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# Exact solutions of a nonlocal nonlinear Schrödinger equation

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In this paper, we study a nonlocal nonlinear Schrödinger equation (NNLSE). The infinitesimal generator, symmetry group and similarity reductions are obtained by the aid of Lie group method. Subsequently, similarity solutions of NNLSE are derived from the reduction equations. Finally, the auxiliary function method gives some exact solutions. Results show that these solutions which we obtain can be used to study relating physical problems.

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## 1. Introduction

Since Zabusky and Kruskal defined the name soliton in 1965, soliton theory has been developed rapidly [1]. The nonlinear optics was best used to reflect the diversity of optical solitons. Optical solitons includes spatial optical solitons, temporal optical solitons, and spatiotemporal optical solitons. It was obtained from the balance between optical pulse broadening caused by dispersion (or diffraction) and optical pulse compression caused by nonlinear effects [2-3]. It is well known that the nonlocal nonlinear Schrödinger equation (NNLSE) plays a very important role in many branches of mathematics and physics. The dynamics of (1+1)-dimensional (one spatial and one temporal variables) spatial optical soliton is the NNLSE in nonlocal nonlinear media. The NNLSE can be regarded as the most important soliton equation. Therefore, the study of the NNLSE has important theoretical and applied significance. Its form is given as follows [4-6]:

$$i\frac{\partial u}{\partial t}+\frac{1}{2}\frac{\partial^2 u}{\partial x^2}+su\int_{-\infty}^{+\infty}R(x^{'}-x)\left|u\left(x^{'},t\right)\right|^2 dx^{'}=0 \tag{1}$$

where $u(x,t)$ is the normalized slowly varying amplitude, s = ± 1 corresponds to focusing ($s$ = +1) or defocusing ($s$ = -1) nonlinearity. According to the width of response function $R(x)$ relative to the width of the optical pulse intensity $|u(x,t)|^2$, the degree of nonlocality is divided into three types: weak, general and strong. The recent studies show that the weakly and strongly nonlocal nonlinear Kerr media is of concern [7-9]. For a strongly nonlocal nonlinear Kerr media [7-9], the nonlinear term in Eq.(1) reduces to $su(a+bx+cx^2)$, $a,b,c$ are real constants, and for a weakly nonlocal nonlinear Kerr media[8], the nonlinear term in Eq.(1) reduces to $su\left[|u|^2+\gamma\partial_x^2\left(|u|^2\right)\right]$, $\gamma=\left(1/2\right)\int_{-\infty}^{+\infty}x^2R\left(x\right)dx$.

In this paper, we consider the (1+1)-dimensional spatial optical soliton in weakly nonlocal nonlinear non-Kerr media with an external potential, for parabolic law nonlinearity or cubic-quintic nonlinearity (polynomial law nonlinearity of second order), its form is given as follows:

$$iu_t+au_{xx}+b|u|^2u+b\beta|u|^4u+c|u|^2_{xx}u+pu=0 \tag{2}$$

where $p$ represents an external potential, $a,b,c$ and $p$ are real constants.

Seeking the exact solutions of the NNLSE has been an interesting and hot topic since a long time ago [10-14]. Similarity solutions and solitary wave solutions are usually used to describe physical phenomena and to check on the reliability and accuracy of numerical algorithm, so getting similarity solutions and solitary wave solutions has a great significance. To the best of our knowledge, related classical Lie group method and auxiliary function method have not been preformed to the NNLSE [11,15].

In this paper, we first perform Lie symmetry analysis [15-23] for the NNLSE (2). Then, we discuss the Lie symmetry group, similarity reductions and reduction equations of Eq.(2). Finally, by using the auxiliary function method, we obtain Hyperbolic function solutions, Elliptic function solutions and so on.

## 2. Lie symmetry analysis and exact solutions of NNLSE

### 2.1. Lie symmetry analysis of NNLSE

In this section, we will perform Lie symmetry analysis for Eq.(2), and obtain its infinitesimal generator and symmetry groups. To this aim, we first use the following transformation

$$u = ve^{i\varphi} \quad , \tag{3}$$

we can get

$$u_t = v_t e^{i\varphi} + i\varphi_t e^{i\varphi} v \quad , \tag{4}$$

$$u_{xx} = v_{xx} e^{i\varphi} + 2ie^{i\varphi}\varphi_x v_x + iv\varphi_{xx} e^{i\varphi} + i^2 e^{i\varphi}\varphi_x^2 v \quad , \tag{5}$$

$$|u|_{xx}^2 = 2v_x^2 + 2vv_{xx} \quad . \tag{6}$$

Substituting (4)-(6) into Eq.(2), separating the real and imaginary part, we can obtain

$$\begin{cases} v_t + 2av_x\varphi_x + av\varphi_{xx} = 0, \\ -\varphi_t v + av_{xx} - a\varphi_x^2 v + 2cvv_x^2 + 2cv^2 v_{xx} \\ +bv^3 + b\beta v^5 + pv = 0. \end{cases} \tag{7}$$

According to the method of determining the infinitesimal generator of NPDEs, we can obtain the infinitesimal generator of Eq.(7) as follows:

$$\underline{V} = \xi(x,t,\varphi,v)\frac{\partial}{\partial x} + \phi(x,t,\varphi,v)\frac{\partial}{\partial t} + \eta_1(x,t,\varphi,v)\frac{\partial}{\partial \varphi} + \eta_2(x,t,\varphi,v)\frac{\partial}{\partial v} \tag{8}$$

where $\xi(x,t,\varphi,v)$ , $\phi(x,t,\varphi,v)$ , $\eta_1(x,t,\varphi,v)$ and $\eta_2(x,t,\varphi,v)$ are coefficient functions of the infinitesimal generator to be determined.

Using the invariance condition $pr^{(2)}\underline{V}(\Delta)|_{\Delta} = 0$ , where $\Delta$ is Eq.(7) and $pr^{(2)}\underline{V}$ is the second prolongation of $\underline{V}$ . Applying the second prolongation of $\underline{V}$ to Eq.(7), and with help of Maple software, we can obtain

$$\xi = c_1 t + c_2, \phi = c_3, \eta_1 = \frac{xc_1}{2a} + c_4, \eta_2 = 0 \tag{9}$$

where $c_1, c_2, c_3, c_4$ are arbitrary constants.

We can obtain the corresponding geometric vector fields of Eq.(7) as follows:

$$V_1 = \frac{\partial}{\partial t}, V_2 = \frac{\partial}{\partial x}, V_3 = \frac{\partial}{\partial \varphi}, V_4 = t\frac{\partial}{\partial x} + \frac{x}{2a}\frac{\partial}{\partial \varphi} \tag{10}$$

Then, all of the infinitesimal generators of Eq.(7) can be expressed as

$$\underline{V} = c_1 V_1 + c_2 V_2 + c_3 V_3 + c_4 V_4. \tag{11}$$

To obtain the group transformation which is generated by the infinitesimal generators $V_i$ for i = 1,2,3,4, we should solve the following initial problems of ordinary differential equations:

$$\begin{cases} \dfrac{d(x,\tilde{t},\tilde{\varphi},\tilde{v})}{d\varepsilon} = (\xi,\phi,\eta_1,\eta_2), \\ (x,\tilde{t},\tilde{\varphi},\tilde{v})|_{\varepsilon=0} = (x,t,\tilde{\varphi},v), \end{cases} \tag{12}$$

where

$$\xi = \xi(x,\tilde{t},\tilde{\varphi},\tilde{v}), \phi = \phi(x,\tilde{t},\tilde{\varphi},\tilde{v}),$$
$$\eta_1 = \eta_1(x,\tilde{t},\tilde{\varphi},\tilde{v}), \eta_2 = \eta_2(x,\tilde{t},\tilde{\varphi},\tilde{v})$$

and $\varepsilon$ is a group parameter.

Exponentiating the infinitesimal symmetries of Eq.(7), we can obtain the one-parameter groups $g_i(\varepsilon)$ generated by $V_i$ for i = 1,2,3,4

$$g_1 : (x,t,\varphi,v) \to (x+\varepsilon,t,\varphi,v),$$

$$g_2 : (x,t,\varphi,v) \to (x,t+\varepsilon,\varphi,v),$$

$$g_3 : (x,t,\varphi,v) \to \left(x+t\varepsilon, t, \frac{x\varepsilon\varphi}{2a} + \varphi, v\right),$$

$$g_4 : (x,t,\varphi,v) \to (x,t,\varepsilon+\varphi,v),$$

where $g_1$ is a space translation, $g_2$ is a time translation, $\varepsilon$ is an arbitrary constant.

Using the above groups $g_i$ (i = 1,2,3,4), if $\varphi = f(x,t), v = h(x,t)$ is a known solution of Eq.(7), we can obtain the corresponding new solutions $\varphi_i, v_i$ (i = 1, 2,3,4) respectively as follows:

$$\varphi_1 = f(x-\varepsilon,t), v_1 = h(x-\varepsilon,t).$$

$$\varphi_2 = f(x,t-\varepsilon), v_2 = h(x,t-\varepsilon).$$

$$\varphi_3 = \left(1+\frac{x\varepsilon}{2a}\right) f(x-t\varepsilon,t), v_3 = h(x-t\varepsilon,t).$$

$$\varphi_4 = f(x,t)-\varepsilon, v_4 = h(x,t).$$

According to the known solution $\varphi = f(x,t), v = h(x,t)$, by using one-parameter symmetry groups $g_i$ (i = 1,2,3,4) continuously, one can get a new solution which can be expressed as the following form:

$$\varphi = \left(1+\frac{x\varepsilon_3}{2a}\right) f(x-\varepsilon_1-t\varepsilon_3, t-\varepsilon_2)-\varepsilon_4,$$

$$v = h(x-\varepsilon_1-t\varepsilon_3, t-\varepsilon_2),$$

where $\varepsilon_i$ (i = 1,2,3,4) are arbitrary constants.

### 2.2. Symmetry reductions and exact solutions of NNLSE

In the section, we will obtain similarity variables and its reduction equations, and get similarity solutions by solving the reduction equations.

**Case 1.** For the infinitesimal generator $V_1 = \frac{\partial}{\partial t}$, the similarity variables are $r = x, F(r) = \varphi, G(r) = v,$ and the group-invariant solution is $\varphi = F(r) = \varphi, v = G(r)$. Substituting the group-invariant solution into Eq.(7), the reduction equation as follows:

$$\begin{cases} 2aG_rF_r + aGF_{rr} = 0, \\ aG_{rr} - aGF_r^2 + 2cGG_r^2 + \\ 2cG^2G_{rr} + bG^3 + b\beta G^5 + pG = 0. \end{cases} \tag{13}$$

**Case 2.** For the infinitesimal generator $V_2 = \frac{\partial}{\partial x}$, the similarity variables are $r = t, F(r) = \varphi, G(r) = v,$ and the group-invariant solution is $\varphi = F(r), v = G(r)$. Substituting the group-invariant solution into Eq.(7), the reduction equation as follows:

$$\begin{cases} G_r = 0, \\ -F_rG + bG^3 + b\beta G^5 + pG = 0. \end{cases} \tag{14}$$

Eq.(7) has a solution $\varphi = c_1, v = c_2,$ where $c_1, c_2,$ are arbitrary constants. Obviously, the solution is not meaningful.

**Case 3.** For the infinitesimal generator $V_4 = t\frac{\partial}{\partial x} + \frac{x}{2a}\frac{\partial}{\partial \varphi}$, the similarity variables are $r = t, \ F(r) = \frac{4at\varphi - x^2}{4at}, \ G(r) = v,$ and the group-invariant solution is $\varphi = F(r) + \frac{x^2}{4at}$, $v = G(r)$. Substituting the group-invariant solution into Eq.(7), the reduction equation as follows:

$$\begin{cases} G_r + \frac{1}{2r}G = 0, \\ -F_rG + bG^3 + b\beta G^5 + pG = 0. \end{cases} \tag{15}$$

**Case 4.** For the infinitesimal generator $V_5 = kV_1 + dV_2 = k\frac{\partial}{\partial t} + d\frac{\partial}{\partial x}$, the similarity variables are $r = kx - dt, \ F(r) = \varphi, \ G(r) = v,$ and the group-invariant solution is $\varphi = F(r), \ v = G(r)$. Substituting the group-invariant solution into Eq.(7), the reduction equation as follows:

$$\begin{cases} -dG_r+2ak^2G_rF_r+ak^2GF_{rr}=0, \\ dGF_r+ak^2G_{rr}-ak^2GF_r^2+2ck^2GG_r^2 \\ +2ck^2G^2G_{rr}+bG^3+b\beta G^5+pG=0. \end{cases} \quad 16)$$

**Case 5.** For the infinitesimal generator $V_6=V_1+V_4=\frac{\partial}{\partial t}+t\frac{\partial}{\partial x}+\frac{x}{2a}\frac{\partial}{\partial \varphi}$, the similarity variables are $r=-2x+t^2$, $F(r)=\frac{(-2x+t^2)t-xt+6a\varphi}{6a}$, $G(r)=v$, and the group-invariant solution is $\varphi=F(r)+\frac{xt-(t^2-2x)t}{6a}$, $v=G(r)$. Substituting the group-invariant solution into Eq.(7), the reduction equation as follows:

$$\begin{cases} 8aG_rF_r+4aGF_{rr}=0, \\ \frac{rG}{4a}+4aG_{rr}-4aGF_r^2+8cGG_r^2+8cG^2G_{rr} \\ +bG^3+b\beta G^5+pG=0. \end{cases} \quad (17)$$

### 3. Auxiliary function method for NNLSE

In this section, solitary wave solutions and similarity solutions of Eq. (2) that will be obtained by auxiliary function method. We first use the following transformation

$$u=\varphi e^{i\lambda t}, \quad (18)$$

substituting (18) into Eq.(2), we can get the following equation

$$\left(a+2c\varphi^2\right)\frac{d^2\varphi}{dx^2}+2c\varphi\left(\frac{d\varphi}{dx}\right)^2+(p-\lambda)\varphi \\ +b\varphi^3+b\beta\varphi^5=0. \quad (19)$$

we solve Eq.(19) by auxiliary function method, and get $\varphi(x)$. We seek solutions of Eq.(19) in a power series of the form

$$\varphi(x)=\varphi(\tau)=\sum_{i=0}^{n}A_i\tau^i, \quad (20)$$

where $\tau=\operatorname{sech}(c_2+c_3x)$, $A_i\ (i=0,1,...,n)$ and $c_i$ (i = 1,2,3) are the constants to be determined. Balancing the highest order item with the nonlinear term in Eq.(20) gives $n=1$.

We can seek the solution of Eq.(20) in the form

$$\varphi(x)=\varphi(\tau)=A_0+A_1\tau \quad (21)$$

Substituting (21) into Eq.(19) and setting the coefficients of $\tau^i$ to zero, we obtain a set of algebraic equations. Solving these algebraic equations with the aid of Maple software, we obtain following solutions

**Case 1.**

(1) When $c_4\beta\neq 0,\ \varphi(x)=c_4\operatorname{sech}(c_2+c_3x)$, (22)

where

$$a=\frac{c\left(2\beta c_4^2+3\right)}{\beta}, b=\frac{6cc_3^2}{\beta c_4^2}, \beta=\beta, c=c,$$

$$\lambda=\frac{2\beta cc_3^2c_4^2+3cc_3^2+\beta p}{\beta}, p=p.$$

(2) When $a(\lambda-p)>0, bc\beta>0$, and

$$12c^2(\lambda-p)-a^2b\beta+3abc=0,$$

$$\varphi(x)=\sqrt{\frac{6c(\lambda-p)}{ab\beta}}\operatorname{sech}\left(x\sqrt{\frac{\lambda-p}{a}}\right), \quad (23)$$

Where $a=1/2, c=s\gamma, b=s+\gamma_1$, $\beta=\gamma_2/b$, nonlocal coefficient $\gamma=\frac{1}{2}\int_{-\infty}^{+\infty}R(x)dx$, third and fifth-order nonlinear coefficient $\gamma_1$ and $\gamma_2$, potential outfield $p$, frequency $\lambda$, amplitude $\varphi(x)$.It is easy to obtain the analytical solution of Eq.(2), we call $u(x,t)$ is an analytical bright soliton solution [15].

Using the similarity method, we can also obtain the following explicit analytic solutions of Eq.(19)

**Case 2.**

When $c_4\beta \neq 0, \varphi(x) = c_4 \tanh(c_2 + c_3 x)$, (24)

where

$$a = \frac{c(4\beta c_4^2 + 3)}{\beta}, b = \frac{-6cc_3^2}{\beta c_4^2}, \beta = \beta, c = c,$$

$$\lambda = \frac{-6\beta cc_3^2 c_4^2 - 6cc_3^2 + \beta p}{\beta}, p = p.$$

**Case 3.**

When $c_4\beta \neq 0, \varphi(x) = c_4 \sec(c_2 + c_3 x)$, (25)

where

$$a = \frac{c(2\beta c_4^2 + 3)}{\beta}, b = \frac{-6cc_3^2}{\beta c_4^2}, \beta = \beta, c = c,$$

$$\lambda = \frac{-2\beta cc_3^2 c_4^2 - 3cc_3^2 + \beta p}{\beta}, p = p.$$

**Case 4.**

(1) When $c_5\beta \neq 0, \varphi(x) = c_5 \text{JacobiS N}(c_2 + c_3 x, i)$,

(26)

where $a = \frac{3c}{\beta}, b = \frac{6cc_3^2}{\beta c_5^2}, \beta = \beta, c = c$,

$$\lambda = 2cc_3^2 c_5^2 + p, p = p.$$

When $c_5\beta \neq 0, \varphi(x) = c_5 \text{JacobiS N}\left(c_2 + c_3 x, i\frac{\sqrt{3}}{2}\right)$,

(27)

where

$$a = \frac{c\left(2\left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)\beta c_5\right) + 3 + 2\beta c_5^2}{\beta},$$

$$b = \frac{6cc_3^2\left(-\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)}{\beta c_5^2}, \beta = \beta, c = c,$$

$$\lambda = \frac{3cc_3^2\left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right) + \beta p}{\beta}, p = p.$$

(2) When

$$c_5\beta \neq 0, \varphi(x) = c_5 \text{JacobiS N}\left(-c_2 + c_3 x, \frac{\sqrt{3}i}{2}\right), \quad (28)$$

where

$$a = \frac{c\left(2\left(\frac{1}{2} - \frac{\sqrt{3}i}{2}\right)\beta c_5\right) + 3 + 2\beta c_5^2}{\beta},$$

$$b = \frac{6cc_3^2\left(-\frac{1}{2} - \frac{\sqrt{3}i}{2}\right)}{\beta c_5^2}, \beta = \beta,$$

$$c = c, \lambda = \frac{3cc_3^2\left(\frac{1}{2} - \frac{\sqrt{3}i}{2}\right) + \beta p}{\beta}, p = p.$$

(3) When

$$c_5\beta \neq 0,$$

$$\varphi(x) = c_5 \text{JacobiS N}\left(c_2 + c_3 x, \frac{\sqrt{3}i}{-2}\right), \quad (29)$$

where $a = -\frac{c\left(2\left(-\frac{1}{2} - \frac{\sqrt{3}i}{2}\right)\beta c_5\right) + 3 - 2\beta c_5^2}{\beta}$,

$$b = -\frac{6cc_3^2\left(-\frac{1}{2} - \frac{\sqrt{3}i}{2}\right)}{\beta c_5^2}, \beta = \beta, c = c,$$

$$\lambda = -\frac{3cc_3^2\left(-\frac{1}{2} - \frac{\sqrt{3}i}{2}\right) - \beta p}{\beta}, p = p.$$

(4) When

$$c_5\beta \neq 0, \varphi(x) = c_5 \text{JacobiS N}\left(c_2 + c_3 x, \frac{\sqrt{3}i}{2}\right),$$

(30)

where

$$a=-\frac{c\left(2\left(\frac{1}{2}+\frac{\sqrt{3}i}{2}\right)\beta c_5\right)-3-2\beta c_5^2}{\beta},$$

$$b=-\frac{6cc_3^2\left(-\frac{1}{2}+\frac{\sqrt{3}i}{2}\right)}{\beta c_5^2},\beta=\beta,c=c,$$

$$\lambda=-\frac{3cc_3^2\left(-\frac{1}{2}+\frac{\sqrt{3}i}{2}\right)-\beta p}{\beta},p=p.\cdot$$

When $c_1\neq 0,$

$$\varphi(x)=c_5\mathrm{JacobiSN}(c_2+c_3x,c_1),\qquad(31)$$

where $$a=\frac{2c_1^4c_5^4b\beta+2c_1^2c_5^4b\beta+3c_1^4c_5^2b\beta}{6c_1^4}+\frac{2b\beta c_5^2+6c_1^4p+3bc_1^2c_5^2}{6c_1^4},p=p.$$

(5) When $2c\beta(a\beta-3c)a\neq 0,$

$$c\beta(a\beta-3c)\left(\begin{array}{l}a^2\beta^2+18c^2-6ac\beta+\\ \sqrt{\begin{array}{l}-3a^4\beta^4+24a^3\beta^3c\\ -72a^2c^2\beta^2+108a\beta c^3\end{array}}\end{array}\right)>0,\quad\text{and}$$

$$-3a^4\beta^4+24a^3\beta^3c-72a^2c^2\beta^2+108a\beta c^3>0,$$

$$\varphi(x)=\frac{-F}{2c\beta(a\beta-3c)}\mathrm{JacobiSN}(c_2+c_3x,c_1),\qquad(32)$$

where $\beta=\beta,c=c,\qquad p=p,a=a,$

$$\lambda=\frac{9c^2c_3^2+ap\beta^2-3cp\beta}{\beta(a\beta-3c)},$$

$$b=\frac{12c^2c_3^2(a\beta-3c)}{a\beta\left(\frac{-a^2\beta^2+\sqrt{\begin{array}{l}-3a^4\beta^4+24a^3\beta^3c\\ -72a^2c^2\beta^2+108a\beta c^3\end{array}}}{2(a\beta-3c)}-\frac{3c(-a^2\beta^2+\sqrt{\begin{array}{l}-3a^4\beta^4+24a^3\beta^3c-\\ 72a^2c^2\beta^2+108a\beta c^3\end{array}})}{2a\beta(a\beta-3c)}+3c\right)}$$

$$F=a\beta(a\beta-3c)\times\sqrt{c\beta(a\beta-3c)\left(\begin{array}{l}a^2\beta^2+18c^2-6ac\beta\\ +\sqrt{\begin{array}{l}-3a^4\beta^4+24a^3\beta^3c-\\ 72a^2c^2\beta^2+108a\beta c^3\end{array}}\end{array}\right)}.$$

**Case 5.**

When $a(\lambda-p)>0,bc\beta<0$, and

$$12c^2(\lambda-p)-a^2b\beta+3abc=0,$$

$$\varphi(x)=\pm\sqrt{\frac{-6c(\lambda-p)}{ab\beta}}\mathrm{csch}\left(x\sqrt{\frac{\lambda-p}{a}}\right),\qquad(33)$$

where $a=1/2,c=s\gamma,b=s+\gamma_1,\quad \beta=\gamma_2/b,$ nonlocal coefficient $\gamma=\frac{1}{2}\int_{-\infty}^{+\infty}R(x)dx,$ third and fifth-order nonlinear coefficient $\gamma_1$ and $\gamma_2$, potential outfield $p$, frequency $\lambda$, amplitude $\varphi(x)$. It is easy to obtain the analytical solution of Eq.(2), we call $u(x,t)$ is an analytical singular solution [15].

**Case 6.**

When $a(\lambda-p)>0,bc\beta<0$, and

$$12c^2(\lambda-p)-a^2b\beta+3abc=0,$$

$$\varphi(x)=\pm\sqrt{\frac{6c(\lambda-p)}{ab\beta}}\mathrm{csc}\left(x\sqrt{\frac{-\lambda+p}{a}}\right),\qquad(34)$$

where $a=1/2,c=s\gamma,b=s+\gamma_1,\quad \beta=\gamma_2/b,$ nonlocal coefficient $\gamma=\frac{1}{2}\int_{-\infty}^{+\infty}R(x)dx,$ third and fifth-order nonlinear coefficient $\gamma_1$ and $\gamma_2$, potential outfield $p$, frequency $\lambda$, amplitude $\varphi(x)$. It is easy to obtain the analytical solution of Eq.(2), we call $u(x,t)$ is a delta singular periodic solution [15].

## 4. Conclusions

In this paper, based on Lie group method, we study the symmetry reductions and exact solutions of a nonlocal nonlinear Schrödinger equation. First, we perform Lie symmetry analysis for the nonlocal nonlinear Schrödinger equation and obtain its infinitesimal generator, symmetry group. Next, using similarity variables to obtain reduction equations, we get similarity solutions of Eq.(7) by solving the reduction equation. In the end, we use auxiliary function method to obtain exact solutions of the nonlocal nonlinear Schrödinger equation. In future work, we will consider the nonlocal nonlinear Schrödinger equation with polynomial law of high order.

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