

Magnetic field wiggy-chain undulator

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The phenomenon of tuned coherent radiation in free-electron lasers is given by undulator – the FEL main component. The radiation is issued by a relativistic electron beam injected in a periodic magnetic field generated by spatially periodic structures made up of permanent magnets or currents. Important difficulties in mathematical evaluation of the magnetic field components occur when using circular undulator cross section. Therefore a substitution of the circular cross section by a polygonal cross section, easier mathematically treatable, is desirable. In this paper a hexagonal cross section was chosen. Each wire of the stack (a frame) is made up of two chains that are connected alternating to the chains of the next frame. Eventually, in the last frame the two chains are connected to each other, closing the circuit. In a frame, each chain has a loop on each of the six sides of the hexagon. Thus the magnetic field alternates on each side of the hexagon. The wiggy-chain undulator magnetic field is numerical evaluated.

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1. Introduction

In free-electron laser (FEL) research and development one of the main trends is the elaboration of the compact devices [1,2,3,4,7]. The undulator is the principal component where the phenomena of coherent radiation take place.

Free-electron lasers (FEL) imply the elaboration of compact devices. The phenomenon of tuned coherent radiation is given by undulator - the FEL principal component. The radiation is obtained by means of a relativistic electron beam injected in a periodic magnetic field produced by spatially asymmetric periodic structures formed by permanent magnets or currents (undulator, wiggler). As a result a coherent radiation is generated in the Z - direction. In the new transversal undulators the Z magnetic field components are periodic with Z and the incoming electrons have transverse momenta.

Current symmetric devices produce magnetic fields which are spatially periodic, symmetric about y-axis and x-axis.

The current has alternating directions in wire frames stack.

The transverse cross section of the device structure is composed by 6 pairs of curve segments. Each pair forms a chain ring. As can be seen in Fig.1 each coil consists of six rings, each placed on the side of a hexagon.

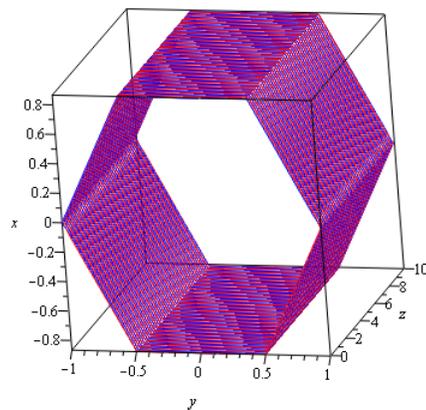


Fig. 1. Example of a wiggy-chain structure.

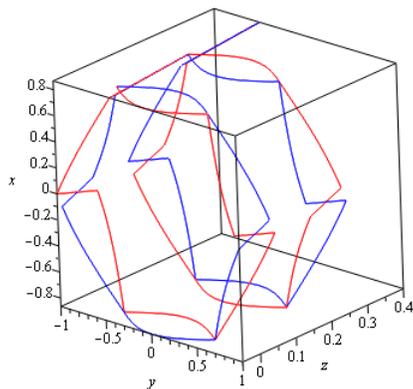


Fig. 2. Detail of first two coils of the structure.

The curve composing a half of a ring is described by cosine function. A detail of the first two coils is presented in Fig. 2 and 3.

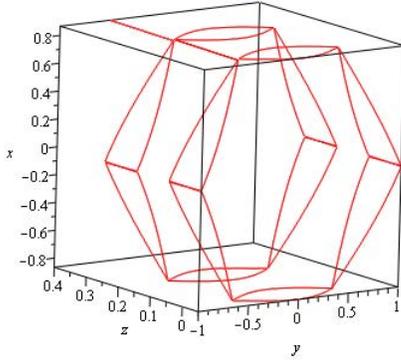


Fig. 3. Side detail of first two coils of the structure.

In order to describe the coil spatial curve equations, we define the next parameters:

m is the total number of coils, six sides, each sampled in $jmax$ points, so we have $nmax = 6 \cdot m \cdot jmax$ points on each of the 2 branches of the coils.

The index of the current point on this curve is denoted i and varies from 0 to $nmax$. The index of the current hexagonal side is n while k is the index of the current coil. The coil pitch is denoted by p . The cosine amplitude on z -direction is h .

With these notations, the coordinates describing the coil spatial curve are:

$$\begin{aligned} x_i &= \sin\left(\frac{n+1}{3}\pi\right) - t \cdot \sin\left(\frac{n}{3}\pi\right) \\ y_i &= t \cdot \cos\left(\frac{n}{3}\pi\right) - \cos\left(\frac{n+1}{3}\pi\right) \\ z_{Ai} &= k \cdot p + (-1)^{n+k} \cdot h \cdot \cos(\pi \cdot y_i) \\ z_{Bi} &= k \cdot p - (-1)^{n+k} \cdot h \cdot \cos(\pi \cdot y_i) \end{aligned}$$

Where t stands for local hexagonal side coordinate. z_A and z_B are the z coordinates of the upper (A) and lower (B) branch of the coil.

2. Methods

The magnetic field of each loop was computed with flows in the Biot - Savart law. The magnetic field components for a segment of the loop are computed with a formula⁶ based on integrals multiplied by the factor $\frac{\mu_0 \mu_r}{4\pi} J$, where μ_0 is the vacuum magnetic permeability,

μ_r is the relative magnetic permeability, J is the current, z is wire position on Z axis, and x and y are the coordinates of the point from the current loop, index i referring to one of the 6 segments:

$$B_{xi} = \frac{\mu_r \mu_0}{4\pi} J \int (dl_{yi} \cdot r_z - dl_{zi} \cdot r_y) / r^3;$$

$$B_{yi} = \frac{\mu_r \mu_0}{4\pi} J \int (dl_{zi} \cdot r_x - dl_{xi} \cdot r_z) / r^3;$$

$$B_{zi} = \frac{\mu_r \mu_0}{4\pi} J \int (dl_{xi} \cdot r_y - dl_{yi} \cdot r_x) / r^3;$$

$$r_x = xp - x, \quad r_y = yp - y, \quad r_z = zp - z;$$

$$r^2 = r_x^2 + r_y^2 + r_z^2$$

Symmetric structures are just particular cases that may be also treated using this model.

Integration is made using numerical methods.

Evaluating B_z component the next integral is obtained:

$$B_z(zp) = \frac{\mu_r \mu_0}{4 \cdot \pi} J \int \left(\frac{F_A}{R_A} - \frac{F_B}{R_B} \right) dt$$

where

$$F_A = (yp - y(t)) \cdot \sin\left(\frac{n\pi}{3}\right) - (zp - z_A(t)) \cdot \cos\left(\frac{n\pi}{3}\right)$$

$$F_B = (yp - y(t)) \cdot \sin\left(\frac{n\pi}{3}\right) - (zp - z_B(t)) \cdot \cos\left(\frac{n\pi}{3}\right)$$

$$R_A = \left((xp - x(t))^2 + (yp - y(t))^2 + (zp - z_A(t))^2 \right)^{1.5}$$

$$R_B = \left((xp - x(t))^2 + (yp - y(t))^2 + (zp - z_B(t))^2 \right)^{1.5}$$

are introduced to simplify the equation. The variable zp takes values from 0 to $m \cdot p$, being the parameter of B_z .

3. Results

A simulation was done for $xp = 0$, $yp = 0$. The number of coils $m = 50$ was kept constant, while four different values ($p = 0.2$, $p = 0.25$, $p = 0.35$ and $p = 0.4$) were given to the coil pitch, resulting zp varying from 0 to 10, respectively 12.5, 17.5 and 20.

The dependence of B_z of zp for these four values of the pitch p is shown in figures 4-7. At small values of the pitch (0.2 for instance) the amplitude of the variations of field intensity is small, the curve being quite smooth in a very large interval. Increasing a little the pitch till 0.25 a slight ripple appears. The maximum and the minimum of B_z remain roughly unchanged.

Rising the value of the pitch till 0.35 the ripple becomes important (its amplitude increased about 8 times) and the maximum and the minimum values of B_z increased (with about 20 %).

Finally, at a pitch of 0.4, the ripple is even larger (about 12 times the value for $p = 0.2$) with the maximum and the minimum values of B_z slightly increased compared to $p = 0.35$.

All these four different configurations have in common the same situation: the maximum and the minimum are located at the ends of the coil (the entrance and the exit) and they are both very large.

There are two solutions to reduce these amplitude variations. A first solution is to cut some sides of the first and the last coil. They may have, for instance, only one or two sides instead of six.

The other solution⁵ is to start with a larger value of the hexagon side for the first coil, then gradually decrease it for the few next coils, then keeping it constant. The same procedure is applied symmetrically at the other end (the last coils).

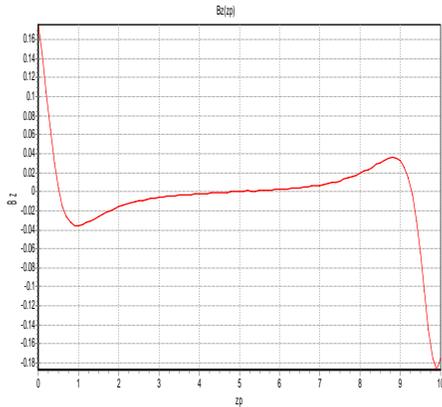


Fig. 4. The undulator z magnetic field component vs. z_p direction for $p = 0.2$.

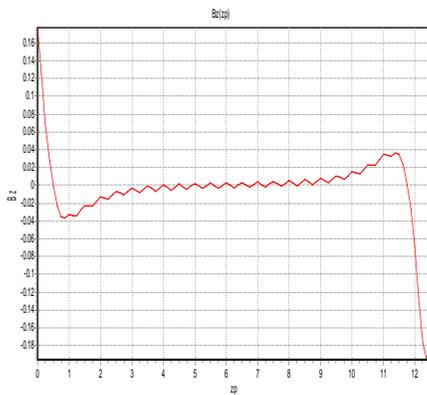


Fig. 5. The undulator z magnetic field component vs. z_p direction for $p = 0.25$.

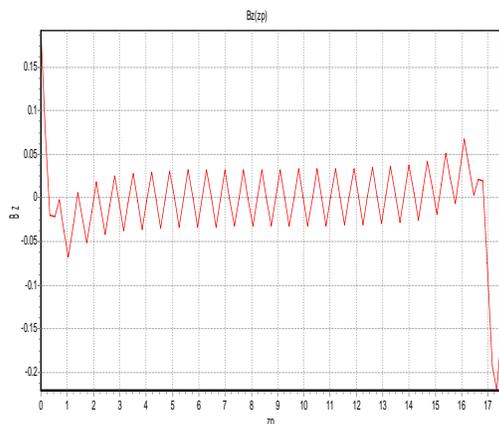


Fig. 6. The undulator z magnetic field component vs. z_p direction for $p = 0.35$.

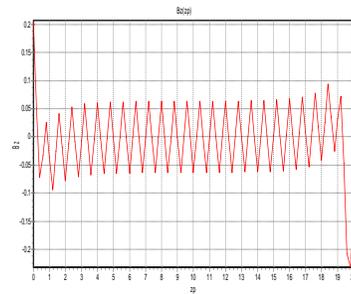


Fig. 7. The undulator z magnetic field component vs. z_p direction for $p = 0.4$.

4. Conclusions

A new model of an undulator for free electron lasers is proposed in the aim of solving important difficulties in mathematical evaluation of the magnetic field components that occur when using circular undulator cross section. Therefore a substitution of the circular cross section by a polygonal cross section, easier mathematically treatable, is desirable. Here a hexagonal cross section was chosen. Each wire of the stack (a frame) is made up of two chains that are connected alternating to the chains of the next frame. Eventually, in the last frame the two chains are connected to each other, closing the circuit. In a frame, each chain has a loop on each of the six sides of the hexagon. Thus the magnetic field alternates on each side of the hexagon. Each chain ring, considered as dipole, creates alternating magnetic field components. The numerical simulation revealed the problems raised by the large intensities at the input and output ends of the coil and suggested appropriate solutions.

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