separation distance, normalized frequency and thermal treatment duration.

3. Results and discussion

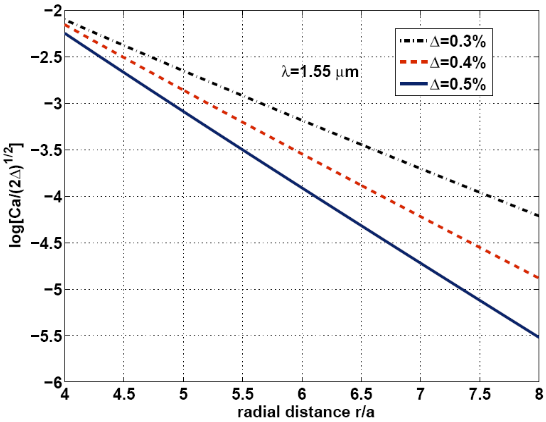
To assess the performance of the TEC fiber coupler for applications in integrated optoelectronic devices, numerical results based on the formulation developed in the previous section are presented. Figs. 3 (a) and (b) shows the variation of normalized coupling coefficient $Ca/\sqrt{2\Delta}$ and beat length L_c of TEC-couplers with respect to fiber-core separation distance d/α for different heating times. As the duration of heat treatment increases, the coupling becomes stronger and the beat length reduces drastically. After 10 h of heat treatment, the beat length can be less than 20 mm in the overall range 4<d/α<8.



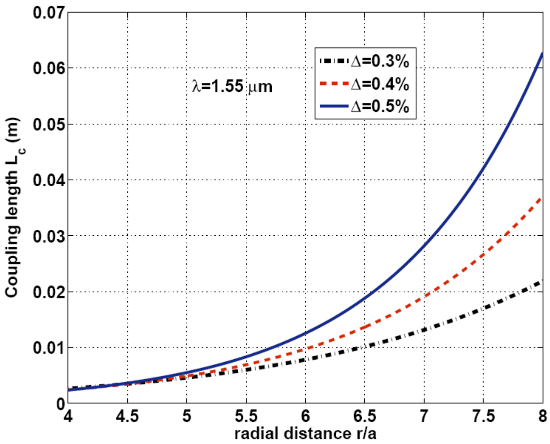
(a) (b)

Fig. 3. Normalized coupling coefficient (a) and beat length (b) of TEC-couplers as a function of radial fiber separation for different heating times.

Figs. 4(a) and (b) show the coupling coefficient and beat length respectively versus fiber-core separation for three increasing values of the relative refractive index change Δ. As it is seen, the beat length is affected by the relative refractive index change Δ for large fiber-core separations d/α >5. Obviously, beat length is mainly affected by fiber separation rather than the refractive index change Δ. For a given fiber-core spacing, the beat length increases with refractive index change and decreases with operating wavelength due to the fact that the field extends deeper into the 'cladding' region of each TEC fiber. The wavelength dependence of coupling properties is illustrated in Figs. 5(a) and (b). Increasing the operating wavelength, coupling becomes stronger and beat length decreases mainly in large fiber-core separations.

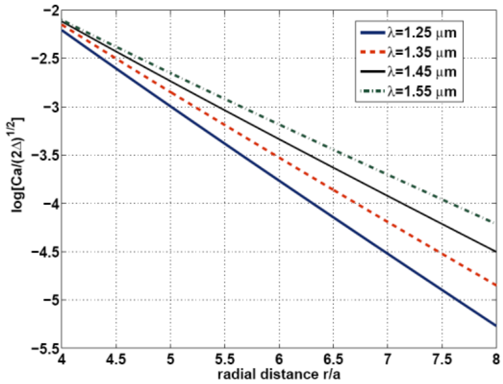


(a)

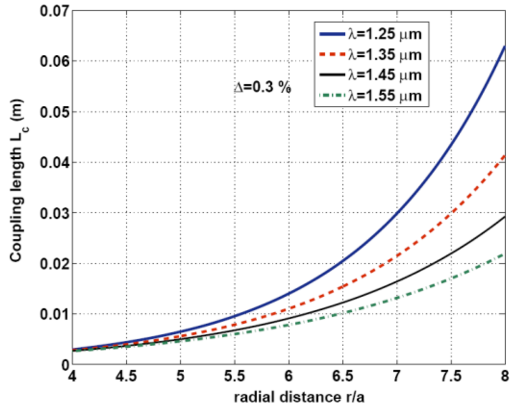


(b)

Fig. 4. Normalized coupling coefficient (a) and beat length (b) of TEC-couplers as a function of radial fiber separation for different relative refractive index changes.



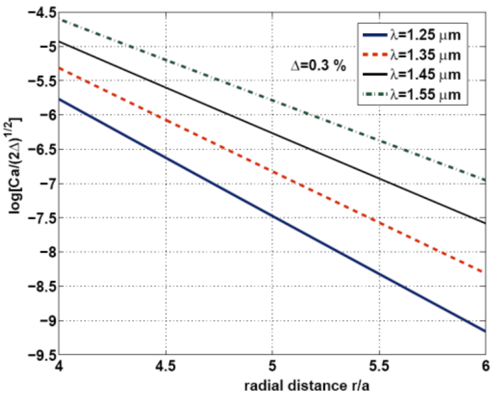
(a)



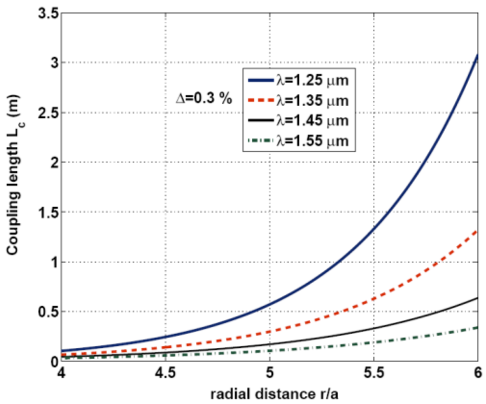
(b)

Fig. 5. Normalized coupling coefficient (a) and beat length (b) of TEC-couplers as a function of radial fiber separation for different wavelengths.

In order to compare the coupling properties of TEC-couplers with those of SI-SMF couplers, we calculate both coupling coefficient and beat length of SI-SMF couplers versus separation distance d/α, for different operating wavelengths, using Eqs. (3) and (4). The parameters of the standard SI-SMF before heat treatment have been used: relative refractive index difference Δ = 0.3%, refractive index of the cladding n_{cl} = 1.46 and core radius α = 4 μm. The results are presented in Figs. 6 (a)-(b). Comparing Figs. (5) and (6), we observe that TEC-couplers have a very short beat length even at large core separations which is very advantageous for integrated optics applications. The coupling coefficient in SI-SMF couplers is strongly wavelength dependent in agreement to previous calculations [20]. As the wavelength decreases, the normalized frequency V is increased and the fields become more tightly confined to the core regions and as a consequence are less influenced by the adjacent fiber core. On the other hand, spectral response of TEC-couplers shows a less wavelength dependence than SI-SMF couplers due to core expansion after long time of thermal treatment.



(a)



(b)

Fig. 6. Normalized coupling coefficient coefficient (a) and beat length (b) of SI-SMF as a function of radial fiber separation for different wavelengths.

4. Conclusions

In conclusion, we have theoretically analyzed the properties of thermally diffused expanded core (TEC) fiber couplers. The coupling coefficient and beat length have been calculated using coupled-mode theory which is accurate for fiber-core separations $d/a \geq 4$ and normalized frequencies $V > 1$. Both the relative refractive index change and operation wavelength dependence of the coupling properties have been investigated. Our results show that the beat length of TEC-couplers can be reduced to millimeter range by an appropriate selection of refractive index difference, core size expansion due to thermal heating, and core separation. The coupling coefficient should be strongly wavelength dependent, however, spectral response shows a less wavelength dependence in the case of TEC-couplers than in conventional SI-SMF couplers. In general TEC-couplers have a very short beat length even at large core separations which is very advantageous for integrated optics applications as tunable filters, switches, power dividers. Moreover, since the beat length is sensitive to temperature or strain, TEC-couplers could be used for designing temperature or strain sensors.

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*Corresponding author: gsksma@hol.gr