Electromagnetic scattering from anisotropic inhomogeneous impedance cylinder of arbitrary shape

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In this study, the electromagnetic scattering from anisotropic inhomogeneous impedance cylinder of arbitrary shape is presented for both TM and TE plane wave illuminations. In the solution of scattering problem, scattered TE and TM fields are expressed as single layer potentials. Using the boundary condition and jump relations of single layer potential on the boundary, boundary integral equation is obtained and solved via Nyström method. Obtained results are compared with those obtained by analytical method for inhomogeneous anisotropic impedance cylinder and good agreements are observed.

1. Introduction

The boundary conditions which electric and magnetic fields have to satisfy on the surfaces of object play an important role in scattering problems. One of these conditions is called as the impedance boundary condition (IBC) which gives a relation between tangential electric and magnetic field vectors on a given surface in terms of coefficient called as surface impedance. This kind of boundary condition are used for modelling coated surfaces, rough surfaces and used firstly by Leontovich [1] and Wait [2]. Generally, the surface impedance used for modelling of the scatterer is assumed to be constant scaler [3, 4]. However, when more accurate model of scatterer is considered, the surface impedance has to be a function of location such as modelling of inhomogeneous earth surface, even may be anisotropic to model anisotropic medium and corrugated surfaces. Therefore, most general IBC such as anisotropic and inhomogeneous IBC is to be considered in order to investigate the scattering from complicated material. Scattering from canonical structures whose surfaces satisfy inhomogeneous isotropic SIBC have been proposed in [5-7], scattering from inhomogeneous isotropic impedance cylinder of arbitrary shape has been investigated by Nyström method in [8] for nonzero surface impedance. Scattering from anisotropic inhomogeneous impedance circular cylinder has been presented in [9] by series expansion method. Scattering from anisotropic inhomogeneous impedance cylinder of arbitrary shape is solved by physical optics (PO) method in [10].

The main objective of this study is to describe method for the solution of the direct scattering problems with objects having arbitrary shape and anisotropic inhomogeneous impedance boundary conditions for both TE and TM plane wave illuminations. Proposed method is based on an integral representation of the scattered TM and TE fields through single layer potential that leads to a boundary integral equation through the jump relations of single layer potentials. This integral equation is well-posed and can be solved numerically through a Nyström method.

In Section 2, the scattering problem is formulated and solved. In Section 3, some examples are given. We also compare the results with those obtained by analytical technique [9] available for anisotropic inhomogeneous impedance cylinder. Both results match accurately. Finally, conclusions and concluding remarks are given in Section 4. A time factor $\exp\{-i\omega t\}$ is assumed and omitted throughout the paper.

2. Formulation and solution of the problem

The geometry of the considered scattering problem and parameters employed in the formulation are shown in Fig. 1. The object is defined by its boundary ∂D and inhomogeneous anisotropic surface impedance $\overline{\overline{Z}}(\vec{r}), \vec{r} \in \partial D$. The exterior environment is taken to be medium with permittivity ε, permeability μ and lossless.
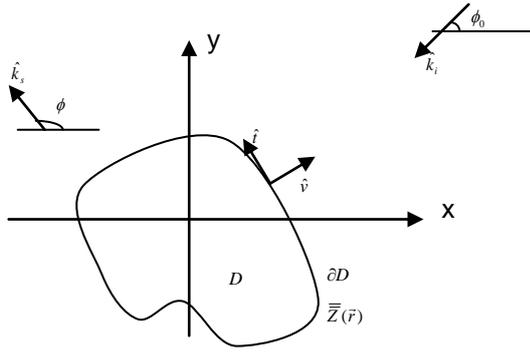
Fig. 1. The geometry of the problem

Cylinder is illuminated by monochromatic plane wave, whose electric field is along z axis that corresponds to TM illumination of the form as,

$$\vec{E}^i(\vec{r}) = (0,0,E_z^i(\vec{r}))$$
$$E_z^i(\vec{r}) = e^{ik\hat{k}_i.\vec{r}} \tag{1}$$

or whose magnetic field is along the z axis that corresponds to TE illumination of the form as,

$$\vec{H}^i(\vec{r}) = (0,0,H_z^i(\vec{r}))$$
$$H_z^i(\vec{r}) = \frac{1}{Z_0} e^{ik\hat{k}_i.\vec{r}} \tag{2}$$

where $Z_0 = \sqrt{\mu/\varepsilon}$ is characteristic impedance of exterior medium and $\hat{k}_i = -\cos\phi_0\hat{u}_x - \sin\phi_0\hat{u}_y$ is the propagation direction of incident field with incidence angle ϕ_0 and, $k = \omega\sqrt{\varepsilon\mu}$ is the wave number of exterior region. Due to the homogeneity of the problem with respect the z-axis, partial derivative with respect to z is zero. Since boundary condition is anisotropic, total field contains both TM ($E_z \neq 0, H_z = 0$) and TE ($E_z = 0, H_z \neq 0$) fields.

The TM and TE fields satisfy the reduced Helmholtz equation as

$$\Delta E_z + k^2 E_z = 0 \tag{3}$$
$$\Delta H_z + k^2 H_z = 0$$

and the inhomogeneous anisotropic IBC [9,10],

$$\hat{v}(\vec{r}) \times (\hat{v}(\vec{r}) \times \vec{E}) = -\overline{\overline{Z}}(\vec{r}).(\hat{v}(\vec{r}) \times \vec{H}(\vec{r})), \quad \vec{r} \in \partial D \tag{4}$$

and radiation conditions as

$$\lim_{r\to\infty} \sqrt{\rho}\left(\frac{\partial E_z^s}{\partial\rho} - ikE_z^s\right) = 0, \quad \rho = |\vec{r}| \tag{5}$$

$$\lim_{r\to\infty} \sqrt{\rho}\left(\frac{\partial H_z^s}{\partial\rho} - ikH_z^s\right) = 0, \quad \rho = |\vec{r}| \tag{6}$$

where $\hat{v}$ is unit normal vector on ∂D, and $\overline{\overline{Z}}(r)$ is inhomogeneous impedance dyadic expressed as,

$$\overline{\overline{Z}}(\vec{r}) = Z_{vv}(\vec{r})\hat{u}_z\hat{u}_z + Z_{zt}(\vec{r})\hat{u}_z\hat{t}(\vec{r}) + Z_{tt}(\vec{r})\hat{t}(\vec{r})\hat{t}(\vec{r}) + \\ + Z_{tz}(\vec{r})\hat{t}(\vec{r})\hat{u}_z, \vec{r} \in \partial D \tag{7}$$

where $\hat{t}(\vec{r}) = \hat{u}_z \times \hat{v}(\vec{r})$ is tangential unit vector on ∂D as depicted Fig. 1. As seen from (7), this is most general IBC and all kind of boundary condition discussed previously in literature can be expressed by appropriate choice of impedance functions. For example, if $\overline{\overline{Z}}(\vec{r}) = 0$, boundary condition described in (7) reduces to perfect electric conductor (PEC) condition. If $Z_{zt} = Z_{tz} = 0$ and $Z_{zz}, Z_{tt} \to \infty$, (7) reduces to perfect magnetic conductor (PMC).
If $Z_{zt} = Z_{tz} = 0$ and $Z_{zz}(\vec{r}) = Z_{tt}(\vec{r}) \neq 0, \vec{r} \in \partial D$, (7) reduces isotropic inhomogeneous IBC which is discussed in [5,8].

Let's represent the fields on the boundary ∂D as

$$\vec{E}(\vec{r}) = E_v(\vec{r})\hat{v}(\vec{r}) + E_t(\vec{r})\hat{t}(\vec{r}) + E_z(\vec{r})\hat{u}_z, \quad \vec{r} \in \partial D \tag{8}$$
$$\vec{H}(\vec{r}) = H_v(\vec{r})\hat{v}(\vec{r}) + H_t(\vec{r})\hat{t}(\vec{r}) + H_z(\vec{r})\hat{u}_z, \quad \vec{r} \in \partial D \tag{9}$$

Substituting (8) and (9) into (4), one obtains boundary conditions as

$$E_t(\vec{r}) = Z_{tz}(\vec{r})H_t(\vec{r}) - Z_{tt}(\vec{r})H_z(\vec{r}), \quad \vec{r} \in \partial D \tag{10}$$
$$E_z(\vec{r}) = Z_{zz}(\vec{r})H_t(\vec{r}) - Z_{zt}(\vec{r})H_z(\vec{r}), \quad \vec{r} \in \partial D \tag{11}$$

By using Maxwell equations, one can obtain as,

$$H_t(\vec{r}) = \frac{i}{kZ_0}\frac{\partial E_z}{\partial v}(\vec{r}), \quad \vec{r} \in \partial D \tag{12}$$

$$E_t(\vec{r}) = \frac{-iZ_0}{k}\frac{\partial H_z}{\partial v}(\vec{r}), \quad \vec{r} \in \partial D \tag{13}$$

Substituting (12) and (13) into (8) and (9), one obtains

$$\frac{-iZ_0}{k}\frac{\partial H_z}{\partial v}(\vec{r}) = Z_{tz}(\vec{r})\frac{i}{kZ_0}\frac{\partial E_z}{\partial v}(\vec{r}) - Z_{tt}(\vec{r})H_z(\vec{r}), \tag{14}$$
$$\vec{r} \in \partial D$$

$$E_z(\vec{r}) = Z_{zz}(\vec{r})\frac{i}{kZ_0}\frac{\partial E_z}{\partial v}(\vec{r}) - Z_{zt}(\vec{r})H_z(\vec{r}), \quad \vec{r} \in \partial D \quad (15)$$

Since E_z^s and H_z^s satisfy Helmholtz equation and radiation condition, they can be expressed by single layer potential on the closed exterior of ∂D respectively as [11, 12]

$$E_z^s(\vec{r}) = (S\Phi)(\vec{r}) = \int_{\partial D}\Phi(\vec{r}')G(\vec{r},\vec{r}')ds(\vec{r}'), \quad \vec{r} \in R^2 \setminus \overline{D} \quad (16)$$

$$H_z^s(\vec{r}) = (S\Psi)(\vec{r}) = \int_{\partial D}\Psi(\vec{r}')G(\vec{r},\vec{r}')ds(\vec{r}'), \vec{r} \in R^2 \setminus \overline{D} \quad (17)$$

where S is single layer integral operator Φ and Ψ are unknown densities and $G(\vec{r},\vec{r}')$ is Green's function of exterior medium given by

$$G(\vec{r},\vec{r}') = \frac{i}{4}H_0^{(1)}(k|\vec{r}-\vec{r}'|) \quad (18)$$

where $H_0^{(1)}(.)$ is the Hankel function of the first kind and of order zero. Representation (16) and (17) can be used to evaluate fields and its normal derivative on boundary ∂D by using jump relation of single layer potential [11, 12] as

$$E_z^s(\vec{r}) = (S\Phi)(\vec{r}) = \int_{\partial D}\Phi(\vec{r}')G(\vec{r},\vec{r}')ds(\vec{r}'), \quad \vec{r} \in \partial D \quad (19)$$

$$H_z^s(\vec{r}) = (S\Psi)(\vec{r}) = \int_{\partial D}\Psi(\vec{r}')G(\vec{r},\vec{r}')ds(\vec{r}'), \quad \vec{r} \in \partial D \quad (20)$$

$$\frac{\partial E_z^s}{\partial v}(\vec{r}) = (K\Phi)(\vec{r}) - \frac{1}{2}\Phi(\vec{r}) = \int_{\partial D}\Phi(\vec{r}')\frac{G(\vec{r},\vec{r}')}{\partial v(\vec{r})}ds(\vec{r}') - \frac{1}{2}\Phi(\vec{r}), \quad \vec{r} \in \partial D \quad (21)$$

$$\frac{\partial H_z^s}{\partial v}(\vec{r}) = (K\Psi)(\vec{r}) - \frac{1}{2}\Psi(\vec{r}) = \int_{\partial D}\Psi(\vec{r}')\frac{G(\vec{r},\vec{r}')}{\partial v(\vec{r})}ds(\vec{r}') - \frac{1}{2}\Psi(\vec{r}), \quad \vec{r} \in \partial D \quad (22)$$

substituting (19),(20),(21) and (22) into (14) and (15), boundary integral equation is obtained as

$$\frac{-iZ_0}{k}((K\Psi)(\vec{r}) - \frac{1}{2}\Psi(\vec{r})) - Z_{tz}(\vec{r})\frac{i}{kZ_0}((K\Phi)(\vec{r}) - \frac{1}{2}\Phi(\vec{r})) + Z_{tt}(\vec{r})(S\Psi)(\vec{r}) = f(\vec{r}), \quad \vec{r} \in \partial D \quad (23)$$

$$(S\Phi)(\vec{r}) - Z_{zz}(\vec{r})\frac{i}{kZ_0}((K\Phi)(\vec{r}) - \frac{1}{2}\Phi(\vec{r})) + Z_{zt}(\vec{r})(S\Psi)(\vec{r}) = g(\vec{r}), \quad \vec{r} \in \partial D \quad (24)$$

where $f(\vec{r})$ and $g(\vec{r})$ are functions depends on illumination. For TM illumination case,

$$f(\vec{r}) = -(Z_{tz}(\vec{r})/Z_0)\hat{v}(\vec{r}).\hat{k}_i e^{ik\hat{k}_i.\vec{r}}, \quad g(\vec{r}) = -((Z_{zz}(\vec{r})/Z_0)\hat{v}(\vec{r}).\hat{k}_i + 1)e^{ik\hat{k}_i.\vec{r}}, \quad \vec{r} \in \partial D \quad (25)$$

For TE illumination case,

$$f(\vec{r}) = -(\hat{v}(\vec{r}).\hat{k}_i + Z_{tt}(\vec{r})/Z_0)e^{ik\hat{k}_i.\vec{r}}, \quad g(\vec{r}) = -(Z_{zt}(\vec{r})/Z_0)e^{ik\hat{k}_i.\vec{r}}, \quad \vec{r} \in \partial D \quad (26)$$

Let's represent the boundary and surface impedance of the object D by parametric equations given by

$$D = \left\{\partial D : r(t) = (x(t), y(t)), \overline{\overline{Z}} : \overline{\overline{Z}}(t), t \in [0, 2\pi]\right\} \quad (27)$$

and unit normal vector on the boundary ∂D can be defined as

$$\hat{v}(t) = \frac{(y'(t), -x'(t))}{|r'(t)|} \quad (28)$$

By using the parametrization in (27), Integral equations (23) and (24) can be written more compact forms respectively as

$$\int_0^{2\pi} L_1(t,\tau)\Phi(\tau)d\tau + \int_0^{2\pi} L_2(t,\tau)\Psi(\tau)d\tau + \frac{iZ_{tz}(t)}{2kZ_0}\Phi(t) + \frac{iZ_0}{2k}\Psi(t) = f(r(t)) \tag{29}$$

$$\int_0^{2\pi} L_3(t,\tau)\Phi(\tau)d\tau + \int_0^{2\pi} L_4(t,\tau)\Psi(\tau)d\tau + \frac{iZ_{zz}(t)}{2kZ_0}\Phi(t) = g(r(t)) \tag{30}$$

where L_1, L_2, L_3, L_4 are Kernels of the integral operators given by

$$L_1(t,\tau) = -\frac{Z_{tz}(t)}{4Z_0}\frac{\hat{v}(t).\{r(t)-r(\tau)\}}{|r(t)-r(\tau)|}H_1^{(1)}(k|r(t)-r(\tau)|)|r'(\tau)| \tag{31}$$

$$L_2(t,\tau) = \frac{iZ_{tt}(t)}{4}H_0^{(1)}(k|r(t)-r(\tau)|)|r'(\tau)| - Z_0\frac{\hat{v}(t).\{r(t)-r(\tau)\}}{4|r(t)-r(\tau)|}H_1^{(1)}(k|r(t)-r(\tau)|)|r'(\tau)| \tag{32}$$

$$L_3(t,\tau) = \frac{i}{4}H_0^{(1)}(k|r(t)-r(\tau)|)|r'(\tau)| - \frac{Z_{zz}(t)}{4Z_0}\frac{\hat{v}(t).\{r(t)-r(\tau)\}}{|r(t)-r(\tau)|}H_1^{(1)}(k|r(t)-r(\tau)|)|r'(\tau)| \tag{33}$$

$$L_4(t,\tau) = -Z_{zt}(t)\frac{i}{4}H_0^{(1)}(k|r(t)-r(\tau)|)|r'(\tau)| \tag{34}$$

In order to implement Nyström method, Kernel of the integrals are decomposed as

$$L_n(t,\tau) = L_n^{(1)}(t,\tau)\ln\left(4\sin^2\frac{t-\tau}{2}\right) + L_n^{(2)}(t,\tau), \quad n=1,..,4 \tag{35}$$

where

$$L_1^{(1)}(t,\tau) = \frac{Z_{tz}(t)}{4\pi i Z_0}\frac{\hat{v}(t)\{r(t)-r(\tau)\}}{|r(t)-r(\tau)|}J_1(k|r(t)-r(\tau)|)|r'(\tau)| \tag{36}$$

$$L_2^{(1)}(t,\tau) = -\frac{Z_{tt}(t)}{4\pi}J_0(k|r(t)-r(\tau)|)|r'(\tau)| + \frac{Z_0}{4\pi i}\frac{\hat{v}(t)\{r(t)-r(\tau)\}}{|r(t)-r(\tau)|}J_1(k|r(t)-r(\tau)|)|r'(\tau)| \tag{37}$$

$$L_3^{(1)}(t,\tau) = -\frac{1}{4\pi}J_0(k|r(t)-r(\tau)|)|r'(\tau)| + \frac{Z_{zz}(t)}{4\pi i Z_0}\frac{\hat{v}(t)\{r(t)-r(\tau)\}}{|r(t)-r(\tau)|}J_1(k|r(t)-r(\tau)|)|r'(\tau)| \tag{38}$$

$$L_4^{(1)}(t,\tau) = \frac{Z_{zt}(t)}{4\pi}J_0(k|r(t)-r(\tau)|)|r'(\tau)| \tag{39}$$

and

$$L_n^{(2)}(t,\tau) = L_n(t,\tau) - L_n^{(1)}(t,\tau)\ln\left(4\sin^2\frac{t-\tau}{2}\right), \quad n=1,..,4 \tag{40}$$

with diagonal terms

$$L_1^{(1)}(t,t) = 0 \tag{41}$$

$$L_2^{(1)}(t,t) = -\frac{Z_{tt}(t)}{4\pi}|r'(t)| \tag{42}$$

$$L_3^{(1)}(t,t) = -\frac{1}{4\pi}|r'(t)| \tag{43}$$

$$L_4^{(1)}(t,t) = \frac{Z_{zt}(t)}{4\pi}|r'(t)| \tag{44}$$

and

$$L_1^{(2)}(t,t) = \frac{Z_{tz}(t)\hat{v}(t).r''(t)}{4\pi i k Z_0 |r'(t)|} \tag{45}$$

$$L_2^{(2)}(t,t) = \frac{Z_{tt}(t)}{2}\left[\frac{i}{2} - \frac{C}{\pi} - \frac{1}{\pi}\ln\frac{k|r'(t)|}{2}\right]|r'(t)| + \frac{Z_0\hat{v}(t).r''(t)}{4\pi i k |r'(t)|} \tag{46}$$

$$L_3^{(2)}(t,t) = \frac{1}{2}\left[\frac{i}{2} - \frac{C}{\pi} - \frac{1}{\pi}\ln\frac{k|r'(t)|}{2}\right]|r'(t)| + \frac{Z_{zz}(t)\hat{v}(t).r''(t)}{4\pi i k Z_0 |r'(t)|} \tag{46'}$$

$$L_4^{(2)}(t,t) = \frac{-Z_{zt}(t)}{2}\left[\frac{i}{2} - \frac{C}{\pi} - \frac{1}{\pi}\ln\frac{k|r'(t)|}{2}\right]|r'(t)| \tag{47}$$

where $C = 0.57721...$ is Euler constant. If the boundary curve and the impedance functions both are analytic, then $L_n^{(1)}$ and $L_n^{(2)}, n = 1,2,3,4$ are also analytic. Therefore, their integrals can be efficiently approximated by trigonometric interpolation quadrature formulas. More precisely, we choose 2N equidistant grid points $t_p = p\pi / N$, $p = 0,1,...2N-1$ and use the quadrature rule for any function $h(t), t \in [0,2\pi)$ that is

$$\int_0^{2\pi} L_n^{(1)}(t_m,\tau)\ln(4\sin^2(\frac{t-\tau}{2}))h(\tau)d\tau \approx \sum_{p=0}^{2N-1} R_{|m-p|}^{(N)} L_n^{(1)}(t_m,t_p)h(t_p) \tag{48}$$

with the quadrature weights

$$R_p^{(N)} = -\frac{2\pi}{N}\sum_{m=1}^{N-1}\frac{1}{m}\cos(\frac{mp\pi}{N}) - \frac{(-1)^p\pi}{N^2} \tag{49}$$

and the trapezoidal rule

$$\int_0^{2\pi} L_n^{(2)}(t_m,\tau)h(\tau)d\tau \approx \frac{\pi}{N}\sum_{p=0}^{2N-1} L_n^{(2)}(t_m,t_p)h(t_p) \tag{50}$$

Nyström method is to use approximations given by (48) and (50) that lead to approximating the integral equation (29) and (30) by solving the linear system

$$\sum_{p=0}^{2N-1}\left\{R_{|m-p|}^{(N)} L_1^{(1)}(t_m,t_p) + \frac{\pi}{N} L_1^{(2)}(t_m,t_p)\right\}\Phi_p^{(N)} + \sum_{p=0}^{2N-1}\left\{R_{|m-p|}^{(N)} L_2^{(1)}(t_m,t_p) + \frac{\pi}{N} L_2^{(2)}(t_m,t_p)\right\}\Psi_p^{(N)}$$
$$+ \frac{iZ_{tz}(t_m)}{2kZ_0}\Phi_m^{(N)} + \frac{iZ_0}{2k}\Psi_m^{(N)} = f(r(t_m)), \quad m = 0,1,2,...2N-1 \tag{51}$$

$$\sum_{p=0}^{2N-1}\left\{R_{|m-p|}^{(N)} L_3^{(1)}(t_m,t_p) + \frac{\pi}{N} L_3^{(2)}(t_m,t_p)\right\}\Phi_p^{(N)} + \sum_{p=0}^{2N-1}\left\{R_{|m-p|}^{(N)} L_4^{(1)}(t_m,t_p) + \frac{\pi}{N} L_4^{(2)}(t_m,t_p)\right\}\Psi_p^{(N)}$$
$$+ \frac{iZ_{zz}(t_m)}{2kZ_0}\Phi_m^{(N)} = g(r(t_m)), \quad m = 0,1,2,...2N-1 \tag{52}$$

for approximations $\Phi_p^{(N)}$ and $\Psi_p^{(N)}$ to the values $\Phi(t_p)$ and $\Psi(t_p)$ respectively of the solution at the grid points. This Nyström method can be shown to converge for continuous $L_n^{(1)}$ and $L_n^{(2)}, n = 1,2,3,4$, i.e., for twice continuously differentiable boundaries and continuous impedance functions. Moreover, for the case of analytic boundary curves and impedances it enjoys an exponential convergence rate, i.e., doubling the number of grid points doubles the number of correct digits in the approximation. For more details on the Nyström method we refer the readers to [11, 12].

Once the boundary integral equations (29) and (30) are solved the near and far fields of the TM and TE scattered waves can be calculated through (19) and (20) respectively. Let's call TM and TE fields as vertically (V) and horizontally (H) waves respectively. Polarimetry scattering width $\sigma_{ab}, a = H,V; b = H,V$ is related to ratio of a polarized scattered power to b polarized incident power and defined by

$$\sigma_{ab}(\phi) = \lim_{\rho\to\infty} 2\pi\rho\frac{|E_a(\rho,\phi)|^2}{|E_b|^2}, \quad a = H,V; b = H,V \tag{53}$$

3. Numerical results

The proposed procedure has been applied to two illustrative examples. In all examples, wave number of exterior medium and incidence angle are chosen as $k = 1$ and $\phi_0 = 0^0$ respectively. Integral equations (51) and (52) are solved for $N = 50$. The first example is dedicated to validate proposed integral equation method for TM illumination. For this purpose, scattering from anisotropic inhomogeneous impedance circular cylinder whose analytical solution is available in [9] is considered. Obtained results are compared with those obtained by analytical method in [9] and good agreements are observed. In the second example, electromagnetic scattering from kite shaped anisotropic inhomogeneous impedance cylinder is solved by proposed method.

Case 1

Scattering from anisotropic inhomogeneous impedance circular cylinder with radius 1m is considered for TM incident case. Parameterization of boundary and anisotropic surface impedances are given respectively as

$$\partial D = \{r(t) = (\cos t, \sin t), t \in [0, 2\pi)\} \text{ and}$$
$$Z_{zz}(t) = 100(1+i)\cos(t) , \; Z_{zt}(t) = 50(1+2i)\sin(2t)$$
$$Z_{tz}(t) = 50(2+i)\cos(2t), Z_{tt}(t) = 100(1+i\sin t) .$$
$$t \in [0, 2\pi).$$

Since the incident field is TM wave, TM and TE scattered widths are called as σ_{VV} and σ_{HV}, depicted in Fig. 2 and Fig. 3 respectively. Obtained results are compared with those obtained by analytical method in [9]. As seen from Fig. 2 and Fig. 3, good agreements are observed.
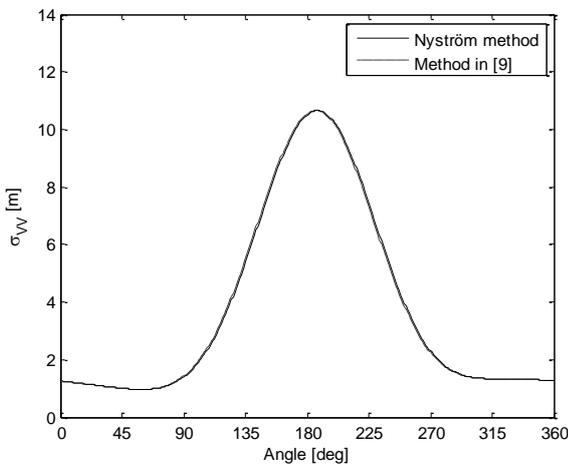
Fig. 2. Scattering width σ_{VV} of the circular cylinder for case 1
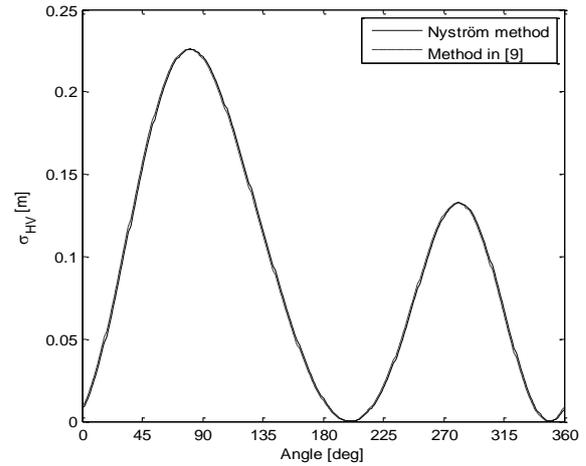
Fig. 3. Scattering width σ_{HV} of the circular cylinder for case 1

Case 2

Proposed method is applied for solution of scattering from anisotropic inhomogeneous impedance kite shaped cylinder for TM and TE incident cases. Parametric representation of boundary and anisotropic surface impedances are given respectively as $\partial D = \{r(t) = (\cos t + 0.65\cos(2t) - 0.65, 1.5\sin t), t \in [0, 2\pi)\}$ and,

$$Z_{zz}(t) = 20(3+4i)\cos(3t), \qquad Z_{zt}(t) = 50(2+i)t\sin(2t),$$
$$Z_{tz}(t) = 100(1+ti)\cos(t), \; Z_{tt}(t) = 30(t+i\sin t)\cos t,$$

$t \in [0, 2\pi)$. Polarimetry scattering widths $\sigma_{VV}, \sigma_{VH}, \sigma_{HV}$ and σ_{HH} based on definition (53) are obtained. σ_{VV} and σ_{HV} are depicted in Fig. 4. σ_{VH} and σ_{HH} are depicted in Fig. 5.
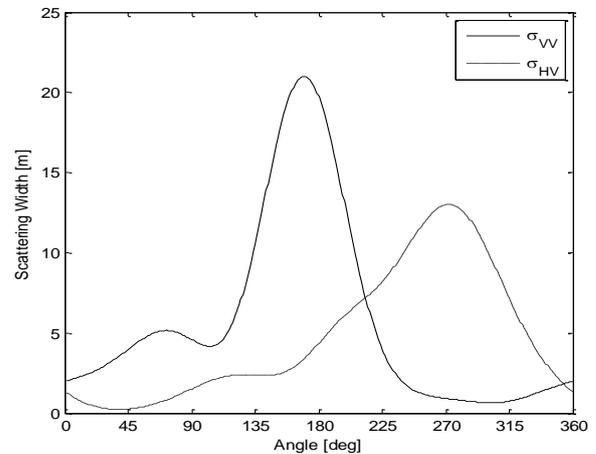
Fig. 4. Polarimetry scattering widths σ_{VV} and σ_{HV} of the kite shaped object for case 2
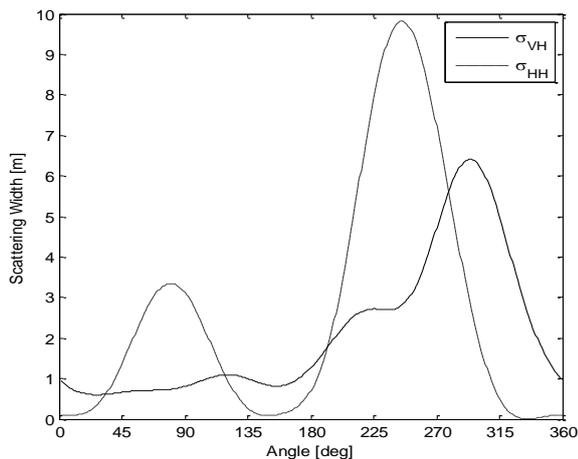
Fig. 5. Polarimetry scattering widths σ_{VH} and σ_{HH}
of the kite shaped object for case 2

4. Conclusions

The surface impedance of scattering object can be a function of location and anisotropic depending on the geometrical and physical properties of the scattering object. The problems involving anisotropic inhomogeneous IBC are important from both mathematical and physical points of view. These problems have practical applications such as antenna design and radar cross section (RCS) reduction for specific purposes. By choosing appropriate anisotropic surface impedance, a certain radiation and polarization patterns of antenna or minimize RCS of the target for particular direction can be obtained.

In this study, electromagnetic scattering from anisotropic inhomogeneous impedance cylinder of arbitrary shape is considered by integral equations which are solved by numerical effective Nyström method for both TM and TE illumination. For this reason, the scattered TM and TE fields are represented by single-layer potentials firstly. Using the jump relations of single layer potentials and its normal derivatives on the boundary, boundary integral equations are obtained and solved by numerical effective Nyström method which has exponential convergence property. Obtained results are compared with those obtained by analytical method for circular cylinder with anisotropic inhomogeneous IBC and good agreements are observed.

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