# Measuring of the refractive index profile gradient index planar microlens by numerical method

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In this paper, the gradient index planar microlens profile shape made by ion exchange process is determined by the semicircular interference pattern obtained from total shearing interferometric. In the following, the focal length and half the width of the wave was calculated with simulation of Gaussian beam propagation through this planar microlens by Optiwave software. It is worth mentioning that the simulation results are in good agreement with the experimental results obtained for the profile.

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### 1. Introduction

At present, the most extensive method used to build the gradient index planar microlens is the process of ion exchange through a circular aperture on the substrate. Effective factors in the ion exchange process and refractive index profile include selecting the type and geometry of the substrate, the type of melt composition, temperature, ion exchange time, and applied electric field.

Due to its low dissipation, immunity against optical damage, compatibility with commercial fiber surfaces and low cost of production, glass is used as a substrate for making this type of lens. Theoretically and experimentally, how is the formation of a three-dimensional distribution of the refractive index in a planar microlens determined in the manufacturing process? In manufacturing gradedindex planar microlens, if the exchange time of ions between composition and substrate is long, the shape of the ion penetration in the substrate will be almost hemispherical. Consequently, the shape of the concentration changes or; in other words, the gradual refractive index profile also has a spherical function. In addition, to make changes in the large refractive index during ion exchange process, the electric field is applied to the substrate [1]. The electric field is formed due to low mobility of some elements such as Tl<sup>+</sup> applied to the other side of the substrate, causing the thermal penetration and mobility of most ions in the glass. The electric field applied to a very small aperture on the substrate often results in a hemispherical refractive index profile. It should be noted that the disturbance in the electric field causes heterogeneity of the refractive index and different spatial conductivity. Graded-index planar microlenses are widely used in imaging systems and in micropaptics; namely, fiber optic communication, electro-optic

instruments, etc [2-4]. In using the graded-index planar microlenses, it is necessary that the refractive index profile and its changes, depending on the wavelength, be measured accurately because the knowledge of the gradual refractive index profile is important not only for determining the performance of optical devices (such as focal length, numerical aperture, aberration correction, diameter of focal beam, etc.), but also for optimizing construction conditions. There are many methods such as interference method [5], near-field method [6], reflection method [7], imaging method [8], and near-field scanning method [9] for measuring the coefficient of refractive index. Due to the complexity of some of these methods, a numerical method is presented here. Transverse interference method is similar to the methods of thermography in the fields of aerodynamics, seismology, plasma physics and astronomy. In these methods, phase displacement of a probe wave such as an optical wave or an earthquake wave is detected, and the symmetrical internal structure of a phase goal is used to calculate it. In the transverse interferometric method, an optical fiber or a rod sample is considered as a phase perturbation, which has an axial symmetric refractive index profile. The phase difference between the incident probe light through fring displacement is detected by a formula. Thus, in calculating the refractive index profile of the fring displacement, it is assumed that the incident probe beam landing on the sample travels a straight path within it. Therefore, it is necessary to derive a precise formula to correct the refraction of the probe beam. It is assumed that a fiber optic or a rod sample in - refractive index matching oil (which has a refractive index close to the refractive index of the sample cladd) is inserted. By using an interference microscope, the cross section is observed. The probe beam is divided into two arms after going through the core by

Mach-Zehnder interferometer, and the beam of one of the arms is displaced by a shear prism. In the first case, when the shear distance is larger than the core size, the flat wavefront of one of the divided beams interferes with another delay wavefront. Therefore, a typical interference pattern (overall shear pattern) is observed. The measured value of the fring displacement indicates the optical crosspath difference of the core from the outer cladd or adaptive oil; and from the measured values of fring displacement, the refractive index pattern is obtained. In other words, when the shear distance becomes small enough in comparison to the core size, the observed fring displacement approximately indicates differential optical path relative to the transverse distance. When the core radius is smaller than a few hundred micrometers, conventional interferometry is more suitable for measurement because in differential interferometry, the shear distance cannot be sufficiently diminished compared to the core radius to achieve a suitable fring displacement. In other words, when the radius of the core is more than one millimeter, as in the case of focal bars and lens, differential interferometry is more suitable for observation since in conventional interferometry, the fring displacement goes beyond the microscope's field due to the difference in the large optical path. For planar microlens with three-dimensional graded-index, refractive index has 16 coefficients as follows:

$$n(r,z)^{2} = n_{\circ}^{2} \left\{ \begin{cases} \left[1,(gz)^{2},(gz)^{4},(gz)^{6}\right] \\ \times \begin{bmatrix} v_{11} & v_{12} & v_{13} & v_{14} \\ v_{21} & v_{22} & v_{23} & v_{24} \\ v_{31} & v_{32} & v_{33} & v_{34} \\ v_{41} & v_{42} & v_{43} & v_{44} \end{bmatrix} \begin{bmatrix} 1 \\ (gr)^{2} \\ (gr)^{4} \\ (gr)^{6} \end{bmatrix} \right\}$$
(1)

Where  $n_0$  is the refractive index in the center of the lens's aperture and g is a normalized constant determining the lens power.  $\mathbf{r} = (x^2 + y^2)^{\frac{1}{2}}$  is the radius of the microlens aperture and z is optical axis of the microlens. In this paper, by overall shearing pattern interferometry [10], a numerical method is presented for measuring three dimensional refractive index profile of planar microlens made by the ion exchange process. Although the shape of the obtained function does not provide a model for solving the Fick's penetrating equation, it is a simple and easy way to calculate the refractive profile in terms of radial and axial coordinates with the number of low coefficients compared to the interference method with 16 fitting coefficients.

#### 2. Fitting method

The Mach-Zehnder interferometer is a two-beam single-pass interferometer that can measure the two-

dimensional optical path difference (OPD) of a sample placed in one arm of the instrument. The schematic illustration of the interference device is shown in Fig. 1. Different monochrome laser sources can be couppled to measure the difference in optical path in the form of a function of wavelength characterizing the color properties of graded materials in the system. The laser source is spatially filtered and parallel, then it is sent to a beam splitter that divides the beam into reference and interfering arms. The sample is located in the test arm and two beams are combined in the splitter of the second beam. Samples and results of the interference pattern are imaged on the detector.



Fig. 1. Schematic of Mach-Zehnder interferometer used for measuring two dimensional refractive index profiles

The overall shear interference pattern for cut piece of the graded-index planar microlens provides half-circle fringes. A sample of graded-index planar microlens and cut piece is shown in Fig. 2.



Fig. 2. Graded-index planar microlens and geometry of sample preparation

The sample is usually assumed to be a phase object so that the optical path difference between the two Mach Zehnder interferometer arms is determined as follows.

$$OPD(r,z) = \frac{\lambda}{2\pi} \varphi(r,z) = m(r,z)\lambda = \Delta n(r,z)d \quad (2)$$

Where  $\lambda$  is the laser wavelength,  $\phi$  is the phase difference between the two interferometer arms, m is the fring order, d is the thickness of the sample, r is the radius of the aperture and  $\Delta n(r, z)$  is the distribution of the three-dimensional refractive index. Therefore, if it is possible to measure the difference in the order of fring as a

fring count with the thickness of the sample, gradual refractive index profile can be determined as follows.

$$\Delta n(r,z) = \frac{\lambda}{2\pi d} m(r,z) \tag{3}$$

From this equation, the refractive index of m-order fring can be determined as follows.

$$n_s + \frac{\lambda m}{d} = n_m \tag{4}$$

Where  $n_s$  is the refractive index of the substrate and the start of the fring count (m) is from outside of the interference pattern. Thus, by interference pattern with m semi-circular fring, a set of  $n_m$  is obtained and  $\Delta n$  can be obtained from them. The uncertainty of the refractive index changes can also be obtained from the following equation:

$$\delta(\Delta n) = \sqrt{\left(\frac{m}{d}\delta\lambda\right)^2 + \left(\frac{\lambda}{d}\delta m\right)^2 + \left(\frac{m\lambda}{d^2}\delta d\right)^2} \quad (5)$$

In addition, due to other constraints of sample preparation such as surface deviations from flat state, the accuracy of measurement decreases. The deformation of the flat surface causes an increase in measurement error. Different methods of cutting, polishing and scouring can help eliminate of wedge and curvature of the surface. The error introduced by the wedge can be minimized by several methods. First, if the sample had a homogeneous region, wedge fringes could be removed in the interferometer or in the data processing. Second, the wedge could be measured by measuring the thicknesses across the sample, and then in the process, its data are deleted. Third, the sample could be swirled in a liquid with a matching refractive index. In all three methods, the error introduced in the measurement of  $\Delta n$  is equal to:

$$\frac{\delta(\Delta n)}{|\Delta n|} = \frac{\delta d}{d} \tag{6}$$

where  $\delta d$  is the thickness change in the sample due to the wedge, d is the average sample thickness and  $|\Delta n|$  is the total change in refractive index of the sample. For a sample in the air, the error reported for the refractive index profile is equal to:

$$\frac{\delta(\Delta n)}{|\Delta n|} = \frac{\delta d(n-1)}{d|\Delta n|} \tag{7}$$

where n is the mean refractive index of the sample. Measurement of a sample in a fluid with an adaptive refractive index results in a significant improvement in the measurement of error. After obtaining  $\Delta n$ , a linear relationship between the index of the refractive index and the distribution of the ionic concentration can be obtained. These measures help to make the manufacturing process optimal, and finally, the desired refractive index profile is reached.

#### 3. Experimental results

In this paper, we have used the specification of planar microlens presented in reference [11] with an aperture diameter of 1.33 mm. This planar microlens is made by the penetration of monocular alkaline ions by applying an electric field in a silicate glass substrate. The interference pattern of this microlens is shown in Fig. 3. In reference [11], the gradual refractive index of planar microlens was measured by shear interference method, but in this paper, the gradual refractive index profile is measured by a simple model with the interference pattern. Focal length and diameter of the focused beam have been earned by simulation of the Gaussian beam propagation (by software optiwave).



Fig. 3. The resulting interference pattern of planar microlens obtained by the total shear interferometer [11]

Interference pattern includes 27 semi-circle fringes. The refractive index of the substrate is  $n_s = 1.537$  and the wavelength used in interferometry measurements is  $\lambda = 0.63 \mu m$  and PML thickness is  $d = 60 \mu m$  [11].

## The fitting of circular freezes to the polynomial equation

Initially, on the interference pattern, the shape of each semi-circle fring is fitted as the following circle equation:

$$z^{2} + r^{2} = (a(m))^{2}, \forall m = 1, 2, ... N_{F}$$
 (8)

where  $N_F$  is the number of fring interference patterns, and

a(m) shows the radius of each m-order fring. In the next step, according to equations (4) and (6), the refractive index of the microlens and the radius of the fringes are a function of fringe order. Thus, we can write the following

relation between the refractive index  $n_m$  and the radius of the fringes a(m):

$$\ln(n_0^2 + n^2) = \ln q + p \ln(a_m^2)$$
(9)

Where q and p are fitting coefficients ( $n_0$  shows the maximum refractive index obtained after penetration). q shows the rate for the refractive index relative to r, z. Finally, the refractive index is obtained by inserting equation 6 in equation 7 as function of the coordinates r and z.

$$n^{2} = n_{0}^{2} - q(z^{2} + r^{2})^{p}$$
(10)

Fig. 4 is the fitting of equation 7 with coefficients, with only three coefficients needed for the best fitting.



Fig. 4. Refractive index profile of planar microlens

By comparing the obtained refractive index profile and the profile presented in reference [11], a good agreement between them is observed. Hence, by obtaining the relationship between fitness coefficients and manufacturing conditions, we can control the factors affecting the production of microlens. From equation 8, it is easy to determine the depth of a planar microlens  $z_{max}$ and the aperture radius after penetration (in terms of the fitting coefficients) as follows:

$$z_{\max} = q \sqrt{\frac{n_0^2 - n^2}{p}}$$
(11)

$$r_{\max} = \sqrt[2q]{\frac{n_0^2 - n^2}{p}}$$
(12)

So, fit coefficients are almost dependent on the shape of the lens. In order to show that this fitted equation can be used as the refractive index profile of planar microlens, we will investigate some optical properties of microlens. For this purpose, we simulated the propagation of the Gaussian laser beam with a half-width of 200 microns in planar microlens (with a gradual refractive index) by the Optiwave software. Fig. 5a shows the two-dimensional distribution of Gaussian beam intensity in a planar microlens environment, and Fig. 5b shows the onedimensional intensity of Gaussian beam simulated along the optical axis and r = 0, and in Fig. 5c, the onedimensional distribution of Gaussian beam intensity in focal point is given. The simulated focal length and focused beam width are 2528  $\mu$ m and 42.7  $\mu$ m, respectively, and focal length in reference [11] is 2479  $\mu$ m. Therefore, a good agreement was obtained between simulation results and experimental data.



Fig. 5. (a) two-dimensional distribution of Gaussian beam intensity in a planar microlens environment (b) one-dimensional intensity of Gaussian beam simulated along the optical axis and r = 0 (c) one - dimensional distribution of Gaussian beam intensity in focal point

### 4. Conclusion

We have presented a method for fitting the refractive index of the planar microlens made by the ion exchange method. In this study, it was shown that the three coefficients p, q and  $n_o$  are sufficient to fit the index of the refractive index, which has an accuracy equivalent to that of the refractive index profile obtained from the complex processing of fringes data obtained by transverse interferometry. The obtained refractive index is in good agreement with the experimental profile. In other words, the optical properties of the simulations performed with this refractive index profile are in good agreement with experimental data. This agreement shows that the method used to determine the refractive index profile has done this with sufficient accuracy. In addition, the duration of the simulation of Gaussian beam propagation is significantly reduced because the planar microlens can be expressed by fewer parameters.

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