Eccentric connectivity polynomial of triangular benzenoid

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The eccentric connectivity index of the molecular graph G, $\xi^c(G)$, was proposed by Sharma, Goswami and Madan. It is defined as $\xi^c(G) = \Sigma_{u\in V(G)}deg_G(u)ecc(u)$, where $deg_G(x)$ denotes the degree of the vertex x in G and ecc(u) = Max{d(x,u) | x ∈ V(G)}. The eccentricity connectivity polynomial of a molecular graph G is defined as $ECP(G,x) = \Sigma_{a\in V(G)}deg_G(a)x^{ecc(a)}$, where ecc(a) is defined as the length of a maximal path connecting a to another vertex of G. In this paper this polynomial is computed for triangular benzenoid graphs.

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1. Introduction

At first we recall some algebraic definitions that will be used in the paper. Throughout this paper, graph means simple connected graph. The vertex and edge sets of a graph G are denoted by V(G) and E(G), respectively. If x, y ∈ V(G) then the distance d(x,y) between x and y is defined as the length of a minimum path connecting x and y. The eccentric connectivity index of the molecular graph G, ξ (G), was proposed by Sharma, Goswami and Madan[1]. It is defined as $\xi(G) = \Sigma_{u\in V(G)}deg_G(u)ecc(u)$, where $deg_G(x)$ denotes the degree of the vertex x in G and ecc(u) = Max{d(x,u) | x ∈ V(G)}, see [2-6] for details. The radius and diameter of G are defined as the minimum and maximum eccentricity among vertices of G, respectively.

We now define the eccentric connectivity polynomial of a graph G, ς(x), as

$$\varsigma(x) = \Sigma_{a\in V(G)}deg_G(a)x^{ecc(a)}.$$

Then the eccentric connectivity index is the first derivative of ς(x) evaluated at $x = 1$.

Herein, our notation is standard and taken from the standard book of graph theory such as [7] and [8-13].

2. Results and discussion

The aim of this section is to compute ς(x), for an infinite family of triangular benzenoid graph. To do this we should to consider the following examples:

Example 1. Consider graph G depicted in Fig. 1. This graph has 13 vertices and 15 edges. This graph has a vertex such as *u*, with ecc(u) = 3, three vertices with eccentricity of 4 and six vertices of eccentricity 5. In other words $\varsigma(x) = 3x^3 + 9x^4 + 18x^5$ and so $\xi(G) = 135$.

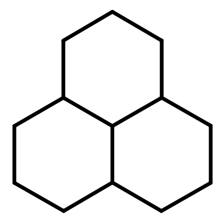


Fig. 1. Graph of triangular benzenoid G[2].

Example 2. Consider graph G[3] depicted in Fig. 2. This graph has 22 vertices and 27 edges. By computing eccentricity polynomial of G[3] it is easy to check that $\varsigma(x) = 3x^4 + 9x^5 + 18x^6 + 24x^7$. Hence $\xi(G) = 333$.

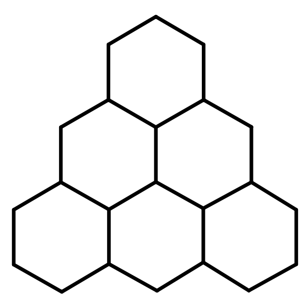


Fig. 2. Graph of triangular benzenoid G[3].

Example 3. Consider graph G[4] depicted in Fig. 3. This graph has 33 vertices and 42 edges. Similar to last examples one can see that $\varsigma(x) = 9x^6 + 18x^7 + 27x^8 + 30x^9$. Hence $\xi(G) = 4678$.

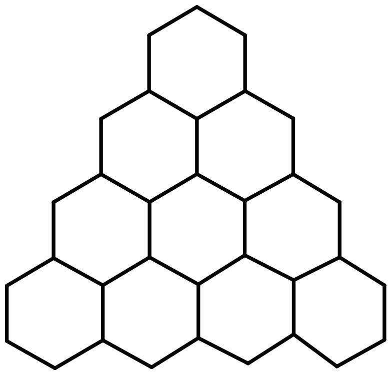


Fig. 3. Graph of triangular benzenoid G[4].

Example 4. Consider graph G[5] depicted in Fig. 4. This graph has 46 vertices and 60 edges. Also, $\varsigma(x) = 3x^7 + 18x^8 + 27x^9 + 36x^{10} + 36x^{11}$ and so $\xi(G) = 10278$.

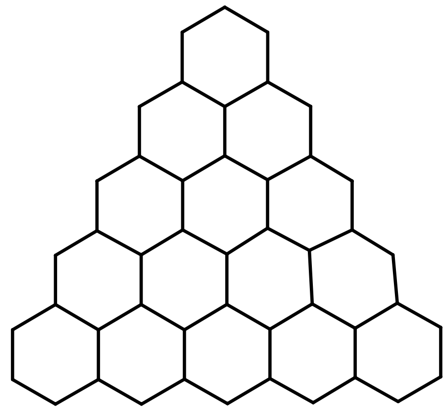
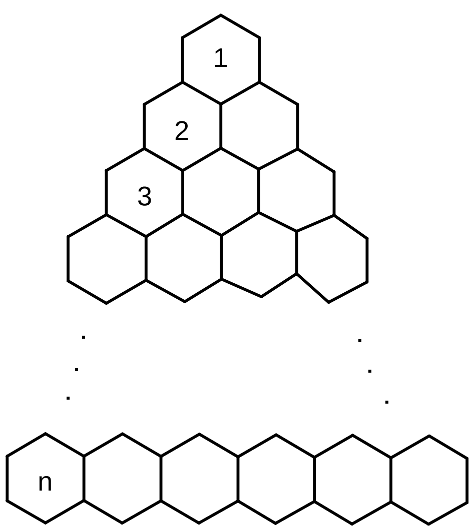


Fig. 4. Graph of triangular benzenoid G[4].

In generally consider graph $G[n]$ depicted in Fig. 4. This graph has $n^2 + 4n + 1$ vertices and $\frac{3(n^2+3n)}{2}$ edges. By continuing above method one can see the eccentric connectivity index is as follows:

$$\varsigma(x) = \begin{cases} 3\sum_{i=2}^{n} a_i x^{n+i} + 2(n+1)x^{2n+1}, a_i \notin \{1,4,\ldots,3i-2,\ldots,3n-2,3n-3\} & n \overset{3}{\equiv} 0 \\ 3\sum_{i=1}^{n} a_i x^{n+i} + 2(n+1)x^{2n+1}, a_i \notin \{2,5,\ldots,3i-1,\ldots,3n-1,3n-3\} & n \overset{3}{\equiv} 1 \\ 3\sum_{i=1}^{n} a_i x^{n+i} + 2(n+1)x^{2n+1}, a_i \notin \{3,6,\ldots,3i,\ldots,3n,3n\} & n \overset{3}{\equiv} 2 \end{cases}$$



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