Evaluation of the negative refractive index by beam deviation measurements

A. BELARDINI*, C. SIBILIA
Dipartimento di Scienze di Base ed Applicate per l'Ingegneria – Sapienza Università di Roma and CNISM
Via A. Scarpa 14, I-00161 Roma, Italy

Here we present an easy experimental method that allows the characterization of the negative refractive index of a isotropic metamaterial in the visible spectral region. The method is based on the measurement of the deviation of a light beam passing through the metamaterial as a function of the incident angle. The theoretical expression was derived in the case of negative refraction. It was shown that such a method can be used also in the realistic case of a thin metamaterial deposited on a thick substrate.

(Received November 7, 2012; accepted April 11, 2013)

Keywords: Negative index materials characterization, Metamaterials characterization, Optical characterization

1. Introduction

In 1968 V. G. Veselago described, from the theoretical point of view, the behavior of negative index materials and envisaged the possibility to their realization [1]. In more recent years J. B. Pendry demonstrate that negative index materials can be used in order to realize perfect lenses with super-resolution [2] and other exotic functionalities such as cloaking or invisibility [3,4]. From the theoretical point of view, light refracted by isotropic negative index materials bents by following negative angles with respect to the path followed in the case of positive materials (Fig. 1a,b) as described by Snell's law

$$n_1 Sin(\phi) = n_2 Sin(\phi') \qquad (1)$$

when one of the two refractive indices, either n_1 or n_2, is negative.

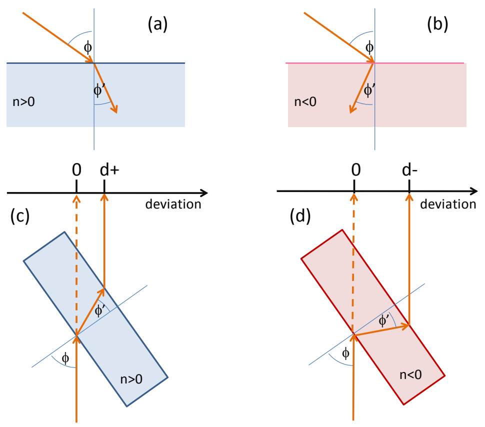

Fig. 1. Illustration of the refracted light in presence of (a) positive index material and (b) negative index material; deviation of a light beam introduced by a positive index material (c) and negative index material (d).

In order to present a negative index it is necessary that both the relative permittivity ε_r and the permeability μ_r of a material are negative [1]. In nature there aren't materials that present at the same time $\varepsilon_r < 0$ and $\mu_r < 0$, so different artificial materials were proposed, the so called metamaterials, in which geometric features (smaller than the wavelength of interest) determine an effective negative index behavior. First examples of real metamaterials were obtained in the microwave regime where the dimensions of the features are in the millimeter scale [5,6], but effort were devoted in order to scale down in the infrared and visible regime [7]. In 2004 J. B. Pendry demonstrated that it is possible to obtain negative index materials by using chiral materials without the stringent condition to have both negative ε_r and μ_r [8]. In that materials chirality gives rise to the phenomenon of optical activity [9] where the speed of light is different for left handed circular polarized light with respect to the right handed one.

On these bases a lot of different metamaterials were realized showing 3D chirality [10], 2D chirality [11,12] or even extrinsic chirality [13-15] in the visible optical range.

From the point of view of the metamaterials characterization, different techniques have been used in order to study the effective behavior of a metamaterial in the optical frequency. For example second harmonic generation in different configurations [16-18] is considered as a very sensitive method in order to retrieve sub-wavelength morphological symmetries, like the presence of an effective optical activity [18-20]. However in order to verify the presence of an effective negative index behavior other complex techniques must be used, like phase delay measurements or coupled transmission and reflection measurements, so that information on phase velocity of the light are obtained [13,21].

Here we want to present a fast and easy way to characterize the negative index behavior of a planar slab of a isotropic metamaterial, by measuring the deviation of a light beam passing through the metamaterial as a function of the incidence angle (see Fig.1c,d). This technique relies on the statement that the light passing in a negative index material must by refracted by following the Snell's law (eq.1) in a reversed way with respect to positive index materials.

2. Method

By looking at Fig. 2 it is possible to calculate the deviation *d* of the light beam introduced by a planar material of thickness *t* and refractive index n_2 as a function of the incidence angle ϕ.

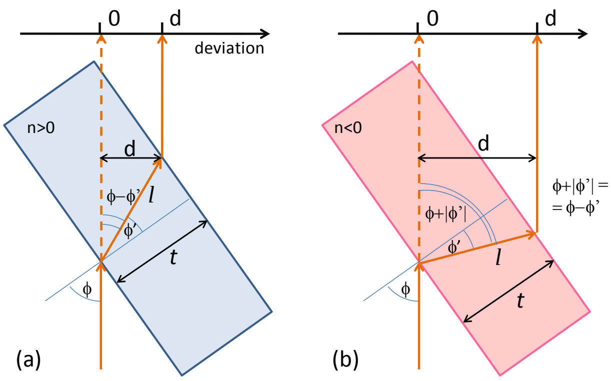

Fig. 2. Schematic of the geometrical parameters used in the calculation of the deviation of light (a) in the case of positive index materials and (b) in the case of negative index materials.

The deviation *d* is calculated with respect to the un-refracted light that can be observed when light impinges at normal incidence ($\phi = 0$). From Fig. 2, in both the cases of positive or negative refractive index n_2, *d* results to be

$$d = l \cdot \sin(\phi - \phi') \quad , \qquad (2)$$

where ϕ' is the internal refracted angle given by eq.1 and *l* is the geometrical length of the beam path inside the material. The relation between *l* and *t* is given, in both cases of positive or negative refractive index, by:

$$t = l \cdot \cos(\phi') = l \cdot \cos(-\phi') \quad . \qquad (3)$$

By substituting the value of *l* found in eq. 3 inside eq.2 it is possible to obtain the relation of the deviation introduced by the planar material:

$$d = t \cdot \frac{\sin(\phi - \phi')}{\cos(\phi')} \quad . \qquad (4)$$

By remembering the trigonometric relation

$$\sin(\phi - \phi') = \sin(\phi)\cos(\phi') - \sin(\phi')\cos(\phi) \qquad (5)$$

the eq.4 can be rewritten as:

$$\frac{d}{t} = \frac{\sin(\phi)\cos(\phi') - \sin(\phi')\cos(\phi)}{\cos(\phi')} = \sin(\phi) \cdot \left[1 - \frac{\sin(\phi')\cos(\phi)}{\sin(\phi)\cos(\phi')}\right] = \sin(\phi) \cdot \left[1 - \frac{1}{n_2/n_1} \frac{\cos(\phi)}{\cos(\phi')}\right] \qquad (6)$$

where it was also used the eq.1.

Since the internal angle ϕ' can lies only in the [-π/2; π/2] range it results that:

$$\cos(\phi') = \sqrt{1 - \sin^2(\phi')} \, , \qquad (7)$$

then, by using again eq.1, it is possible to rewrite eq.6 by explicating the dependence of *d* (normalized to the thickness *t*) with respect to the ratio of the refractive indexes $n = n_2/n_1$ and the external incidence angle ϕ :

$$\frac{d}{t} = \sin(\phi) \cdot \left[1 - \frac{1}{n_2/n_1} \frac{\cos(\phi)}{\sqrt{1 - \sin^2(\phi')}}\right] = \sin(\phi) \cdot \left[1 - \frac{1}{n_2/n_1} \frac{\cos(\phi)}{\sqrt{1 - \frac{\sin^2(\phi)}{(n_2/n_1)^2}}}\right] =$$

$$= \sin(\phi) \cdot \left[1 - \frac{1}{\frac{n_2/n_1}{|n_2/n_1|}} \frac{\cos(\phi)}{\sqrt{(n_2/n_1)^2 - \sin^2(\phi)}}\right] = \sin(\phi) \cdot \left[1 \mp \frac{\cos(\phi)}{\sqrt{(n_2/n_1)^2 - \sin^2(\phi)}}\right] \quad , \qquad (8)$$

where the minus sign holds for positive value of the ratio $n = n_2/n_1$ and the plus sign holds for negative values. As it is known from the internal total reflection phenomenon, eq.8 is valid only when: $(n_2/n_1)^2 > \sin^2(\phi)$, therefore it is always valid in the case of $|n_2/n_1| \geq 1$, meanwhile in the case when $|n_2/n_1| < 1$, it is valid only when the incidence angle is below the critical angle $\phi_c = \arcsin(n_2/n_1)$.

In Fig. 3 the eq. 8 is shown for different values of the ratio $n = n_2/n_1$ ranging in both positive and negative region (in Fig. 3a is shown the eq.8 in the case of $|n| \geq 1$, in Fig. 3b is shown the eq.8 in the case $|n| \leq 1$). The graph is shown only for positive incidence angle ϕ, because eq. 8 is anti-symmetric, thus $d/t\{\phi\} = -d/t\{-\phi\}$. By comparing the measured deviation obtained from a sample of a metamaterial with the curves in Fig. 3 or by fitting the data with eq. 8, it is possible to retrieve the effective index when the thickness of the sample is known.

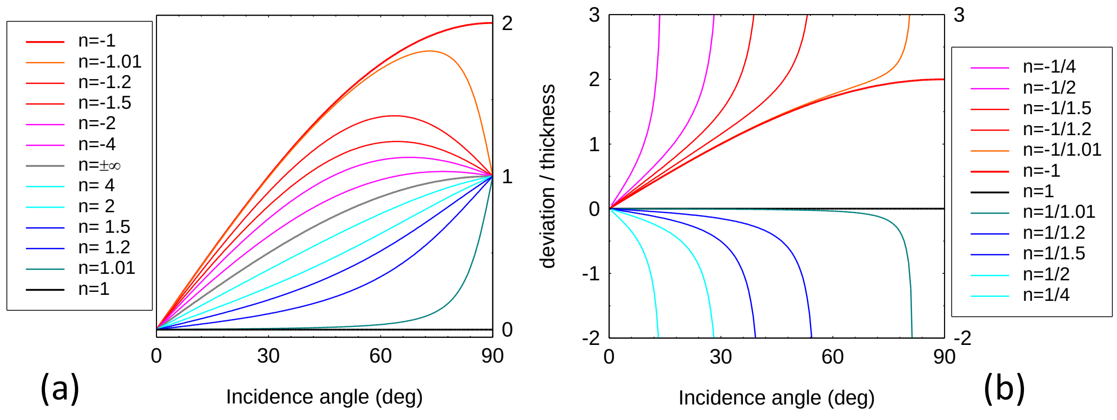

Fig. 3. Graph of the deviation normalized to the sample thickness as a function of the incidence angle (a) for $|n| \geq 1$ *and (b) for* $|n| \leq 1$.

What it is worth of interest is the fact that, in the case of negative index ($n < -1$) the curves of Fig. 3a present a relative maximum in the deviation that is achieved for incidence angles lesser than 90°. This allows to know immediately if we are in the presence of a negative index metamaterial even if its thickness is unknown.

Usually metamaterials in optical regime are fabricated on dielectric substrate (see Fig. 4a). The Snell's law in eq.1 applied to two (or more) layers states that:

$$n_0 \sin(\phi) = n_1 \sin(\phi') = n_2 \sin(\phi'') \quad , \quad (9)$$

So that the refracted angle inside the metamaterial depends only on the external incident angle and not on the index of refraction of the substrate.

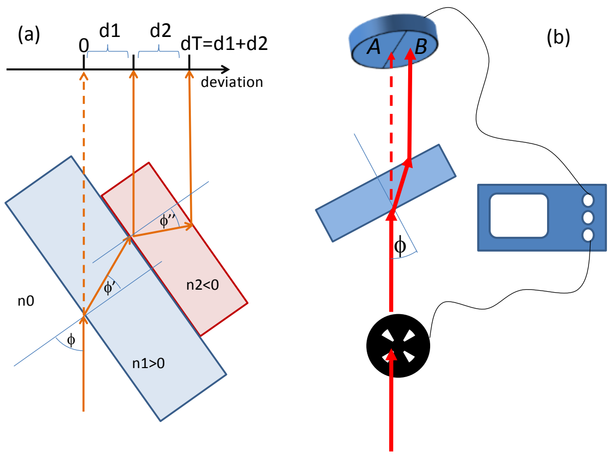

Fig. 4. (a) deviation introduced by two layers; (b) schematic of a potential experimental set-up.

In this case the total deviation d_T results to be the sum of the deviation obtained on the bare substrate d_1 and the deviation obtained on the metamaterial d_2. This fact can be very useful in practical cases where it is possible to measure directly only the total deviation and the deviation given by the bare substrate. The deviation pertinent to the metamaterial will result simply by:

$$d_2 = d_T - d_1 \quad . \quad (10)$$

Then the refractive index of the metamaterial can be obtained by fitting the data obtained from eq.10.

3. Experimental set-up

The above considerations allow us to propose a possible scheme of a experimental set-up, similar to the one described in ref. 22, that allows to measure the deviation of the light (see Fig. 4b). A collimated light beam is modulated by a chopper, than the light impinges on the sample that is posed on a motorized rotation stage. The light at the output of the sample is detected by a photodiode dived in sectors (labeled *A* for the left side sector and *B* for the right side sector in Fig. 4b). The output of the sensor was analysed by a position detector circuit giving the normalised difference between the power of the light in the sector *A* and the power of the light in sector *B*:

$$X = \frac{P_A - P_B}{P_A + P_B} \quad . \quad (11)$$

The normalization is necessary in order to remove the effect of different light intensities, due, for example, to the different transmitted signal for *s* and *p* polarization state. The *X* value is proportional to the displacement of the beam with respect to the centre of the detector and was supplied in volts at the output of the circuit. Measurements conversion from the circuit signal (in volts) to the real displacement (in mm) is retrieved by a calibration measurement performed, for example, onto a glass slide of known thickness. In Fig. 5 we show the experimental measurements of the deviation introduced by a 1 mm thick microscope glass slide, when the incoming light (λ=400nm) is polarized in *s* state (black triangles) and *p* state (red squares).

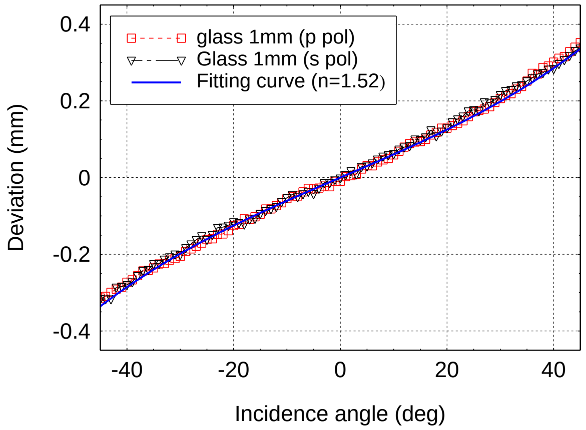

Fig. 5. Experimental deviation of a 1mm glass slide for s polarized light (black triangles) and p polarized light (red triangles). The blue curve is the fitting curve obtained by using eq. 8 with the index of refraction n as the free parameter.

Both curves can be fitted by eq.8 (blue line in Fig. 5) resulting the refractive index of glass *n*=1.52 with a standard deviation of σ_n=0.00546. With this kind of set-up we have already demonstrated that it is possible to obtain sensitivity better than 100 nm [22].

4. Conclusions

The method here presented is an easy and sensitive way useful in the evaluation of the negative refractive index of a isotropic metamaterial. The general relation that links deviation and index of refraction that we retrieved, can be used both for positive and negative index materials. In particular the method can be used in the case of large interest when the metamaterial is fabricated on a thicker substrate.

Acknowledgements

This work has been performed in the framework of the project “MARINE” granted by Italian Ministry of Defense.

References

[1] V. G. Veselago, Soviet Physics Uspekhi **10**, 509 (1968).
[2] J. B. Pendry, Phys. Rev. Lett. **85**, 3966 (2000).
[3] J. B. Pendry, D. Schurig, D. R. Smith, Science **312**, 1780 (2006).
[4] D. Schurig, J. J. Mock, B. J. Justice, S. A. Cummer, J. B. Pendry, A. F. Starr, D. R. Smith, Science **314**, 977 (2006).
[5] R. Shelby, D. R. Smith, S. Schultz, Science **292**, 77 (2001).
[6] D. R. Smith, J. B. Pendry, M. C. K. Wiltshire, Science **305**, 788 (2004).
[7] C. M. Soukoulis, S. Linden, M. Wegener, Science **315**, 47 (2007).
[8] J. B. Pendry, Science **306**, 1353 (2004).
[9] E. Fazio, W. Ramadan, A. Belardini, A. Bosco, M. Bertolotti, A. Petris, V. I. Vlad, Phys. Rev. E **67**, 026611 (2003).
[10] J. K. Gansel, M. Thiel, M. S. Rill, M. Decker, K. Bade, V. Saile, G. von Freymann, S. Linden, M. Wegener, Science **325**, 1513 (2009).
[11] A. S. Schwanecke, A. Krasavin, D. M. Bagnall, A. Potts, A. V. Zayats, N. I. Zheludev, Phys. Rev. Lett. **91**, 247404 (2003).
[12] V. K. Valev, N. Smisdom, A. V. Silhanek, B. De Clercq, W. Gillijns, M. Ameloot, V. V. Moshchalkov, T. Verbiest, Nano Lett. **9**, 3945 (2009).
[13] E. Plum, X.-X. Liu, V. A. Fedotov, Y. Chen, D. P. Tsai, N. I. Zheludev, Phys. Rev. Lett. **102**, 113902 (2009).
[14] S. N. Volkov, K. Dolgaleva, R. W. Boyd, K. Jefimovs, J. Turunen, Y. Svirko, B. K. Canfield, M. Kauranen, Phys. Rev. A **79**, 043819 (2009).
[15] A. Belardini, M. C. Larciprete, M. Centini, E. Fazio, C. Sibilia, D. Chiappe, C. Martella, A. Toma, M. Giordano, F. Buatier de Mongeot, Phys. Rev. Lett. **107**, 257401 (2011).
[16] T. Verbiest, M. Kauranen, Y. Van Rompaey, A. Persoons, Phys. Rev. Lett. **77**, 1456 (1996).
[17] T. Verbiest, M. Kauranen, A. Persoons, Phys. Rev. Lett. **82**, 3601 (1999).
[18] M. C. Larciprete, F. A. Bovino, M. Giardina, A. Belardini, M. Centini, C. Sibilia, M. Bertolotti, A. Passaseo, V. Tasco, Opt. Lett. **34**, 2189 (2009).
[19] T. Verbiest, K. Clays, V. Rodriguez, CRC Press, NY (USA), 2009.
[20] G. D’Aguanno, M. C. Larciprete, N. Mattiucci, A. Belardini, M. J. Bloemer, E. Fazio, O. Buganov, M. Centini, C. Sibilia, Phys. Rev. A **81**, 013834 (2010).
[21] D. R. Smith, S. Schultz P. Markos, C. M. Soukoulis, Phys. Rev. B **65**, 195104 (2002).
[22] A. Belardini, F. Pannone, G. Leahu, M. C. Larciprete, M. Centini, C. Sibilia, C. Martella, M. Giordano, D. Chiappe, F. Buatier de Mongeot, Appl. Phys. Lett. **100**, 251109 (2012).

*Corresponding author: alessandro.belardini@uniroma1.it