

# Numerical study of index guiding quasi-crystal fibers by employing FEM method

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In this paper quasi photonic crystal fibers (QPCFs) based on 8-fold and Penrose symmetries are analyzed. The confinement loss of both proposed QPCFs with employing perfect matching layer by finite element method has been calculated. It is shown for both presented structures, the confinement loss is negligible relative to Rayleigh scattering loss. The birefringence of Penrose tile QPCF, is 80 times greater than that of Amnan Beenker quasicrystal fiber. We present numerically the existence of endlessly single mode fiber and zero birefringence QPCFs as advantage of employing novel rotational symmetry.

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## 1. Introduction

The modified index guiding fiber is a class of optical fibers representing extraordinary dispersion and nonlinearity due to the wavelength dependent cladding of photonic crystal fibers (PCFs). The mechanism of index guiding Photonic crystal fiber (PCF) is similar to the conventional fibers as it is composed of a silica core and a cladding that has an average refractive index less than core which guarantees the light propagation in the core [1-6].

Among several motivations to study the modified index guiding fibers, we can recall high nonlinearities, ultra-flattened dispersion, endlessly single mode, high or zero birefringence regimes [7-10]. Beside all their abilities tuning dispersion, wide propagating wavelength range and other optical properties with controlling on geometry is intriguing and extremely advantageous. It is worth noting that use of PCF for wide range of single mode transmission regime is very promising [7]. On the other hand flattened dispersion is guaranteed for long range transmission without guiding dispersion [11]. In the context of Index guiding photonic crystal fibers (IGPCFs), single mode regime has been widely discussed, but numerical existence of endlessly single mode fiber is not reported up to now [7].

Fabrication of the PCFs is well developed and adopted with any desired structure of them. Stack and draw technique is quite flexible to build up many different geometrical arrangements.

On the other hand Bravais lattices as periodic structures are limited to few geometries consequently symmetries, as they are limited to few rotational symmetries such as triangular and square lattices [1, 12].

In addition several structures with richer rotational symmetries with  $n$ -fold symmetry where  $n > 6$  and  $n = 5$

(known as quasi-crystals) has presented the ability of their light propagation with photonic band gap mechanism, but only few works can be found in literature [15].

The Quasi Crystal Index Guiding Fibers have not been well studied. We can mention only few works in the literature, A 12-fold index guiding photonic-quasi-crystal fiber is analyzed for its single mode regime [17].

We can mention many of symmetries such as 8-fold, 5-fold (and different Penrose tiling) which are promising for their rich rotational symmetry.

In this work we develop a FEM method to analyze optical fiber with quasicrystal structure working via modified index guiding mechanism. In this paper we take in calculation two different rotational symmetries to study the impact of this parameter on the fiber properties. The report is organized as follows, the geometry of 8-fold and Penrose structures are described in section 2. In section 3, calculation method and results of both introduced structures are presented by mean of FEM. The paper is enclosed with conclusion on the very promising result of endlessly single mode regime and zero birefringence which can be achieved regarding to new geometries.

## 2. Geometrical structures

An optical fiber is a Z-translation invariant structure. Photonic crystal fibers are optical fibers with two dimensioned symmetric crystals in the transverse plane and a dislocation which is known as the core of the PCF. The fiber with high index material filled core is called index guiding photonic crystal fiber. The total internal reflection is the physical principles of light guiding in these fibers. Constructive interference of reflected waves by different

periodic of the cladding in the PCF core causes guiding in the band gap PCFs. An optical fiber with quasi-crystal structure in the transverse plane is called quasi photonic crystal fiber (QPCF). However in addition to richer rotational symmetry a translational symmetry with larger period can be found in the quasi-crystal structures. In the following the QPCFs with 8-fold and 5-fold geometry are introduced.

### 2.1. 8-fold structure

An 8-fold structure (called Amnan-Beenker) is chosen due to high level of symmetry. This structure is made of tiles of rhombi of acute angle  $\pi/4$  and squares. Fig. 1 represents the fiber structure proposed for index guiding,  $r$  represents the air holes radius and  $r/\Lambda=0.37$  where  $\Lambda$  is quasi-crystal constant. The material is considered glass with refractive index of  $n=1.46$ .

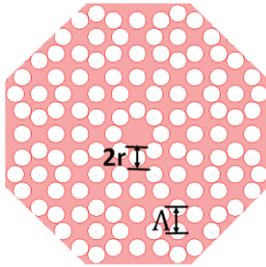


Fig. 1. Cross-section of an 8-fold Amnan-Beenker QC fiber

We consider the central hole of 8-fold filled with glass as core with radius of  $\Lambda$ . The 8-fold symmetry holds a  $\pi/8$  radian rotational symmetry, which can affect on the birefringence due to the symmetry on the  $\frac{\pi}{2}$  radian.

### 2.2. Penrose tile with 5-fold symmetry

We have chosen a 5-fold structure (called Penrose) that can be made by 5 vectors with angle of  $\frac{2\pi}{5}$ . Fig. 2 represents the cross sectional presentation of fiber structure proposed for index guiding QPCF,  $r$  represents the air holes radii and  $r/\Lambda=0.308$  where  $\Lambda$  is quasi crystal constant. The material characteristic is chosen the same as previous structure.

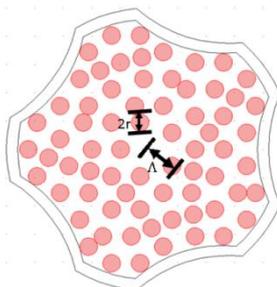


Fig. 2. Cross section of a Penrose tile QC fiber

We consider the central hole of 5-fold filled with glass as core and a core size of equal to  $\Lambda$  and due to the kind of symmetry we expect very high birefringence.

### 3. Calculation approach and results analysis

Quasi-crystal fibers can be considered as an in-homogeneous media for electromagnetic wave, propagation and the Maxwell equation (governing equation) is reduced to [19]:

$$\nabla \times (\epsilon_r^{-1} \nabla \times H) = K_0^2 \mu_r H \quad (1)$$

Where  $\epsilon_r$  and  $\mu_r$  are the relative dielectric permittivity and magnetic permeability tensors respectively,  $K_0 = \frac{2\pi}{\lambda}$  is the wave number in the vacuum,  $\lambda$  is the wavelength and  $H$  is the magnetic field. The finite size of clad causes the propagation constant  $\beta (= K_0 n_{eff} - i\alpha)$  to be a complex number.  $n_{eff}$  is the effective refractive index of the medium. That means modes are leaky and eigenvalues are complex, which the real part of eigenvalue gives the mode refractive index and its imaginary part gives the fiber mode confinement loss (CL). The propagation of electromagnetic field variation in the  $z$ -direction along the optical fiber is  $H = h e^{-i\beta z}$  where the magnetic field is in the transverse plane. To discrete of the eigenvalue Eq. (1) the triangular mesh is employed. By employing the variation finite element method Eq. (1) is reduced to a finite dimensional generalized eigenvalue equation [19].

$$A \vec{h} = \left(\frac{\gamma}{K_0}\right)^2 B \vec{h} \quad (2)$$

Where  $A$  and  $B$  are finite dimensional sparse and symmetric matrices. In this work it is assumed that the medium is isotropic,  $\epsilon_r$  and  $\mu_r = 1$  are scalars and Eq. (2) is reduced to a simple eigenvalue equation. To enclose the computational domain without affecting the numerical solution, a perfectly matched layer (PML) is placed around the outer boundary of the QPCF structure. To accurately describe all the regions in the transverse plane, the dimension of the triangular elements are chosen properly. Fiber symmetry group can be employed to reduce the computation domain hence both memory and time require computation, without affecting the accuracy of the computed fields of the fiber modes and eigenvalues, as the fiber modes effective refractive indices.

In step index fibers the  $V = K_0 a \sqrt{n_{c_0}^2 - n_d^2}$  parameter is employed as a parameter to distinguish the boundary of single mode or multimode regime of operation of the optical fiber [2]. In this analysis the  $V$ -parameter which is proposed for triangular PCFs [6] is also employed for the QPCFs [17]

$$V = \frac{2\pi}{\lambda} \Lambda \sqrt{n_{fgm}^2 - n_{fcm}^2} \quad (3)$$

Where  $n_{fgm}$  and  $n_{fcm}$  are effective refractive indices of the fundamental guided mode (core mode) and of the fundamental (cladding) mode in the cladding respectively.

### 3.1. Amnan-Beenker index guiding quasi-crystal fiber

Eigenvectors of Eq. (2) are employed to determine the z-component of Poynting vector of the 8-fold QPCF and the results for the fundamental modes of core and cladding are presented in Fig. 3, where  $\frac{\Lambda}{\lambda} = 0.6$ .

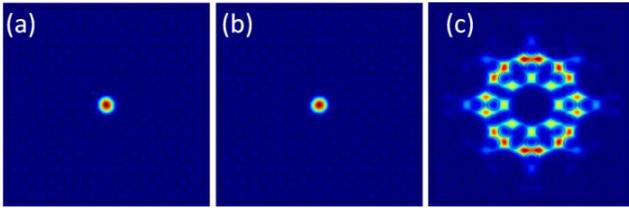


Fig. 3. Poynting Fundamental modes for (a), (b) core and (c) cladding

Fig. 3 (a) and (b) representing the fundamental modes of core as it is shown the modes based on the symmetry are completely round shape and centered in the core. On the other hand fundamental mode of cladding represents a distribution of electromagnetic field with representing 8-fold symmetry.

The effective refractive indices of fundamental mode of core and clad versus normalized frequency  $\frac{\Lambda}{\lambda}$  is shown. In Fig. (4a) and Fig. (4b) and corresponding V-parameter versus the normalized frequency is presented in Fig. (5) the same calculation is done for calculating V parameter of the Penrose tile fiber dotted line Fig. (5).

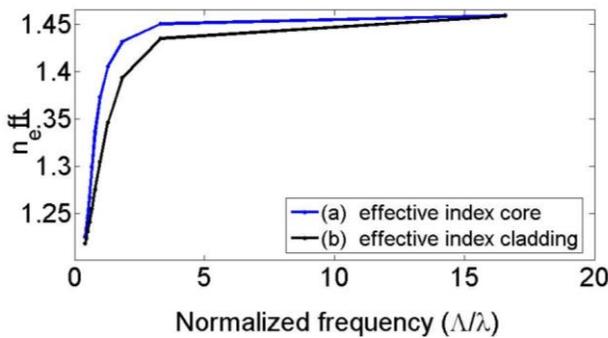


Fig. 4(a) and 4(b) effective index of core (blue curve) and cladding (black curve) vs. Normalized wavelength for 8-fold QPCF

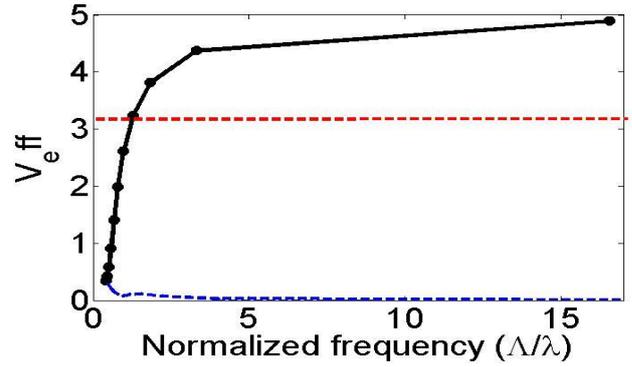


Fig. 5. Effective V parameter vs. Normalized frequency for Penrose (blue curve) and 8-fold quasi-crystal fiber (black curve)

To analysis the V-parameter, the region of  $V_{eff} < \pi$  is corresponding to single mode regime [6, 17]. Therefore for 8-fold fiber presents the single mode regime for  $\frac{\Lambda}{\lambda} < 2.1$ .

The electric field vectors of two different linearly polarized fundamental modes at  $\frac{\Lambda}{\lambda} = 0.6$  are depicted in Fig. 6(a) and 6(b). The existence of these modes and the difference in refractive index causes birefringence.

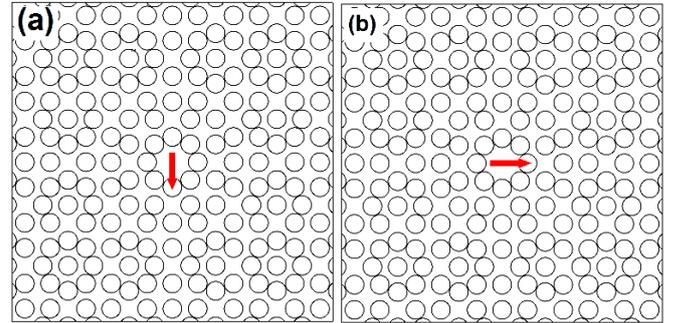


Fig. 6. (a) Fundamental Y-polarized mode, (b) Fundamental X-polarized mode

One of the important parameters in fiber studies is birefringence due to its sensing and telecommunication applications as defined by the following relation [19]:

$$B = |n_{eff}^x - n_{eff}^y| \quad (4)$$

Where  $n_{eff}^x$  and  $n_{eff}^y$  are fundamental mode effective refractive indices of x- and y-polarizations. The study of birefringence in the fiber represents results which meets to the symmetry of the fiber. As depicted in Fig. 7 the birefringence is negligible and it is due to 8-fold symmetry and actually it was expected to be degenerate states. This is shown that they are degenerate states and zero difference between X- and Y- polarized mode birefringency vs normalized wavelength, as it is shown in Fig. 7, Inset.

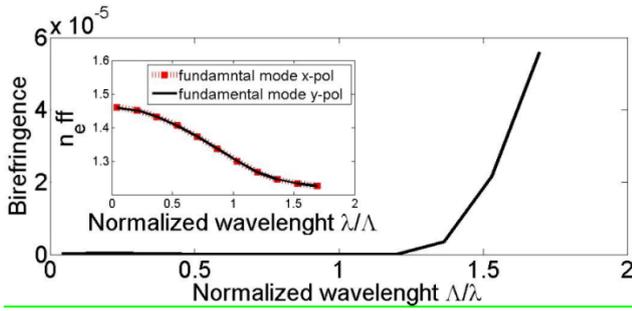


Fig. 7. Birefringence of 8-fold Index guiding Quasi-crystal fiber, (inset) effective refractive indices of x- and y-polarized modes

The maximum birefringence is in the order of  $10^{-5}$  and minimum of birefringence goes to  $1.3 \times 10^{-7}$ . In scale of birefringence they are negligible in comparison with the symmetry breaking fibers, where birefringence increased up to  $10^{-2}$  [20,21]. The increase of wavelength induces the larger effective area for the propagated wave, on the other hand the structure shows poor symmetry in the higher area, which can induce higher birefringence.

The imaginary part of eigenvalues determines the confinement loss of each mode of QPCF. Fig. (8) shows the confinement loss of 8-fold QPCF versus the normalized wavelength. Because of the energy distribution in the cladding, higher order modes are closed to the outer boundary and the confinement loss increases for these modes. As it is shown in Fig. (8), the level of leakage loss is of order  $10^{-17}$  for  $\frac{\lambda}{\Lambda} < 1$  and it reaches  $10^{-5}$  when the normalized wavelength is in the range  $1 < \frac{\lambda}{\Lambda} < 1.6$ . The Rayleigh scattering and absorption losses of QPCFs are similar to those of standard step index fiber.

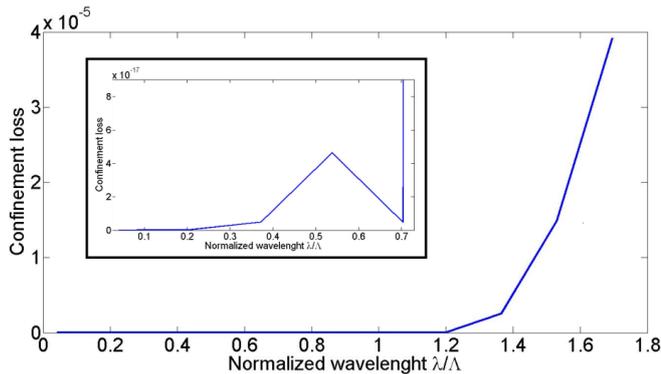


Fig. 8. Represents leak loss for the 8-fold structure, (inset) the loss behavior at  $\lambda/\Lambda < 1$

### 3.2. Penrose tile index guiding quasi-crystal fiber modes

The z-component of Poynting vector of the fundamental mode of core and cladding for Penrose tile structure is presented in Fig. 9. It is found that 6 modes are presented. The energy distribution of electromagnetic field of Fig. 9a and Fig. 9b are concentrated in the core region

with perpendicular polarizations, hence there are two non degenerate core modes with different effective refractive indices. The electromagnetic energy of modes presented in Fig. 9c – Fig. 9f is concentrated in the space filled of the clad, these modes are called the cladding modes. The effective refractive index of modes versus normalized wavelength is depicted in Fig. 10. V-parameter of the Penrose tile and results is shown in Fig. 5.

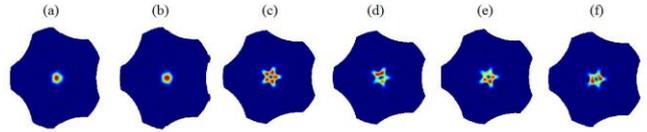


Fig. 9. (a), (b) fundamental mode, (c), (d), (e), and (f) higher order modes

Based on Fig. 5 the Penrose lattice is showing an endlessly single mode regime all over the wavelength range. The physics behind the single mode regime is; cladding modes are enough confined in the fiber structure which can be excited by core mode or vice versa, so it is expected to have a numerically and hence experimentally endlessly single mode fiber.

Fig. 10 is illustrating the effective index of all modes related to Fig. 9 versus normalized wavelength. The calculation shows that all presented modes in Fig. 9 have similar dispersion behavior.

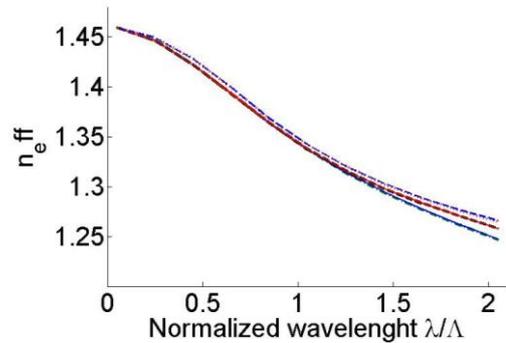


Fig. 10. The effective index vs. normalized wavelength for the first 3 pairs of modes, 9a, 9c, 9e are shown by dashed solid, dotted line respectively

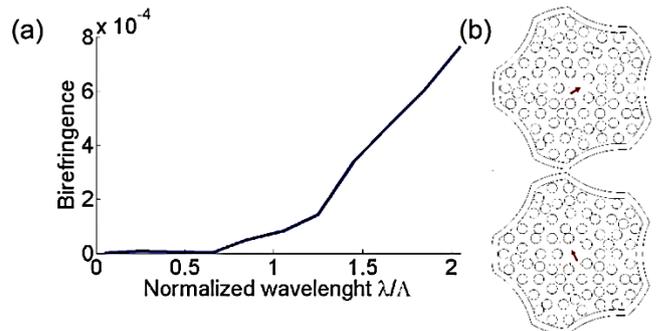


Fig. 11. (a) Birefringence of Penrose lattice vs. normalized wavelength, (b) electric field profiles of the fundamental modes

The symmetry of the Penrose fiber is on the basis of vectors with  $\frac{2\pi}{5}$ , therefore the structure is highly birefringence while cannot present degeneracy on the Penrose structure for perpendicular modes. The birefringence versus normalized wavelength is presented in Fig. 11 (a). The order of birefringence magnitude is 80 times more than 8-fold and it is expected regarding to its symmetry point of view.

The novel geometry, which allow us to develop high birefringence and endlessly single mode fiber, with assistant of high rotational symmetry. The QPCFs look very promising for their rich symmetries.

The Penrose fiber is high birefringence with order of  $10^{-3}$  as it is only due to confinement loss for Penrose tile proposed fiber has been studied and the confinement loss versus  $\frac{\lambda}{\Lambda}$  is represented in Fig. 12.

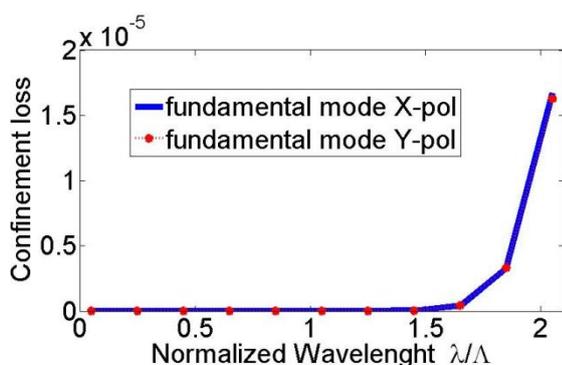


Fig. 12. Confinement loss for the first two fundamental modes of the Penrose fiber

The leak loss for  $\frac{\lambda}{\Lambda} < 1$  is in the order of  $10^{-15}$  and again presented a negligible loss in compare with intrinsic losses of the fiber structure. Therefore in both cases we can neglect confinement loss completely.

#### 4. Conclusion

In conclusion, we have demonstrated that the effective formula defines the V parameter of photonic crystal fibers can be applied for quasi-crystal structures, such as the 8-fold and Penrose tile. Single mode regime for 8-fold determined and we numerically proved endlessly single mode fiber can be made with Penrose fiber.

Confinement loss of both structures is presented and birefringence has been analyzed. 8-fold structure's birefringence due to symmetry of  $\pi/8$  is almost zero, where comparing with Penrose, so the effect of rotational symmetry and its importance on fiber properties is shown. The Penrose tile with some modification can be employed for polarization maintaining applications.

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