OPTOELECTRONICS AND ADVANCED MATERIALS – RAPID COMMUNICATIONS Vol. 16, No. 7-8, July-August 2022, p. 342 - 347

Cubic–quartic optical soliton perturbation with Chen–Lee–Liu equation by sine-Gordon equation approach

YAKUP YILDIRIM[1], ANJAN BISWAS[2,3,4,5,6,*], HASHIM M. ALSHEHRI[3], MILIVOJ R. BELIC[7]
[1]*Department of Computer Engineering, Biruni University, 34010 Istanbul, Turkey*
[2]*Department of Mathematics and Physics, Grambling State University, Grambling, LA—71245, USA*
[3]*Mathematical Modeling and Applied Computation (MMAC) Research Group, Department of Mathematics, King Abdulaziz University, Jeddah–21589, Saudi Arabia*
[4]*Department of Applied Sciences, Cross-Border Faculty, Dunarea de Jos University of Galati, 111 Domneasca Street, Galati-800201, Romania*
[5]*Department of Mathematics and Applied Mathematics, Sefako Makgatho Health Sciences University, Medunsa–0204, Pretoria, South Africa*
[6]*Department of Applied Mathematics, National Research Nuclear University, 31 Kashirskoe Hwy, Moscow–115409, Russian Federation*
[7]*Institute of Physics Belgrade, Pregrevica 118, 11080 Zemun, Serbia*

This paper addresses a modified version of the regular known Chen–Lee–Liu equation where the familiar chromatic dispersion is replaced by a collective count of third–order dispersion as well as fourth–order dispersion effects to compensate the low count of chromatic dispersion. The model is studied for scalar case as well as birefringent fibers for the first time. Hamiltonian type perturbation terms are considered with maximum intensity. Bright, dark and singular solitons with the newly formulated model are extracted by the aid of sine–Gordon equation approach for the first time in the field of nonlinear optics.

(Received January 20, 2022; accepted August 10, 2022)

Keywords: Cubic-quartic, Perturbation, Chen–Lee–Liu equation, Optical solitons

1. Introduction

There are a wide variety of models that govern the propagation of solitons through optical fibers, metamaterials and other such form of waveguides. The most viable one is the nonlinear Schrödinger's equation (NLSE). This model has been extensively studied for several forms of nonlinear refractive index structures. There are three forms of equations that emerge from this model and are referred to as derivative NLSE and are sequentially referred to as DNLSE–I, DNLSE–II and DNLSE–III. These models are mostly studied in the context of Plasma Physics but they can be treated as a viable model to handle optical solitons as well [1–5]. The model that will be studied in the current paper is Chen–Lee–Liu (CLL) equation and is occasionally referred to as DNLSE–II equation. A plethora of pre-existing work has been already reported in regards to this model.

Optical solitons fail to exist when the balance between chromatic dispersion (CD) and self–phase modulation (SPM) is compromised or CD runs low. In these cases, mechanisms have been reported to overcome this crisis situation. One of these mechanisms is the introduction of third–order dispersion (3OD) that leads to the formation of Schrödinger–Hirota equation. Another mechanism is the introduction of Bragg gratings that introduce dispersive reflectivity and compensate this low count of CD. Also, cubic-quartic (CQ) solitons have been introduced, where CD is completely discarded and is replaced by third-order dispersion (3OD) and fourth-order dispersion (4OD). Therefore, today's study is a modified version of the regular known CLL equation where the familiar CD is replaced by a collective count of 3OD as well as 4OD effects. Together, 3OD and 4OD compensate for the low count of CD. This leads to the formulation of CQ version of CLL equation which will be the focus of the current paper. There are a few perturbation terms that are also taken into account which appear with maximum intensity. The model will be studied for scalar case as well as for birefringent fibers by the aid of sine–Gordon equation approach for the first time in the field of nonlinear optics. The soliton solutions are thus recovered and classified. The details are furnished in the rest of the work.

2. Scalar model

The CQ-CLL equation with nonlinear perturbation terms in polarization preserving reads as:

$$iu_t + iau_{xxx} + bu_{xxxx} + ic|u|^2u_x = i[\alpha u_x + \lambda(|u|^{2m}u)_x + \mu(|u|^{2m})_x u], \quad (1)$$

where a, b, c, α, λ and μ are real-valued constants. The complex valued function $u(x,t)$ denotes optical soliton in polarization-preserving fibers. x and t are the non-dimensional distance and time in dimensionless form, respectively. The first term is linear temporal evolution and $i=\sqrt{-1}$. The constants a and b imply to the coefficients of 3OD and 4OD, respectively. The coefficient c constitutes nonlinear dispersion term. Lastly, α, λ and μ are the coefficients of inter-modal dispersion, self-steepening term and higher-order dispersion, respectively.

To obtain optical solitons in polarization-preserving fibers with the perturbed CQ-CLL equation (1), we consider the traveling wave solution

$$u(x,t)=U(\xi)e^{i\varphi(x,t)},$$

$$\varphi(x,t)=-\kappa x+\omega t+\theta_0,$$

$$\xi=x-vt. \tag{2}$$

Here κ, v, ω and θ_0 are real-valued constants. ξ is the wave variable, κ is the frequency, v is the velocity, ω is the wave number and θ_0 is the phase constant. Also, the real-valued function $U(\xi)$ stands for the amplitude component of the soliton and the function $\varphi(x,t)$ is referred to as the phase component of the soliton.

Inserting the traveling wave solution (2) into the model equation (1), we arrive at the real part

$$bU^{(iv)}+6b\kappa^2U''-(\omega+3b\kappa^4+\alpha\kappa)U$$

$$+(c\kappa-\kappa\lambda)U^3=0, \tag{3}$$

by the usage of the constraint conditions

$$m=1, \tag{4}$$

$$a=4b\kappa, \tag{5}$$

$$c-2\mu-3\lambda=0, \tag{6}$$

and the velocity

$$v=-\alpha-8b\kappa^3. \tag{7}$$

The adopted integration scheme that is the sine-Gordon equation algorithm presumes the formal solution [6-9]

$$U(\xi)=\sum_{k=1}^{N}[A_k\cos(V(\xi))$$

$$+B_k\sin(V(\xi))]\cos^{k-1}(V(\xi))+A_0, \tag{8}$$

by the aid of the ordinary differential equation (ODE)

$$V'(\xi)=\sin(V(\xi)), \tag{9}$$

and the analytical solutions

$$\sin(V(\xi))=\mathrm{icsh}(\xi),$$

$$\sin(V(\xi))=\mathrm{sech}(\xi),$$

$$\cos(V(\xi))=\coth(\xi),$$

$$\cos(V(\xi))=\tanh(\xi), \tag{10}$$

where A_k and B_k $(k=1,2,\cdots,N)$ are real-valued constants, while N is the positive valued integer number that is retrieved by the usage of the balance principle.

Balacing $U^{(iv)}$ with U^3 yields $N=2$. Thus, Eq. (8) changes to the solution

$$U(\xi)=A_0+A_1\cos(V(\xi))+B_1\sin(V(\xi))$$

$$+\cos(V(\xi))\begin{pmatrix}A_2\cos(V(\xi))\\+B_2\sin(V(\xi))\end{pmatrix}. \tag{11}$$

Plugging the solution (11) along with the ODE (9) into the equation (3), we arrive the results:

Case-1:

$$\kappa=\pm\frac{\sqrt{15}}{3},\omega=-\alpha\kappa+\frac{8b}{3},A_1=0,$$

$$A_0=0,A_2=0,B_1=0,B_2=\pm6\sqrt{\frac{2\kappa b}{c-\lambda}}. \tag{12}$$

Inserting the coefficients (12) together with the analytical solutions (10) into the solution (11), one retrieves

$$u(x,t)=\pm6\sqrt{\frac{2\kappa b}{c-\lambda}}\tanh(x+(\alpha+8b\kappa^3)t)$$

$$\times\,\mathrm{sech}(x+(\alpha+8b\kappa^3)t)e^{i(-\kappa x+\omega t+\theta_0)}, \tag{13}$$

and

$$u(x,t)=\pm6\sqrt{-\frac{2\kappa b}{c-\lambda}}\coth(x+(\alpha+8b\kappa^3)t)$$

$$\times\,\mathrm{csch}(x+(\alpha+8b\kappa^3)t)e^{i(-\kappa x+\omega t+\theta_0)}. \tag{14}$$

Eq. (13) signifies combo dark-bright soliton by virtue of the constraint

$$\kappa b(c-\lambda)>0,$$

while Eq. (14) corresponds to combo singular soliton with the help of the condition

$$\kappa b(c-\lambda)<0.$$

Case-2:

$$\omega = -22b\kappa^2 - \alpha\kappa + \frac{16b}{3},$$

$$A_0 = \pm\frac{3\kappa^2 - 20}{10}\sqrt{\frac{b\kappa(3\kappa^2 - 10)}{c - \lambda}}, A_1 = 0,$$

$$B_1 = 0, A_2 = \pm 3\sqrt{\frac{b\kappa(3\kappa^2 - 10)}{c - \lambda}},$$

$$B_2 = 0, 9\kappa^4 - 30\kappa^2 - 40 = 0. \tag{15}$$

Plugging the parameters (15) along with the analytical solutions (10) into the solution (11) yields

$$u(x,t) = 3\sqrt{\frac{b\kappa(3\kappa^2 - 10)}{c - \lambda}} \times \left\{\frac{3\kappa^2 - 50}{30} + \operatorname{sech}^2(x + (\alpha + 8b\kappa^3)t)\right\} \times e^{i(-\kappa x + \omega t + \theta_0)}, \tag{16}$$

and

$$u(x,t) = 3\sqrt{\frac{b\kappa(3\kappa^2 - 10)}{c - \lambda}} \times \left\{\frac{3\kappa^2 - 50}{30} - \operatorname{csch}^2(x + (\alpha + 8b\kappa^3)t)\right\} \times e^{i(-\kappa x + \omega t + \theta_0)}. \tag{17}$$

Eq. (16) signifies bright soliton, while Eq. (17) represents singular soliton by the aid of the constraint

$$b\kappa(3\kappa^2 - 10)(c - \lambda) > 0.$$

Case-3:

$$\omega = -\frac{11b\kappa^2}{2} - \alpha\kappa + \frac{b}{3},$$

$$A_0 = \pm\frac{6\kappa^2 - 25}{10}\sqrt{\frac{b\kappa(12\kappa^2 - 10)}{c - \lambda}}, A_1 = 0,$$

$$B_1 = 0, A_2 = \pm 3\sqrt{\frac{b\kappa(12\kappa^2 - 10)}{c - \lambda}},$$

$$B_2 = \pm 3\sqrt{-\frac{b\kappa(12\kappa^2 - 10)}{c - \lambda}},$$

$$18\kappa^4 - 15\kappa^2 - 5 = 0. \tag{18}$$

Substituting the parameters (18) together with the analytical solutions (10) into the solution (11) leads to

$$u(x,t) = 3\sqrt{\frac{b\kappa(12\kappa^2 - 10)}{c - \lambda}} \times \left\{\begin{array}{c} \frac{6\kappa^2 - 55}{30} - \operatorname{csch}^2(x + (\alpha + 8b\kappa^3)t) \\ +\coth(x + (\alpha + 8b\kappa^3)t) \\ \times \operatorname{csch}(x + (\alpha + 8b\kappa^3)t) \end{array}\right\} \times e^{i(-\kappa x + \omega t + \theta_0)}. \tag{19}$$

Eq. (19) stands for combo singular soliton by virtue of the condition

$$b\kappa(12\kappa^2 - 10)(c - \lambda) > 0.$$

3. Birefringent fibers

The CQ-CLL equation with nonlinear perturbation terms in birefringent fibers reads as:

$$iq_t + ia_1 q_{xxx} + b_1 q_{xxxx} + i(c_1|q|^2 + d_1|r|^2)q_x = i[\alpha_1 q_x + \lambda_1(|q|^2 q)_x + \mu_1(|q|^2)_x q], \tag{20}$$

and

$$ir_t + ia_2 r_{xxx} + b_2 r_{xxxx} + i(c_2|r|^2 + d_2|q|^2)r_x = i[\alpha_2 r_x + \lambda_2(|r|^2 r)_x + \mu_2(|r|^2)_x r], \tag{21}$$

where $q(x,t)$ and $r(x,t)$ are the complex valued functions that are referred to as solitons in birefringent fibers. a_l and b_l $(l = 1, 2)$ purport the coefficients of 3OD and 4OD, respectively. c_l represent the coefficients of self-phase modulation (SPM), while d_l correspond to the coefficients of cross-phase modulation (XPM). Lastly, α_l, λ_l and μ_l signify the coefficients of inter-modal dispersion, self-steepening term and higher-order dispersion, respectively.

To obtain optical solitons with the perturbed CQ-CLL equation in birefringent fibers, we address the traveling wave solutions

$$q(x,t) = U_1(\xi)e^{i\varphi(x,t)},$$

$$r(x,t) = U_2(\xi)e^{i\varphi(x,t)},$$

$$\varphi(x,t) = -\kappa x + \omega t + \theta_0,$$

$$\xi = x - vt. \tag{22}$$

Inserting the traveling wave solutions (22) into the model equations (20) and (21), the real parts are yielded by

$$b_l U_l^{(iv)} + (3\kappa a_l - 6\kappa^2 b_l)U_l''$$
$$+(\kappa^4 b_l - \omega - \kappa^3 a_l - \kappa\alpha_l)U_l$$
$$+(\kappa c_l - \kappa\lambda_l)U_l^3 + \kappa d_l U_l U_{\tilde{l}}^2 = 0, \quad (23)$$

while the imaginary parts are retrieved by virtue of

$$(a_l - 4\kappa b_l)U_l'''$$
$$+(4\kappa^3 b_l - v - 3\kappa^2 a_l - \alpha_l)U_l'$$
$$+(c_l - 3\lambda_l - 2\mu_l)U_l^2 U_l' + d_l U_{\tilde{l}}^2 U_l' = 0, \quad (24)$$

where $\tilde{l} = 3 - l$ and $l = 1, 2$. Eqs. (23) and (24) turn into

$$b_l U_l^{(iv)} + 6\kappa^2 b_l U_l'' - (\omega + 3\kappa^4 b_l + \kappa\alpha_l)U_l$$
$$+(\kappa c_l - \kappa\lambda_l + \kappa d_l)U_l^3 = 0, \quad (25)$$

with the help of the constraints

$$U_{\tilde{l}} = U_l, \quad (26)$$

$$a_l = 4\kappa b_l, \quad (27)$$

$$c_l - 3\lambda_l - 2\mu_l + d_l = 0, \quad (28)$$

and the velocity

$$v = -\alpha_l - 8\kappa^3 b_l. \quad (29)$$

Using the balancing principle in Eq. (25), Eq. (8) becomes the solution

$$U_l(\xi) = A_0 + B_1 \sin(V_l(\xi)) + A_1 \cos(V_l(\xi))$$
$$+\cos(V_l(\xi))\begin{pmatrix} B_2 \sin(V_l(\xi)) \\ +A_2 \cos(V_l(\xi)) \end{pmatrix}. \quad (30)$$

Plugging the solution (30) together with the ODE (9) into the equation (25), one retrieves the results:

Case-1:

$$\kappa = \pm\frac{\sqrt{15}}{3}, \omega = \frac{8\kappa b_l - 5\alpha_l}{3\kappa}, A_1 = 0,$$

$$A_0 = 0, A_2 = 0, B_1 = 0, B_2 = \pm 6\sqrt{\frac{2\kappa b_l}{c_l + d_l - \lambda_l}}. \quad (31)$$

Inserting the coefficients (31) together with the analytical solutions (10) into the solution (30), we arrive

$$q(x,t) = \pm 6\sqrt{\frac{2\kappa b_1}{c_1 + d_1 - \lambda_1}}$$
$$\times \tanh(x + (\alpha_1 + 8\kappa^3 b_1)t)$$
$$\times \operatorname{sech}(x + (\alpha_1 + 8\kappa^3 b_1)t)$$
$$\times e^{i(-\kappa x + \omega t + \theta_0)}, \quad (32)$$

$$r(x,t) = \pm 6\sqrt{\frac{2\kappa b_2}{c_2 + d_2 - \lambda_2}}$$
$$\times \tanh(x + (\alpha_2 + 8\kappa^3 b_2)t)$$
$$\times \operatorname{sech}(x + (\alpha_2 + 8\kappa^3 b_2)t)$$
$$\times e^{i(-\kappa x + \omega t + \theta_0)}, \quad (33)$$

$$q(x,t) = \pm 6\sqrt{-\frac{2\kappa b_1}{c_1 + d_1 - \lambda_1}}$$
$$\times \coth(x + (\alpha_1 + 8\kappa^3 b_1)t)$$
$$\times \operatorname{csch}(x + (\alpha_1 + 8\kappa^3 b_1)t)$$
$$\times e^{i(-\kappa x + \omega t + \theta_0)}, \quad (34)$$

$$r(x,t) = \pm 6\sqrt{-\frac{2\kappa b_2}{c_2 + d_2 - \lambda_2}}$$
$$\times \coth(x + (\alpha_2 + 8\kappa^3 b_2)t)$$
$$\times \operatorname{csch}(x + (\alpha_2 + 8\kappa^3 b_2)t)$$
$$\times e^{i(-\kappa x + \omega t + \theta_0)}. \quad (35)$$

Eqs. (32) and (33) signify combo dark-bright solitons by the aid of the constraint

$$\kappa b_l(c_l + d_l - \lambda_l) > 0,$$

while Eqs. (34) and (35) represent combo singular solitons with the help of the condition

$$\kappa b_l(c_l + d_l - \lambda_l) < 0.$$

Case-2:

$$\omega = -22\kappa^2 b_l - \kappa\alpha_l + \frac{16 b_l}{3}, A_1 = 0,$$

$$B_1 = 0, 9\kappa^4 - 30\kappa^2 - 40 = 0,$$

$$A_0 = \pm\frac{3\kappa^2 - 20}{15}\sqrt{-\frac{30 b_l}{\kappa(c_l + d_l - \lambda_l)}},$$

$$A_2 = \pm 2\sqrt{-\frac{30 b_l}{\kappa(c_l + d_l - \lambda_l)}},\ B_2 = 0. \quad (36)$$

Inserting the coefficients (36) together with the explicit solutions (10) into the solution (30), one recovers

$$q(x,t) = 2\sqrt{-\frac{30b_1}{\kappa(c_1+d_1-\lambda_1)}}$$

$$\times\left\{\frac{3\kappa^2-50}{30}+\operatorname{sech}^2(x+(\alpha_1+8\kappa^3 b_1)t)\right\}$$

$$\times e^{i(-\kappa x+\omega t+\theta_0)}, \tag{37}$$

$$r(x,t) = 2\sqrt{-\frac{30b_2}{\kappa(c_2+d_2-\lambda_2)}}$$

$$\times\left\{\frac{3\kappa^2-50}{30}+\operatorname{sech}^2(x+(\alpha_2+8\kappa^3 b_2)t)\right\}$$

$$\times e^{i(-\kappa x+\omega t+\theta_0)}, \tag{38}$$

$$q(x,t) = 2\sqrt{-\frac{30b_1}{\kappa(c_1+d_1-\lambda_1)}}$$

$$\times\left\{\frac{3\kappa^2-50}{30}-\operatorname{csch}^2(x+(\alpha_1+8\kappa^3 b_1)t)\right\}$$

$$\times e^{i(-\kappa x+\omega t+\theta_0)}, \tag{39}$$

$$r(x,t) = 2\sqrt{-\frac{30b_2}{\kappa(c_2+d_2-\lambda_2)}}$$

$$\times\left\{\frac{3\kappa^2-50}{30}-\operatorname{csch}^2(x+(\alpha_2+8\kappa^3 b_2)t)\right\}$$

$$\times e^{i(-\kappa x+\omega t+\theta_0)}. \tag{40}$$

Eqs. (37) and (38) signify bright solitons, while Eqs. (39) and (40) represent singular solitons by the aid of the constraint

$$b_l\kappa(c_l+d_l-\lambda_l)<0.$$

Case-3:

$$A_0 = \pm\frac{6\kappa^2-25}{30}\sqrt{-\frac{30b_l}{\kappa(c_l+d_l-\lambda_l)}},$$

$$A_2 = \pm\sqrt{-\frac{30b_l}{\kappa(c_l+d_l-\lambda_l)}},$$

$$B_2 = \pm\sqrt{\frac{30b_l}{\kappa(c_l+d_l-\lambda_l)}},$$

$$\omega = -\frac{11\kappa^2 b_l}{2}-\kappa\alpha_l+\frac{b_l}{3}, A_1 = 0,$$

$$B_1 = 0, 18\kappa^4-15\kappa^2-5=0. \tag{41}$$

Plugging the coefficients (41) along with the exact solutions (10) into the solution (30) leads to

$$q(x,t) = \sqrt{-\frac{30b_1}{\kappa(c_1+d_1-\lambda_1)}}$$

$$\times\left\{\begin{array}{c}\frac{6\kappa^2-55}{30}-\operatorname{csch}^2(x+(\alpha_1+8\kappa^3 b_1)t)\\ +\coth(x+(\alpha_1+8\kappa^3 b_1)t)\\ \times\operatorname{csch}(x+(\alpha_1+8\kappa^3 b_1)t)\end{array}\right\}$$

$$\times e^{i(-\kappa x+\omega t+\theta_0)}, \tag{42}$$

$$r(x,t) = \sqrt{-\frac{30b_2}{\kappa(c_2+d_2-\lambda_2)}}$$

$$\times\left\{\begin{array}{c}\frac{6\kappa^2-55}{30}-\operatorname{csch}^2(x+(\alpha_2+8\kappa^3 b_2)t)\\ +\coth(x+(\alpha_2+8\kappa^3 b_2)t)\\ \times\operatorname{csch}(x+(\alpha_2+8\kappa^3 b_2)t)\end{array}\right\}$$

$$\times e^{i(-\kappa x+\omega t+\theta_0)}, \tag{43}$$

Eqs. (42) and (43) signify combo singular solitons by the usage of the condition

$$b_l\kappa(c_l+d_l-\lambda_l)<0.$$

4. Conclusions

The current work reveals soliton solutions with the model equation that is the perturbed CLL model both in scalar case as well as in birefringent fibers for the first time. Hamiltonian type perturbation terms are considered with maximum intensity. The results are thus very promising to venture further in this direction. One of the avenues to look into is the aspect of conservation laws. Next, the soliton perturbation theory comes into play. These results are yet to be disclosed with time. Next, it is necessary to move up to the case of dispersion–flattened fibers that would shed further light into this concept. The model will be further studied using the algorithms. This is just a tip of the iceberg.

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*Corresponding author: biswas.anjan@gmail.com