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Edge detour index of $TUC_4C_8(S)$ nanotube

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The detour index is equal to the sum of distances between all pairs of vertices of the connected graph on the longest path between corresponding vertices. The edge-detour index is conceived the same way as the sum of distances between all pairs of edges of the connected graph on the longest path between corresponding edges. In this paper we computed the two type of edge detour index for $TUC_4C_8(S)$.

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1. Introduction

The detour matrix is one of the particularly important distance matrices which are based on the topological distance for vertices in a graph. It was introduces into the mathematical literature in 1969 by Frank Harary [1] and it was discussed in 1990 by Buckley and Harary [2]. The detour matrix was introduced into the chemical literature in 1994 under the name *"the maximum path matrix of a molecular graph"* [3-7] and theoretical graph theory contribution to finding the some interest in chemistry [8-16]. During these works, the ordinary (vertex) version of detour index has been defined for a connected graph G as follows:

$$D = \frac{1}{2}\sum_{j}\sum_{i} d(i,j)$$

where $d(i,j)$ is the longest path between vertices i and j. In [17-21], some work has been done on detour index.

Let $e,f \in E(G)$ and $e = (u,v)$ and $f = (x,y)$. Distance between two edge is defined as follows:

$$d_1(e,f) = \max\{d(u,x), d(v,x), d(u,y), d(v,y)\}$$

and $d_2(e,f) = \min\{d(u,x), d(v,x), d(u,y), d(v,y)\}$

$$d_3(e,f) = \begin{cases} d_1(e,f) & if\ e \neq f \\ 0 & if\ e = f \end{cases}$$

And

$$d_4(e,f) = \begin{cases} d_2(e,f)+1 & if\ e \neq f \\ 0 & if\ e = f \end{cases}$$

So the edge versions of detour index is defined as follows [22]:

$$D_{e0}(G) = \frac{1}{2}\sum_{e,f} d_3(e,f)$$

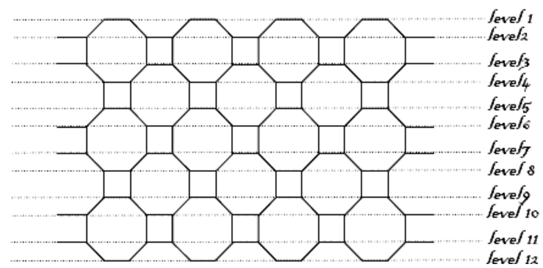
and

$$D_{e1}(G) = \frac{1}{2}\sum_{e,f} d_4(e,f).$$

The aim of this paper is computing of the edge Detour index of $TUC_4C_8(S)$ nanotube.

2. Results and discussion

A C_4C_8 net is a trivalent decoration made by alternating squares and octagons C_8. Let us denote by p the number of squares at first row in the tube and by q, m, k the various levels (i.e., the length) of the tube (Fig. 1).



$p = 4,\ q = 6$ *lattice with* $TUC_4C_8(S)$ *Fig. 1.*

In [21] we computed the detour index of $TUC_4C_8(S)$ nanotube. In Fig. 2 and Fig. 3 we show the detour between vertex v and other vertices. In Figs. 2 and 3, the number m over any vertex, which means $4pq - m$ is the detours from v. If v be an arbitrary vertex in level 1, Fig. 2 shows Detours between v and other vertices,

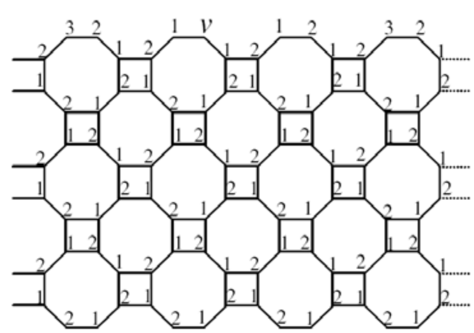


Fig. 2. Detours from vertex v *to vertices lying at levels* $k = 1, 2, .., 12$.

And if be an arbitrary vertex in level i ($2 \le i \le q-1$), Fig. 3 shows Detours between v and other vertices,

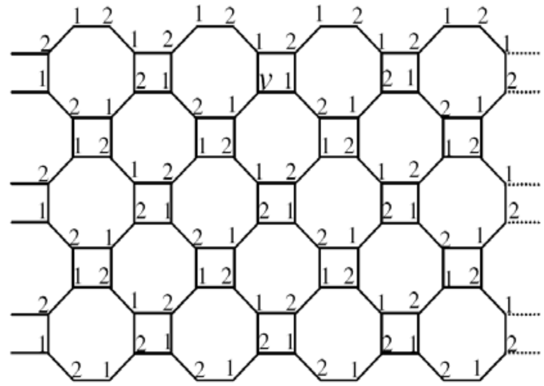


Fig. 3. Detours from vertex v *to vertices lying at levels* $k = 1, 2, .., 12$.

We define:

$A = \{e \in E(G) | e \text{ is an horizontal edge in first or last level}\}$

$B = \{e \in E(G) | e \text{ is an oblique edge between level 1 and level 2 or } e \text{ is an oblique edge between level q} - 1 \text{ and level q}\}$

Now lets e be an horizontal edge in first level.

If f be other horizontal edge in first level then we have: $d_2(e,f) = 4pq - 3$ and $d_1(e,f) = 4pq - 1$ or $d_1(e,f) = 4pq - 2$ according to Fig. 4.

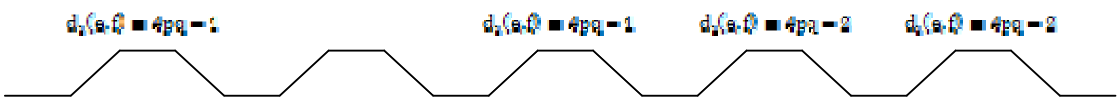


Fig. 4

If f be other edge we have: $d_2(e,f) = 4pq - 2$ and $d_1(e,f) = 4pq - 1$

So the sum of Edge Detours between e and other edges is given as:

$$st_1(e,f) = \sum_{e \in A} d_3(e,f) = \begin{cases} (6pq - 2p - 1) \times (4pq - 1) & p \le 3 \\ (6pq - 2p - 1) \times (4pq - 1) - p + 3 & p > 3 \end{cases}$$

3wseand $si_1(e,f) = \sum_{e \in A} d_4(e,f) = (6pq - 2p - 1) \times (4pq - 1)$.

Since there is $2p$ horizontal edge in first and last level then $s_1 = 2p \times st_1(e,f)$ and $s_{1e} = 2p \times si_1(e,f)$.

If e be another edge that do not belong to A then

$$st_2(e,f) = \sum_{\substack{e \text{ is an edge} \\ e \notin A}} d_3(e,f) = (6pq - 2p - 1) \times (4pq - 1)$$

Since there exist $6pq - 4p$ edge of this type then $s_2 = (6pq - 4p) \times st_2(e,f)$.

If $e \in B$ then $si_2(e,f) = \sum_{e \in B} d_4(e,f) = (6pq - 2p - 1) \times (4pq - 1) - (p - 2)$

Since there exist $4p$ edge of this type then $s_{2e} = 4p \times si_2(e,f)$.

And if e be an other edge that don't belong to sets A and B then

$$sl_3(e,f) = \sum_{\substack{e \text{ is an edge} \\ e \notin A \\ e \notin B}} d_4(e,f) = (6pq - 2p - 1) \times (4pq - 1)$$

Since there exist $6pq - 8p$ edge of this type then $s_{3e} = (6pq - 3p) \times sl_3(e,f)$.

And the Edge Detour index of is as follow:

$$D_{e0}(G) = \frac{1}{2}(s_1 + s_2) = \begin{cases} p(6pq - 2p - 1)(4pq - 1)(3q - 1) & p \leq 3 \\ p(-48p^2q^2 + 72p^2q^3 - 30pq^2 + 16pq + 8p^2q - 3p + 5q + 2 & p > 3 \end{cases}$$

$$D_{e1}(G) = \frac{1}{2}(s_{1e} + s_{2e} + s_{3e}) = p(-48p^2q^2 + 72p^2q^3 - 30pq^2 + 16pq + 8p^2q - 4p + 3q + 3).$$

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