Dispersive optical solitons with DWDM technology by modified simple equation method

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This paper applies modified simple equation method to obtain dispersive soliton solutions to DWDM systems that are modeled by Schrödinger-Hirota equation in Kerr law medium. Dark and singular soliton solutions are retrieved. The corresponding constraint conditions for the existence of these solitons are also presented.

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1. Introduction

Dispersive optical solitons are governed by Schrödinger-Hirota equation (SHE). This model can be obtained from the usual nonlinear Schrödinger's equation (NLSE) through Lie Transform [14]. When group velocity dispersion (GVD) or spatio-temporal dispersion (STD) are small, it is the higher order dispersion terms, such as third order dispersion (3OD) and nonliner dispersion, that kick in to provide the necessary delicate balance to maintain the stable propagation of solitons. This paper studies SHE with DWDM technology that provides performance enhancement with parallel progation of soliton molecules through the channels in an optical fiber. Such a model is non-trivial and is inherently difficult to handle mathematically. There are several mathematical algorithms that can integrate various forms of nonlinear evolution equations [1-30]. This paper will employ one such mathematical scheme to retrieve dispersive soliton solutions for DWDM systems. This is the simple equation method. The subsequent section recapitulates this scheme and is succesfully applied to retrieve soliton solutions to the model.

2. The modified simple equation method

This section is a quick review of the integration algorithm, namely the modified simple equation method. Suppose we have a nonlinear evolution equation as [3-5]:

$$P\left(u,u_t,u_x,u_{xx},u_{tt},u_{tx},\ldots\right)=0, \qquad (1)$$

where P is a polynomial in $u(x,t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following, we give the basic algorithms of the method [1,2].

Step-1: We use the transformation

$$u(x,t)=u(\xi),\ \xi=x-ct, \qquad (2)$$

where c is a constant to be determined, to reduce Eq. (1) to the following ordinary differential equation:

$$Q(u,u',u'',\ldots)=0, \qquad (3)$$

where Q is a polynomial in $u(\xi)$ and its total derivatives, while $'=\dfrac{d}{d\xi}$.

Step-2: We assume that Eq. (3) has the formal solution.

$$u(\xi)=\sum_{l=0}^{N}a_l\left(\frac{\psi'(\xi)}{\psi(\xi)}\right)^l, \qquad (4)$$

where a_l are constants to be determined, such that $a_N\neq 0$, and $\psi(\xi)$ is an unknown function to be determined later.

Step-3: To determine the positive integer N in Eq. (4), we consider the homogeneous balance between the highest order derivatives and the nonlinear terms in Eq. (3).

Step-4: Substituting (4) into (3), we calculate all the necessary derivatives $u', u'', \ldots$ of the unknown function $u(\xi)$ and we account the function $\psi(\xi)$. As a result of this substitution, we get a polynomial of $\psi'(\xi)/\psi(\xi)$ and its derivatives. In this polynomial, we collect all terms with like powers of $\psi^{-j}(\xi), j = 0,1,2,\ldots$ and its derivatives, and we equate all of the coefficients of this polynomial to zero. This results in a system of equations which can be solved to find a_k and $\psi(\xi)$. Thus, we can get the exact solutions of Eq. (1).

3. Application to DWDM system

The dimensionless form of the perturbed SHE with STD and Kerr law nonlinearity is given by [14]

$$iq_t + aq_{xx} + bq_{xt} + c|q|^2 q + i\left(\gamma q_{xxx} + \sigma |q|^2 q_x\right) = i\alpha q_x + i\lambda\left(|q|^2 q\right)_x + i\nu\left(|q|^2\right)_x q. \tag{5}$$

In equation (5), the first term is the linear temporal evolution, a and b are coefficients of GVD and STD respectively. The nonlinear term is with c that is for self-phase modulation (SPM) with Kerr law. The third order dispersion (3OD) is with γ and σ is nonlinear dispersion. On the right hand side α is with inter-modal dispersion, λ is accounted for self-steepening and finally ν is due to the other form of a nonlinear dispersion.

For DWDM system, Eq. (5) generalizes to, after neglecting the effects of self-steepening and nonlinear dispersion due to the coefficient of ν

$$iq_t^{(l)} + i\alpha_l q_x^{(l)} + a_l q_{xx}^{(l)} + b_l q_{xt}^{(l)} + i\gamma q_{xxx}^{(l)} + \left\{c_l \left|q^{(l)}\right|^2 + \sum_{n\neq l}^{N} d_{ln}\left|q^{(n)}\right|^2\right\} q^{(l)} + i\left\{\xi_l \left|q^{(l)}\right|^2 + \sum_{n\neq l}^{N} \eta_{ln}\left|q^{(n)}\right|^2\right\} q_x^{(l)} = 0. \tag{6}$$

Here $1 \le l \le N$. The first term in (6) on left hand side is the linear temporal evolution term, while a_l represents the coefficient of GVD and the coefficient of b_l is the spatio-temporal dispersion. Then, the coefficient of γ_l is the third order dispersion. Also, c_l is the SPM while d_{ln} is from cross-phase modulation. Finally, ξ_l and η_{ln} are from nonlinear dispersions.

In order to solve (6) for solitons, the following solution structure is taken into consideration:

$$q^{(l)}(x,t) = P_l(\xi) e^{i\Phi_l(x,t)}, \tag{7}$$

where the wave variable ξ is given by

$$\xi = k(x - vt). \tag{8}$$

Here, $P_l(\xi)$ represents the amplitude component of the soliton solutions and v is the speed of the soliton, while the phase component $\Phi_l(x,t)$ is defined as

$$\Phi_l(x,t) = -\kappa_l x + \omega_l t + \theta_l, \tag{9}$$

where $1 \le l \le N$. Here $P_l(x,t)$ represents the amplitude portion of the soliton and from the phase component, κ_l is the frequency of the soliton, ω_l is the wave number of the soliton and finally θ_l is the phase constant. Substituting (7) into (6) and decomposing into real and imaginary parts lead to

$$\left(\alpha_l\kappa_l + \omega_l(b_l\kappa_l - 1) - a_l\kappa_l^2 - \gamma_l\kappa_l^3\right)P_l + (c_l + \kappa_l\xi_l)P_l^3 + k^2(a_l - b_l v + 3\gamma_l\kappa_l)P_{l''} + \sum_{n\neq l}^{N}\left\{P_n^2 P_l(d_{ln} + \eta_{ln}\kappa_l)\right\} = 0, \tag{10}$$

and

$$k(\alpha_l + b_l\omega_l + 2a_l\kappa_l - 3\gamma_l\kappa_l + (b_l\kappa_l - 1)v)P_{l'} + k^3\gamma_l P_{l'''} + k\left(\xi_l P_l^2 + \sum_{n\neq l}^{N}\eta_{ln}P_n^2\right)P_{l'} = 0. \tag{11}$$

Using the balancing principle leads to

$$P_n = P_l. \tag{12}$$

Consequently, Eqs. (10) and (11) reduce to

$$\left(\alpha_l\kappa_l + \omega_l(b_l\kappa_l - 1) - a_l\kappa_l^2 - \gamma_l\kappa_l^3\right)P_l + \left\{c_l + \kappa_l\xi_l + \sum_{n\neq l}^{N}(d_{ln} + \eta_{ln}\kappa_l)\right\}P_l^3 + k^2(a_l - b_l v + 3\gamma_l\kappa_l)P_{l''} = 0, \tag{13}$$

and

$$k(\alpha_l+b_l\omega_l+2a_l\kappa_l-3\gamma_l\kappa_l+(b_l\kappa_l-1)v)P_{l'}+k\left(\xi_l+\sum_{n\neq l}^{N}\eta_{ln}\right)P_l^2P_{l'}+k^3\gamma_lP_{l'''}=0. \tag{14}$$

Integrating Eq. (14) with respect to ξ with zero constant of integration, we get

$$k(\alpha_l+b_l\omega_l+2a_l\kappa_l-3\gamma_l\kappa_l+(b_l\kappa_l-1)v)P_l+\frac{k}{3}\left(\xi_l+\sum_{n\neq l}^{N}\eta_{ln}\right)P_l^3+k^3\gamma_lP_{l''}=0. \tag{15}$$

Comparing Eqs. (13) and (15), we deduce that

$$\begin{aligned}\frac{\alpha_l\kappa_l+\omega_l(b_l\kappa_l-1)-a_l\kappa_l^2-\gamma_l\kappa_l^3}{k(\alpha_l+b_l\omega_l+2a_l\kappa_l-3\gamma_l\kappa_l+(b_l\kappa_l-1)v)}&=\frac{c_l+\kappa_l\xi_l+\sum_{n\neq l}^{N}(d_{ln}+\eta_{ln}\kappa_l)}{\frac{k}{3}\left(\xi_l+\sum_{n\neq l}^{N}\eta_{ln}\right)}\\&=\frac{k^2(a_l-b_lv+3\gamma_l\kappa_l)}{k^3\gamma_l}.\end{aligned} \tag{16}$$

From (16), we obtain

$$v=\frac{\sum_{n\neq l}^{N}(a_l\eta_{ln}-3\gamma_ld_{ln})+a_l\xi_l-3\gamma_lc_l}{b_l\left(\sum_{n\neq l}^{N}\eta_{ln}+\xi_l\right)}, \tag{17}$$

$$\omega_l=\frac{a_l\left(v(b_l\kappa_l+1)-\alpha_l-4\gamma_l\kappa_l^2\right)+\kappa_l\left(v^2b_l^2-2a_l^2\right)}{b_l(a_l-vb_l)+\gamma_l(2b_l\kappa_l+1)}-\frac{vb_l\left(6\gamma_l\kappa_l^2+v-\alpha_l\right)+\gamma_l\kappa_l\left(8\gamma_l\kappa_l^2+3v-2\alpha_l\right)}{b_l(a_l-vb_l)+\gamma_l(2b_l\kappa_l+1)}. \tag{18}$$

Now, we can solve Eq. (15) under the constraint condition (16).

Balancing P_l'' with P_l^3 in Eqs. (15), then we get $N=1.$ Consequently, we reach

$$P_l(\xi)=a_0+a_1\left(\frac{\psi'(\xi)}{\psi(\xi)}\right),\ a_1\neq 0. \tag{19}$$

Substituting Eq. (19) in Eq. (15) and then setting the coefficients of $\psi^{-j}(\xi),\ j=0,1,2,3,$ to zero, then we obtain a set of algebraic equations involving $a_0,a_1,k,\kappa_l,\gamma_l,\alpha_l,b_l,v$ and ω_l as follows:

ψ^{-3} **coeff.:**

$$\frac{1}{3}a_1k\left(a_1^2\left(\sum_{n\neq l}^{N}\eta_{ln}+\xi_l\right)+6k^2\gamma_l\right)\psi'^3=0, \tag{20}$$

ψ^{-2} **coeff.:**

$$a_1k\left(a_0a_1\psi'\left(\sum_{n\neq l}^{N}\eta_{ln}+\xi_l\right)-3k^2\gamma_l\psi''\right)\psi'=0, \tag{21}$$

ψ^{-1} **coeff.:**

$$a_1k\left(\left(\begin{array}{l}a_0^2\left(\sum_{n\neq l}^{N}\eta_{ln}+\xi_l\right)+2a_l\kappa_l+\\ v(b_l\kappa_l-1)+b_l\omega_l+\alpha_l-3\gamma_l\kappa_l^2\end{array}\right)\psi'+k^2\gamma_l\psi'''\right)=0, \tag{22}$$

ψ^{0} **coeff.:**

$$\frac{1}{3}a_0k\left(3\left(\begin{array}{l}2a_l\kappa_l+v(b_l\kappa_l-1)+\\ b_l\omega_l+\alpha_l-3\gamma_l\kappa_l^2\end{array}\right)+a_0^2\left(\sum_{n\neq l}^{N}\eta_{ln}+\xi_l\right)\right)=0. \tag{23}$$

Solving this system, we obtain

$$a_0=\sqrt{\frac{3\left(-2a_l\kappa_l+v(1-b_l\kappa_l)-b_l\omega_l-\alpha_l+3\gamma_l\kappa_l^2\right)}{\sum_{n\neq l}^{N}\eta_{ln}+\xi_l}},\quad a_1=\mp\sqrt{-\frac{6k^2\gamma_l}{\sum_{n\neq l}^{N}\eta_{ln}+\xi_l}} \tag{24}$$

$$\psi''=\pm\sqrt{-\frac{2\left(\sum_{n\neq l}^{N}\eta_{ln}+\xi_l\right)a_0^2}{3k^2\gamma_l}}\psi' \tag{25}$$

$$\psi'''=-\frac{2\left(\sum_{n\neq l}^{N}\eta_{ln}+\xi_l\right)a_0^2}{3k^2\gamma_l}\psi' \tag{26}$$

From Eqs. (25) and (26), we can deduce that

$$\psi'(\xi)=\pm\sqrt{-\frac{3k^2\gamma_l}{2\left(\sum_{n\neq l}^{N}\eta_{ln}+\xi_l\right)a_0^2}}c_1e^{\pm\sqrt{-\frac{2\left(\sum_{n\neq l}^{N}\eta_{ln}+\xi_l\right)a_0^2}{3k^2\gamma_l}}\xi}, \tag{27}$$

$$\psi(\xi)=-\frac{3k^2\gamma_l}{2\left(\sum_{n\neq l}^{N}\eta_{ln}+\xi_l\right)a_0^2}c_1 e^{\pm\sqrt{-\frac{2\left(\sum_{n\neq l}^{N}\eta_{ln}+\xi_l\right)a_0^2}{3k^2\gamma_l}}\xi}+c_2, \quad (28)$$

where c_1 and c_2 are constants of integration. Substituting Eq. (27) and Eq. (28) into Eq. (19), we obtain following the following exact solution to Eq. (6).

$$q^{(l)}(x,t)=\left\{a_0-\sqrt{-\frac{6k^2\gamma_l}{\sum_{n\neq l}^{N}\eta_{ln}+\xi_l}}\frac{\sqrt{-\frac{3k^2\gamma_l}{2\left(\sum_{n\neq l}^{N}\eta_{ln}+\xi_l\right)a_0^2}c_1 e^{\pm\sqrt{-\frac{2\left(\sum_{n\neq l}^{N}\eta_{ln}+\xi_l\right)a_0^2}{3k^2\gamma_l}}(k(x-vt))}}}{-\frac{3k^2\gamma_l}{2\left(\sum_{n\neq l}^{N}\eta_{ln}+\xi_l\right)a_0^2}c_1 e^{\pm\sqrt{-\frac{2\left(\sum_{n\neq l}^{N}\eta_{ln}+\xi_l\right)a_0^2}{3k^2\gamma_l}}(k(x-vt))}+c_2}\right\} e^{i(-\kappa_l x+\omega_l t+\theta_l)}. \quad (29)$$

If we set

$$c_1=-\frac{2\left(\sum_{n\neq l}^{N}\eta_{ln}+\xi_l\right)a_0^2}{3k^2\gamma_l}e^{\pm\sqrt{-\frac{2\left(\sum_{n\neq l}^{N}\eta_{ln}+\xi_l\right)a_0^2}{3k^2\gamma_l}}\xi_0}, \quad c_2=\pm 1,$$

We obtain:

$$q^{(l)}(x,t)=\pm\sqrt{-\frac{3\left(2a_l\kappa_l-v(1-b_l\kappa_l)+b_l\omega_l+\alpha_l-3\gamma_l\kappa_l^2\right)}{\sum_{n\neq l}^{N}\eta_{ln}+\xi_l}}\times \tanh\left[\sqrt{\frac{2a_l\kappa_l-v(1-b_l\kappa_l)+b_l\omega_l+\alpha_l-3\gamma_l\kappa_l^2}{2k^2\gamma_l}}(k(x-vt)+\xi_0)\right] e^{i(-\kappa_l x+\omega_l t+\theta_l)}, \quad (30)$$

$$q^{(l)}(x,t)=\pm\sqrt{-\frac{3\left(2a_l\kappa_l-v(1-b_l\kappa_l)+b_l\omega_l+\alpha_l-3\gamma_l\kappa_l^2\right)}{\sum_{n\neq l}^{N}\eta_{ln}+\xi_l}} \times\coth\left[\sqrt{\frac{2a_l\kappa_l-v(1-b_l\kappa_l)+b_l\omega_l+\alpha_l-3\gamma_l\kappa_l^2}{2k^2\gamma_l}}(k(x-vt)+\xi_0)\right] e^{i(-\kappa_l x+\omega_l t+\theta_l)}, \quad (31)$$

where v and ω_l are given by (17) and (18) respectively. Solutions (30) and (31) are valid when

$$2a_l\kappa_l-v(1-b_l\kappa_l)+b_l\omega_l+\alpha_l-3\gamma_l\kappa_l^2>0, \quad \gamma_l>0, \quad (32)$$

$$\sum_{n\neq l}^{N}\eta_{ln}+\xi_l<0. \quad (33)$$

4. Conclusions

This paper obtained dark and singular optical soliton solutions to DWDM systems that stem from SHE with Kerr law nonlinearity. Dark and singular optical soliton solutions are retrieved from the adopted integration algorithm of this paper that is the modified simple equation method. The constraint conditions for the existence of these solitons naturally emerged from the solution strucure of the solitons. This scheme fails to obtain bright soliton solutions to the model and is thus a limitation to the integration process. Nevertheless, this scheme is hopeful for additional applications in fiber-optic studies. These include optical couplers, limiters, switching, Bragg gratings and several other systems. These results will be gradually and sequentially reported with time.

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