Distributed Raman amplifier crosstalk effect in different kind of modulation for forward, backward and bidirectional pumping

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This paper study the variance crosstalk standard deviation in a closed formula for Self-Phase Modulation (SPM) and Cross-Phase Modulation (XPM), which is very important parameters in nonlinear effect phenomena which affect the system performance. The model is used to analysis the best performance with minimum crosstalk in WDM system and the direct effect of increasing the input power signal in the system with respect to the bit rate, pulse shape, modulation formula and the multi-pumping distributed Raman amplifier (forward, backward and bi-directional) pumping.

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1. Introduction

Nonlinear fiber optics plays an important role in the design of WDM transmission system [1]. By studding the nonlinear effect in optical fiber in the high speed, small spacing channels high crosstalk probability is taking place, it is very important to study the parameters which affect these phenomena to be controlled. The main scope of this research is to discuss two important parameters, SPM and XPM, in two different kinds of modulation, on-off key (*OOK*) and differential phase shift key (*DPSK*), at different duty cycle and different bite rate, in single span WDM system.

In N-channels WDM system used lumped amplifiers as erbium-doped fiber amplifier (EDFA) and distributed amplifiers such as Raman amplifier (RA) [2]. The transmitted signals are amplified using high power pumps which co-propagate (forward pumping) or counter propagate (backward pumping) with the signals but in the nonlinear fiber optics we interest in RA because it distributes the power on whole the span network which called Distributed Raman Amplifier (DRA) and by that way of distribution which give the availability to solve soliton communication problem [3]. The most important advantage provided by distributed Raman amplification is, of course, its improved noise performance when compared to traditional lumped amplification [4]. This improved noise performance comes from the actual distribution of gain along the transmission link [5].

This paper is studied the variance standard deviation crosstalk in a closed formula for SPM and XPM and neglect other effects such as four wave mixing (FWM), stimulated Raman scattering (*SRS*) and stimulated Brillouin scattering (*SBS*) are to be studied for future work. The paper shows the variance crosstalk with respect to bit rate (B_T) and pulse shape at two different modulation formulas in multi-pumping DRA (Forward, Backward and Bi-directional Pumping), most of the papers study the crosstalk variance which degraded the system performance [6].

This paper is organized as follows; in Sec. 2, the mathematical theory of a single span WDM DRA model which divides in two subsections. A, the power spectral density parameters which have a direct impact effects on the performance of SPM and XPM induced crosstalk. B, the modulation techniques which are used in the transmission WDM system. In Sec. 3, the results and conclusions of the simulation output and the analysis for every result. This is followed by conclusion of this investigation in Sec. 4.

2. Mathematical model

For N-channels of WDM system the general form for phase modulation of i^{th} channel induced by j^{th} channel due to XPM in the bi-directional of DRA can be written as

$$\varphi_{ij}(t) = 2\gamma \int_0^L P_n\left(0, t - d_{ij}z'\right)\ e^{-\alpha z'} g_F(z')\ g_B(z') dz' \quad (1)$$

Where φ_{ij} is the phase modulation in Bi-directional DRA between two channels i^{th} and j^{th}, γ is the nonlinear coefficient, $P_n\left(0, t - d_{ij}z'\right)$ is the transmitted power of

i^{th} channel as a function of length z' and time t. and d_{ij} is propagation time different between the two channels (walk-off parameter), also the group velocity $d_{ij} \approx D\,\Delta\lambda$ as D is the dispersion coefficient and $\Delta\lambda$ is the walk-off difference between two channels [7].

The DRA forward amplification gain can be write as [7,8]

$$g_F(z') = exp\left(\frac{g_R\left(\sum\frac{\lambda_s}{\lambda_p} P_{S0} + P_{P0}\right)}{A_{eff}\alpha}\left(1 - e^{-\alpha z'}\right)\right) \quad (2)$$

Where g_R is the Raman peak gain which equal to 6.5×10^{-14} m/W, λ_s and λ_p are signal and pump wavelengths, respectively, P_{S0} and P_{P0} are the initial signal and pump power. In this case RA considers the signal to be traveling in one direction, the forward direction. Because of Rayleigh reflections, a fraction of the signal is scattered and after this half of the Rayleigh scattered signal-light propagate in the same direction as the signal and the other half in the opposite direction [9].

For backward amplification gain

$$g_B(z') = exp\left(\frac{g_R\left(\sum\frac{\lambda_s}{\lambda_p} P_{S0} + P_{PL}\right)}{A_{eff}\alpha}\left(e^{-\alpha(z'-L)} - e^{-\alpha L}\right)\right) \quad (3)$$

Where P_{PL} is the initial backward pumping power for a distance (L) between the two Raman amplifiers.

As shown in Fig. 1 the gain behavior in Raman amplifier in forward, backward and bi-directional. We can see the main function for Raman amplifier is to distribute the Power all over the band along the distance (L) is accomplished that's why DRA is used in long haul WDM system to minimize number of amplifiers.

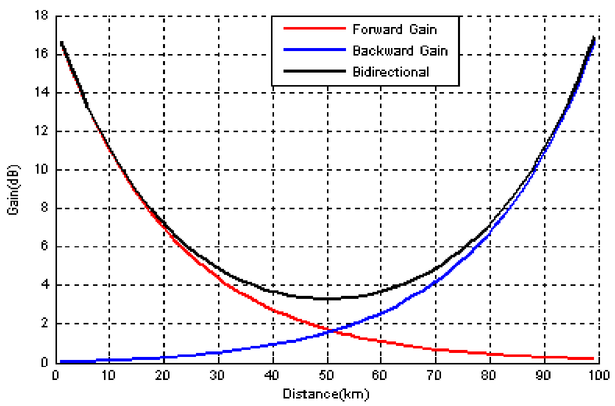


Fig. 1. DRA gain (dB) versus distance (km)

A. Power spectral density

Let $S_{\varphi_{ij}}(\omega)$ denote to the power spectral density of the output random process $\varphi_{ij}(t)$ obtained by passing the random process $P_n(0,t)$ through a linear filter of frequency response $H_{ij}(\omega)$. By definite that the power spectral density (*PSD*) of a random process is equal to the Fourier Transform (*FT*) of its auto-correlation function and by substitute in Eq. (1) we obtain

$$S_{\varphi_{ij}}(\omega) = S_{P_{ij}}(\omega)\left|H_{ij}(\omega)\right|^2 \quad (4)$$

For *RZ* duty cycle we can consider *T* value equal to *2* and *3* for *50%* and *33%* duty cycle respectively, so if we consider the *NRZ* rectangular pulse then

$$p(t) = \begin{cases} 2P_0 & \left(|t| < \frac{T}{2}\right) \\ 0 & \left(|t| > \frac{T}{2}\right) \end{cases} \quad (5)$$

by applying the FT we get

$$|P(\omega)| = 2P_0T\,\frac{\sin(\omega T/2)}{\omega T/2} \quad (6)$$

Similarly, *RZ* pulse but we will apply the desire duty cycle we required. Now we can write the general form of $S_{P_{ij}}(\omega)$ as follow

$$S_{P_{ij}}(\omega) = \frac{1}{4T}|P(\omega)|^2 + \frac{1}{4T^2}\sum_{k=-\infty}^{\infty}\left|P\left(\frac{k}{T}\right)\right|^2 \delta\left(\omega - \frac{k}{T}\right) \quad (7)$$

where $P(\omega)$ is the FT of the rectangular pulse shape as shown in Eq. (6) [9], and K is given by

$$K = \begin{cases} k = \frac{g_R}{\psi A_{eff}}\frac{f_i}{f_j} & \left|f_i - f_j\right| \le 15\,THZ \\ 0 & \left|f_i - f_j\right| > 15\,THZ \end{cases} \quad (8)$$

K parameter sign can be positive or negative, if the optical power increases or not but if $K = 0$ when $f_i = f_j$ that mean that there is no power exchange between channels 0. The transfer function $H_{ij}(\omega)$ which is the mathematical representation to describe inputs and outputs can be written as follow.

$$H_{ij}(\omega) = 2\gamma\int_0^L exp\left(-jd_{ij}z'\omega\right)exp(-\alpha z')\, exp\left(K'\left(1 - e^{-\alpha z'}\right)\right) exp\left(K''\left(e^{-\alpha(z'-L)} - e^{-\alpha L}\right)\right)dz' \quad (9)$$

where $exp(-jd_{ij}z'\omega)$ is the transfer function for the n^{th} channel in the WDM system, $exp\left(K'\left(1-e^{-\alpha z'}\right)\right)$ and $\exp\left(K''\left(e^{-\alpha(z'-L)}-e^{-\alpha L}\right)\right)$ is the transfer function for the forward and backward amplification gain respectively as

$$K' = \frac{g\left(\sum \frac{\lambda_s}{\lambda_p} P_{S0} + P_{P0}\right)}{A_{eff}\alpha} \qquad (10)$$

$$K'' = \frac{g\left(\sum \frac{\lambda_s}{\lambda_p} P_{S0} + P_{PL}\right)}{A_{eff}\alpha} \qquad (11)$$

By solving the exponential integral equation in Eq. (9) and substitute in the integral limits, $H_{ij}(\omega)$ will be as follow

$$\begin{aligned} H_{ij}(\omega) &= 2\gamma\left\{\frac{1-e^{-(\alpha+jd_{ij}\omega)L}}{\alpha+jd_{ij}\omega}\right\} \\ &- 2\gamma K'\left\{\frac{1-e^{-(2\alpha+jd_{ij}\omega)L}}{2\alpha+jd_{ij}\omega}\right\} \\ &+ 2\gamma K'' e^{-\alpha L}\left\{\frac{1-e^{-(jd_{ij}\omega)L}}{jd_{ij}\omega}\right\} \end{aligned} \qquad (12)$$

The previous equations were solved in mathematical form to be reused in the crosstalk analysis with different kind of modulation technique and according to that results minimum crosstalk can be chosen. The crosstalk equation is depending on $S_{\varphi_{ij}}(\omega)$ which is changing according to the modulation technique.

B. Modulation techniques

The simplest and most widely used modulation scheme in optical communication is *OOK*, by take $T_b = T$ as one symbol duration (T_b) is equal to the bit duration (T) 0. The nonlinear phase shift which has been represented in Eq. (1), $\Delta\varphi_{ij}(L,t)$ will be added to Eq. (4)

$$S_{\Delta\varphi_{ij}}(\omega)|_{OOK} = S_{P_{ij}}(\omega)\left|H_{ij}(\omega)\right|^2 \qquad (13)$$

and the variance of *XPM* crosstalk is given by

$$\sigma_{XPM}^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{\Delta\varphi_{ij}}(\omega)|_{OOK}\, d\omega \qquad (14)$$

The other modulation technique is *DPSK* as it is an example of non-coherent orthogonal modulation 0. Combining two basic operations at the transmitter differential encoding of the input binary wave and phase shift keying, one bit is encoded onto 2π phase change and zero bit is represented by the absence of the phase change.

The nonlinear phase shift which has been represented in Eq. (1), $\Delta\varphi_{ij}(L,t)|_{DPSK} = \varphi_{ij}(L,t) - \varphi_{ij}(L,t-T)$ will be added to Eq. (4) and the power spectral density will be

$$S_{\Delta\varphi_{ij}}(\omega)|_{DPSK} = 4S_{P_{ij}}(\omega)\left|H_{ij}(\omega)\right|^2 sin^2\left(\omega T/2\right) \qquad (15)$$

and the Variance of XPM crosstalk is given by

$$\sigma_{XPM}^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{\Delta\varphi_{ij}}(\omega)|_{DPSK}\, d\omega \qquad (16)$$

The variance SPM induced in crosstalk is calculated by taking the inter-channel separation (δ λ) equal to zero in the variance expression of XPM 0.

$$\sigma_{SPM}^2 = \left(\frac{1}{2}\right)^2 \sigma_{XPM}^2 \qquad (17)$$

The factor ½ comes from the fact that the phase shift due to the XPM is twice as large as SPM. Thus the total variance of XPM and SPM induced for the crosstalk is given as

$$\sigma_X^2 = \sigma_{SPM}^2 + \sigma_{XPM}^2 \qquad (18)$$

3. Results and discussions

Considering WDM system as shown in Fig. 2 with bidirectional multi-pumped DRA of 6 pumps, uses of more pump wavelengths can prove gain flatness known as ”14xx *nm*” pump laser which typically required to have 100 to 300 *mW* for each pump 0. The wavelength range of 1420 - 1470 *nm* with spacing of 10 *nm*. N is number of channel which is equal to 60 channels in the C-band starting with 1514 *nm* with spacing 1 *nm* which are multiplexing on single DRA span of length L with input signal power range of 0 - 18 *dBm* at standard single mode fiber (*SMF*) parameters calculation given in Table 1, each channel in the network is amplify by the DRA and then demultiplexing at the receiver end.

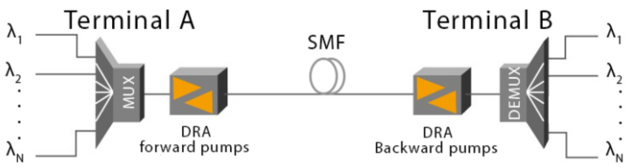


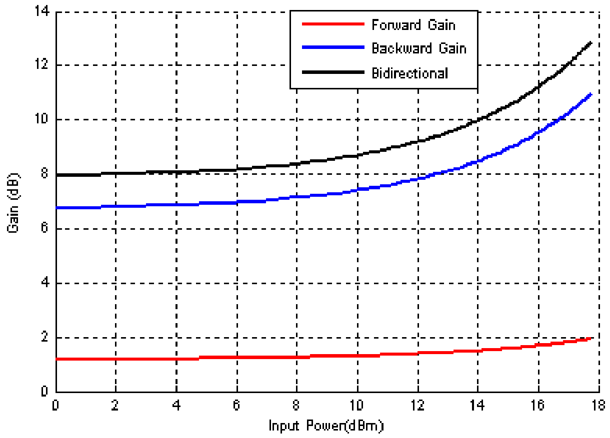
Fig. 2. Single span WDM system using DRA

MATLAB version 7.9.0.529 (R2009b) is used to perform all the simulation and calculation proposed in this paper.

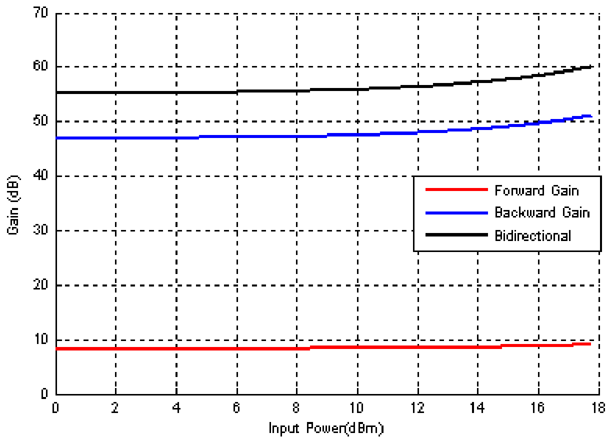
Table 1. DRA System Parameter for SMF

Parameter	Value
Area Effective (A_{eff})	80 μm^2
Attenuation coefficient (α)	0.2 *dB/km*
Length (L)	100 *km*
Non-linear coefficient (γ)	1.18 $W^{-1}km^{-1}$
Dispersion Slop	0.058 x 10^3 *ps/nm/km*
Zero Dispersion Wavelength	1265.5 *nm*
Polarization constant (ψ)	2

Fig. 3 shows The analysis for N-channel WDM system gain with respect to the input signal power in a power transmitting range from 0 - 18 *dBm* in two different pumping power (a) 100 *mW*, (b) 700 *mW*, showing that the huge increase which is almost 85% and flatting in the backward gain at the two different pumping power, on the other hand we didn't observe much increase in the forward gain however or any great enhancement in the flatting gain.



(a) Pumping power 100 *mW*.

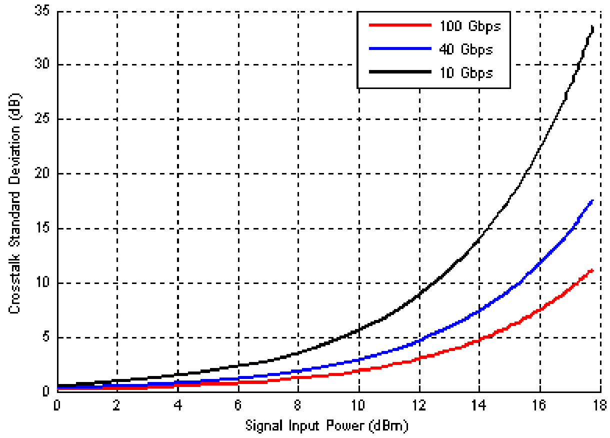


(b) Pumping power 700 *mW*.

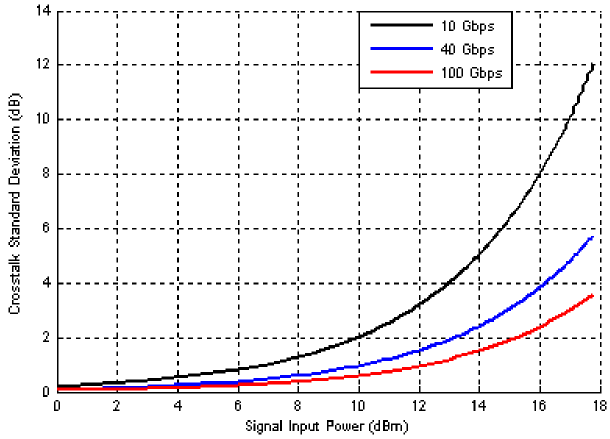
Fig. 3. Gain (dB) versus input signal power (dBm) in all case forward, backward and bi-directional

In practical field Fig. 3(b) is used for Raman amplifier order (1) as the Pumping power is tending to 1 *W* but actually we work in a range from 700 *mW* to 900 *mW* according to the span distance, one can say that increasing the pumping power give better performance as we increase number of electrons in the conduction band.

The variance crosstalk standard deviation *XPM* and *SPM* induced crosstalk with the input signal power for B_T of 10 *Gbps*, 40 *Gbps* and 100 *Gbps* for *NRZ-OOK* in both cases forward and backward pumping DRA is shown in Fig. 4. It can be seen that whenever we increase the B_T the signal crosstalk decrease and minimum crosstalk observed for 100 *Gbps NRZ-OOK* that because of walk-off length which is $L_W = T/|d_{ij}|$ or what is called walk-off phenomena which is related to the group velocity which has a varies inversely relation with dispersion coefficient and B_T that's can explain why when we increase the bite rate the crosstalk is decrease as the walk-off time will be short so nonlinear interaction between two pulses will be very small [7].



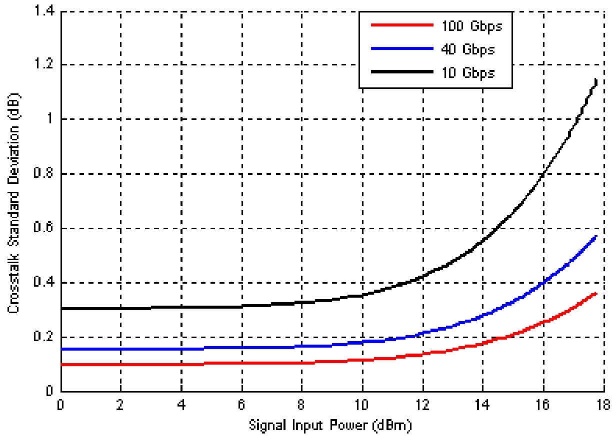
(a) Forward pumping DRA.



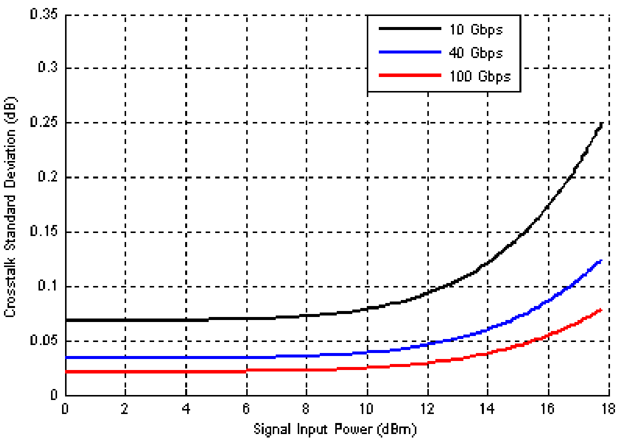
(b) Backward pumping DRA

Fig. 4. Variance crosstalk standard deviation (dB) versus input signal power (dBm) for OOK modulation at bit rate(B) = 10 Gbps, 40 Gbps and 100 Gbps

The same parameters as in Fig. 4, changing the modulation technique which will be *DPSK* in Fig. 5. It can be noticed that variance crosstalk (*dB*) is decreased by 68.4% compared with the *OOK* techniques. The modulation in optical transmitting can be defined as the average number of photon per bit 0. *DPSK* offers as the average power remains constant for every bit-period the signal becomes less affected to *SPM* and *XPM* compared to *OOK* Key.



(a) Forward pumping DRA



(b) Backward pumping DRA

Fig. 5. Variance crosstalk standard deviation (dB) versus input signal power (dBm) for DPSK modulation at bit rate(B) = 10 Gbps, 40 Gbps and 100 Gbps

Fig. 6 shows the variance of SPM and XPM induced crosstalk (*dB*) with respect to signal input power (*dBm*) for forward, backward and bidirectional DRA pumping at *DPSK* at B_T 100 *Gbps* as the backward pumping gives us the minimum value of induced crosstalk compared to forward pumping with content value of crosstalk from 0 to 12 *dBm*.

Based on the previous analysis Fig. 7 shows the minimum crosstalk at B_T 100 *Gbps* for different duty cycle *RZ* (duty cycle *33%*), *RZ* (duty cycle *50%*) and *NRZ*. We can be observed that *RZ - DPSK* (duty cycle *33%*) have the minimum SPM and XPM induced crosstalk as the crosstalk is varies inversely with the duty cycle (τ_b) consequently *RZ* has the less crosstalk compared with *NRZ* because the probability of overlap in one bit with its neighbor channel decrease. We advise the signal power region between 0 - 10 (*dBm*) for stable crosstalk as there is no huge difference in the value.

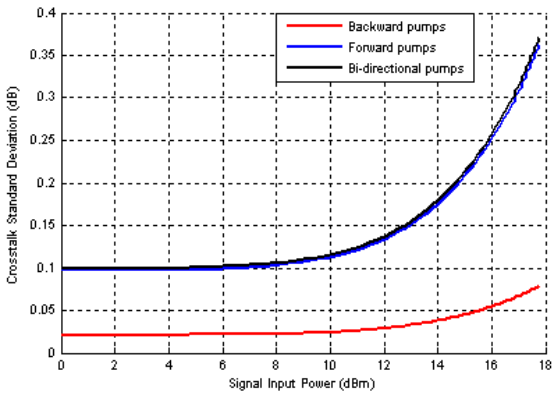


Fig. 6. Variance crosstalk standard deviation (dB) versus input signal power (dBm) for forward, backward and bi-directional DRA pumping at DPSK at B_T = 100 Gbps

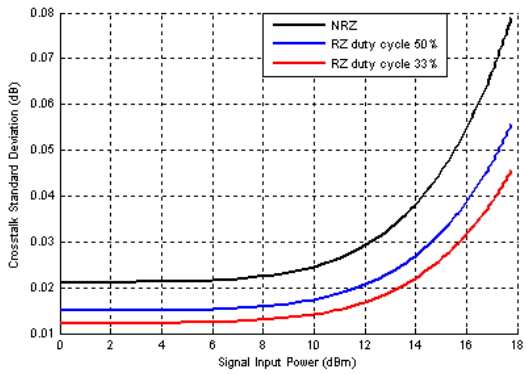


Fig. 7. Variance crosstalk standard deviation (dB) versus input signal power (dBm) for backward DRA pumping at DPSK at duty cycle = 33 % RZ, 50% RZ and NRZ at B_T = 100 Gbps

4. Conclusion

Closed form formulas are derived for SPM and XPM induced crosstalk with respect to the input power signal in single span *WDM DRA* system. The performance has been compared between two different kinds of modulation (*OOK* and *DPSK* with different duty cycle (*RZ* duty cycle *33%*, *RZ* duty cycle *50%* and *NRZ*) in the forward, backward and bidirectional pumped *DRA*. It has been found that the greater B_T of the system the smaller is the SPM and XPM induced crosstalk. The smaller duty cycle of the pulse the minimum crosstalk value is present. It has been found that in backward pumped DRA at B_T 100 *Gbps* *RZ - DPSK* (*33 %* duty cycle) gives best SPM and XPM induced crosstalk performance.

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APPENDIX A

Dealing with infinite equation as Eq. (14, 16) which cannot be implemented in the simulation program the expression variance (σ_X^2) for *OOK* and *DPSK* signals as follows

$$\sigma_{XPM}^2|_{OOK} = (F(T) + F_1(T) + F_2(T) + F_3(T) + F_4(T) + F_5(T)) \qquad (A.1)$$

where *T* value is depend on the duty cycle *33% RZ*, *50% RZ* or *NRZ*, and all the following integration has been performed using the basic trigonometric and algebraic functions 0.

$$\sigma_{XPM}^2|_{DPSK} = (F(T) + F_1(T) + F_2(T) + F_3(T) + F_4(T) + F_5(T)) - (1/4) * (F(2T) + F_1(2T) + F_2(2T) + F_3(2T) + F_4(2T) + F_5(2T)) \qquad (A.2)$$

The integration the following trigonometric formulate and integrals are used

$$\int_0^\infty \frac{sin^2 \alpha x \sin bx}{x(x^2+c^2)}\, dx = \frac{\pi}{8\,|c|^2}\, sign\,(b)\left(2 - 2e^{|b||c|}\right) + sign\,(2a-b)\left(1 - e^{-|2a-b||c|}\right) - sign\,(2a-b)\left(1 - e^{-|2a+b||c|}\right) \qquad (A.3)$$

$$\int_0^\infty \frac{sin^2 \alpha x}{x^2(x^2+c^2)}\, dx = \frac{\pi}{4\,|c|^3}\left(e^{-2|a||c|} + 2|a||c| + 1\right) \qquad (A.4)$$

$$\int_0^\infty \frac{sin^2 \alpha x \sin bx}{x^3}\, dx = \pi\, sign(b+2a)\left(\frac{b^2}{8} + \frac{ab}{2} + \frac{a}{2}\right) - \pi\, sign(b)\frac{b^2}{4} + \pi\, sign\,(-b + 2a)\left(\frac{b^2}{8} + \frac{ab}{2} - \frac{a^2}{2}\right) \qquad (A.5)$$

$$\int_0^\infty \frac{sin^2\, ax\, sin^2 bx}{x^4} = \frac{\pi}{6}\, min(a^2, b^2)\{3 \max(a,b) - \min(a,b)\} \qquad (A.6)$$

and finally

$$(\sin\alpha \sin\beta)^2 = \frac{1}{4}\,[2\, sin^2\alpha + 2\, sin^2\beta - sin^2(\alpha+\beta) - sin^2(\alpha-\beta)] \qquad (A.7)$$

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