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Numerical investigation on the bistability of nonlinear Bragg gratings using elliptic integration

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Based on the coupled mode theory, the bistability performance of nonlinear Bragg gratings(NLBG) is analyzed theoretically in terms of elliptic integration, the analytical expression describing the relation between the input intensity and the output intensity is presented. As a result, the dependence of the bistability on the gratings parameters is investigated numerically. The results show that, the bistability performance of the nonlinear Bragg gratings will be influenced greatly by the couple coefficient, the grating length and detuning respectively.

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1. Introduction

Nonlinear Bragg grating (NLBG) has exhibited wide potential applications in optical limiting, optical switching, soliton communication and so on [1-2], due to its particular properties. With the rapid improvement of the material growth techniques, precise control of the grating length and refractive index modulation amplitude becomes possible, as a result, nonlinear phenomena can occur unrequired very high incident power despite the relatively weak nonlinearities optical materials. Inside photonic band gap (PBG), optical bistability occurs when a positive feedback loop (among inner optical intensity, nonlinear refractive index, and Bragg resonance frequency) causes the Bragg wavelength shift to longer or shorter wavelength [3-5]. Coupled mode equation are extensively used for the analysis of distributed-feedback structure because of its simplicity and flexibility, until now, its main numerical methods include Transfer matrix method and Runge-Kutta method, which have disadvantage that it can be hard to find out general optical properties. In addition, most reports on the switching performance of fiber Bragg grating, in general, regard the couple coefficient and the grating length as a whole to facilitate discussion [3-12]. In fact, in the literature [12], the results have implied the single dependent relationship between the switching performance and the grating length.

In this paper, based on the coupled mode theory, we have presented the analytical expression describing the relation between the input intensity and the output intensity, and investigated the influence of grating inner parameters on the bistability performance of nonlinear Bragg gratings in terms of elliptic integration.

2. Theoretical model

Inside fiber gratings, the z -axial distribution of refractive index can be described by

$$n(z)=n_0+n_1(z)\cos[2\beta_0 z+\varphi]+n_2\left|E(z)\right|^2, \qquad (1)$$

where $E(z)$ is the inner electric field of grating, , the Bragg wave vector β_0 is given by $\beta_0=\pi/\Lambda$, and Λ is the grating period , $\Lambda=\lambda_0/2n_0$, λ_0 is the Bragg wavelength , ϕ is the constant phase. n_0 , $n_1(z)$, and n_2 denote the effective mode refractive index, linear refractive index modulation amplitude, and nonlinear refractive index coefficient, respectively.

The inner electric field can be expressed by

$$E(z)=E_f\exp[i\beta z]+E_b\exp[-i\beta z], \qquad (2)$$

where $\beta=n_0\omega/c$, and ω is the carrier angular frequency, c is the speed of light in vacuum, E_f and E_b represent the slowly varying amplitude of forward and backward wave, respectively. In this paper, we assume that the incident wave is continuous wave. Substituting Eqs. (1) and (2) into the wave equation, one can obtain the following static state nonlinear coupled mode equations [8]

$$\frac{dE_f}{dz}=ikE_b\exp[-i(2\delta z-\phi)]+i\Gamma(\left|E_f\right|^2+2\left|E_b\right|^2)E_f, \qquad (3a)$$

$$\frac{dE_b}{dz}=-ikE_f\exp[i(2\delta z-\phi)]-i\Gamma(2\left|E_f\right|^2+\left|E_b\right|^2)E_b, \qquad (3b)$$

where δ , Γ and k account for the detuning, nonlinear coefficient, and coupling coefficient, respectively, which can be expressed by

$$\delta = \beta - \beta_0 \ , \ \Gamma = \frac{\pi n_2}{\lambda_0}, \quad k(z) = \frac{\pi n_1(z)}{\lambda_0}. \quad (4)$$

In order to solve the eqs. (3a) and (3b), we separate the magnitude and the phase of the counterpropagating field as

$$E_f = |E_f|\exp(i\phi_f) \ , \ E_b = |E_b|\exp(i\phi_b). \quad (5)$$

Substituting Eqs. (5) into Eqs. (3), one can obtain the following conserved quantities

$$E_T^{\ 2} = E_f^{\ 2} - E_b^{\ 2},$$

$$G = |E_f||E_b|\cos\psi(z) + (2\delta + 3\Gamma|E_b|^2)|E_f|^2 / 2k \ , \quad (6)$$

the quantity E_T^2 can be interpreted as the transmitted flux in the entire structure. The phase $\psi(z)$ appears in Eq. (6) is given by

$$\psi(z) = 2\delta z + \phi_f - \phi_b - \phi, \quad (7)$$

Using the conserved quantities E_T^2 and G, we can obtain the following equation

$$\left(\frac{L}{2}\frac{dy}{dz}\right)^2 = (y-J)[(kL)^2 y - (y-J)(\delta L + 2y)^2] = Q(y) \quad (8)$$

where

$$y = |E_f|^2 / |E_c|^2, J = |E_T|^2 / |E_c|^2,$$

$$|E_c|^2 = 4n_0\lambda_0 / 3\pi n_2 L. \quad (9)$$

The boundary conditions are given by

$$z = 0: E_f(0) = E_i, \quad E_r(0) = E_b(0), \quad (10a)$$

$$z = L: E_b(L) = 0, \ E_t = E_f(L), \quad (10b)$$

where E_i 、 E_r and E_t are the slowly varying amplitudes of the incident, reflected and transmitted wave, respectively. Combination of Eq. (8) and Eqs. (10) enables us to construct the following equivalent relationship:

$$z = 0 \ , \ y(0) = I = E_i^{\ 2} / E_c^{\ 2} \ ;$$

$$z = L \ , \ E_f^{\ 2}(L) - E_b^{\ 2}(L) = E_f^{\ 2}(L) = E_T^{\ 2} \ ,$$

$$y(L) = E_f^{\ 2}(L) / E_c^{\ 2} = E_t^{\ 2} / E_c^{\ 2} = E_T^{\ 2} / E_c^{\ 2} = J \ . \quad (11)$$

As a result, the integration of Eq. (8) yields the the analytical expression describing the relation between the input intensity and the output intensity. The integration is the standard elliptical problem whose solution depends on the relation between limits to be applied in the integration and the zeros of the polynomial $Q(y)$, where

$$y_1 = J \ ,$$

$$y_2 = \sqrt[3]{-q/2 + \sqrt{(q/2)^2 + (p/3)^3}} + \sqrt[3]{-q/2 - \sqrt{(q/2)^2 + (p/3)^3}} - \frac{s}{3} \ ,$$

$$y_3 = m \times \sqrt[3]{-q/2 + \sqrt{(q/2)^2 + (p/3)^3}} + m^2 \times \sqrt[3]{-q/2 - \sqrt{(q/2)^2 + (p/3)^3}} - \frac{s}{3} \ ,$$

$$y_4 = m^2 \times \sqrt[3]{-q/2 + \sqrt{(q/2)^2 + (p/3)^3}} + m \times \sqrt[3]{-q/2 - \sqrt{(q/2)^2 + (p/3)^3}} - \frac{s}{3} \ ,$$

$$m = \frac{-1 + i\sqrt{3}}{2} \ , \quad m^2 = \frac{-1 - i\sqrt{3}}{2}, \quad i^2 = -1,$$

$$p = -(\delta L)^2 / 12 - (\delta L)J / 3 - (kL)^2 / 4 - J^2 / 3,$$

$$s = \delta L - J \ ,$$

$$q = -(\delta L)^3 / 108 - (\delta L)^2 J / 18 - (\delta L)J^2 / 9 - 2J^3 / 27 + (kL)^2(\delta L) / 12 - (kL)^2 J / 12 \ , \quad (12)$$

The integration results are as follows:

$$\int_{y(0)}^{y(L)} \frac{dy}{\sqrt{Q(y)}} = \int_I^J \frac{dy}{\sqrt{Q(y)}} = \frac{2}{L}\int_0^L dz, \quad (13)$$

1. $y_1 \geq I, J > y_2 > y_3 > y_4$,

the integration on the left-hand side of Eq. (8) can be written as

$$\int_I^J \frac{dy}{\sqrt{Q(y)}} = -sn^{-1}(\sin\theta, v) / u, \quad (14)$$

where $sn(\sin\theta, v)$ is a Jacobian sine elliptic function, and

$$\sin\theta = [\frac{(y_1 - y_3)(I - y_2)}{(y_1 - y_2)(I - y_3)}]^{1/2},$$

$$u = 2[(y_1 - y_3)(y_2 - y_4)]^{1/2},$$

$$v = 2[(y_1 - y_2)(y_3 - y_4)]^{1/2} / u . \quad (15)$$

Inversion of Eq. (14) leads to

$$I = y_3 - \frac{y_3 - y_2}{1 - \frac{y_1 - y_2}{y_1 - y_3} sn^2(u, v)}, \quad (16)$$

2. $y_1 \geq I, J > y_2$, and $y_3 = y_4^*$

the integration on the left-hand side of Eq. (8) is now solved as

$$\int_I^J \frac{dy}{\sqrt{Q(y)}} = -cn^{-1}(\cos\theta, v)/u , \quad (17)$$

where $cn(\cos\theta, v)$ is a Jacobian cosine elliptic function, and

$$\cos\theta = \frac{(y_1 - I)B - (I - y_2)A}{(y_1 - I)B + (I - y_2)A}, \ u = 4\sqrt{AB} ,$$

$$v^2 = [(y_1 - y_2)^2 - (A - B)^2] / 4AB , A = |y_1 - y_3|,$$

$$B = |y_2 - y_3| \quad (18)$$

Inversion of Eq. (17) can obtain

$$I = y_2 + \frac{y_1 - y_2}{1 + \frac{|y_1 - y_3|}{|y_2 - y_3|}\frac{1 + cn(u,v)}{1 - cn(u,v)}}, \quad (19)$$

3. $y_1 > y_2 > y_3 \geq I, J > y_4$

the modulus v and the argument u are unchanged and gived by Eq. (15),

$$\sin\theta = [\frac{(y_1 - y_3)(I - y_4)}{(y_3 - y_4)(y_1 - I)}]^{1/2},$$

$$I = y_1 + \frac{y_1 - y_4}{1 - \frac{\frac{y_3 - y_4}{y_3 - y_1}}{} sn^2(u, v)}, \quad (20)$$

3. Results and discussions

To facilitate description, the input and output light intensity I_i , I_t are normalized as I_i / I_c , I_t / I_c respectively in following discussions, where $I_c = E_c^2$.

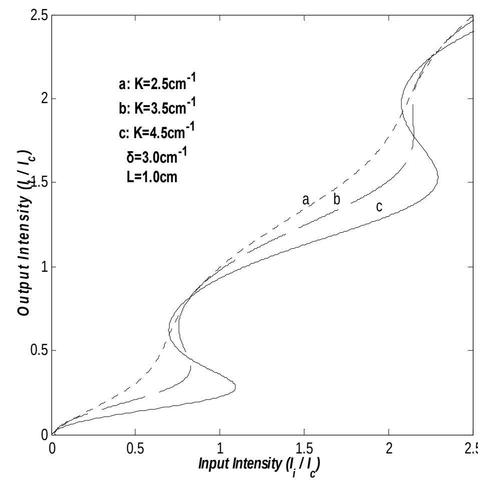

Fig. 1. Stable input-output characteristics of NLBG for various couple coefficient.

Fig. 1 shows the steady-state input-output characteristics of NLBG for various couple coefficient. From the figure, it can summarize the dependence of the bistability characteristics on the couple coefficient for a certain frequency incident light and grating length. For low values of k the feedback is insufficient to create bistability, the optical transmission mode of operation occurs at $k = 2.5cm^{-1}$. With increasement of couple coefficient, the grating internal feedback enhanced, a hysteresis loop is traced out at $k = 3.5cm^{-1}$, and larger values of k lead to multiple-phenomena.

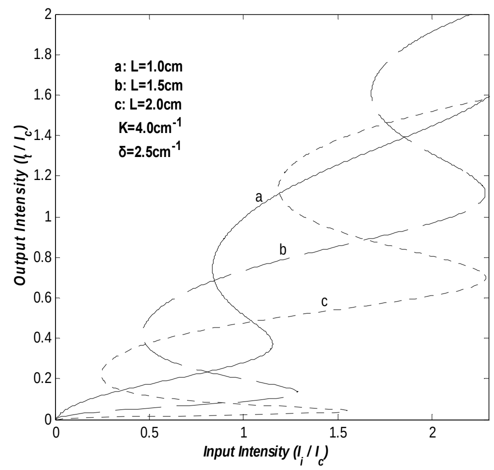

Fig. 2. Stable input-output characteristics of NLBG for various grating length.

Fig. 2 shows the steady-state input-output characteristics of NLBG for various grating length, where $\delta = 2.5cm^{-1}$ $k = 4.0cm^{-1}$. From the figure, it can be seen that , the grating length have obvious influence on the bistable characteristics, such as the switching-on threshold, the on-off ratio, the width of the hysteresis and the transmittance of the upper branch. When length is smaller, no bistable phenomena occurs. With the gradual increasement of length, the bistable effect begin to occurs, moreover the width of the hysteresis increase rapidly, for the bigger length, it exists two hysteresis.

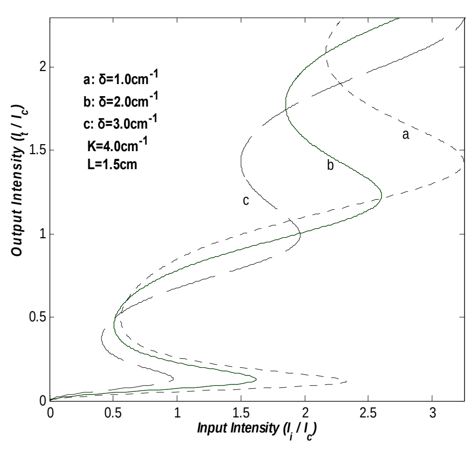

Fig. 3. Stable input-output characteristics of NLBG for various initial detuning.

Fig. 3 shows the steady-state input-output characteristics of NLBG for various initial detuning. From the figure, it can be seen that, the detuning (the incident wavelength) have obvious influence on the bistable characteristics: When the detuning decreases, and the incident wavelength shifts to higher values of the Bragg wavelength, then the lasing threshold increases significantly, even the bistable phenomena vanishes with increasement of the detuning. These features may be understood as follows: In the case of smaller detuning, the frequency of the incident light lies in the centre of reflection spectrum, and the transmitted light is “stopped” when the input intensity is lower, therefore, the required switching-on threshold to excite bistability is larger, and the on-off switching ratio is also large.

4. Conclusions

Based on the nonlinear coupled mode theory, this paper has demonstrated the bistability performance of nonlinear Bragg gratings in terms of elliptic integration, and presented the analytical expression describing the relation between the input intensity and the output intensity. Both theoretical analysis and numerical simulations show that, the bistability performance of the nonlinear Bragg gratings has significant dependence on the the couple coefficient, the grating length and the detuning respectively. The results may provide an instructive insight from a practical viewpoint.

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