On optical solitons of the non-local NLSE with time dependent coefficients

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This paper integrates non-local nonlinear Schrödinger equation (NNLSE) with time dependent coefficients. The first integral method (FIM) is applied to report the optical soliton solutions of NNLSE with parabolic law nonlinearity and time dependent coefficients which are the terms of velocity dipersion, linear and nonlinear terms and also non-local one.

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1. Introduction

The nonlinear Schrödinger's equation (NLSE) governs the dynamics of optical solitons propagating through optical fibers for trans-continental and trans-oceanic distances.

With the help of multiple-scale perturbation analysis, this NLSE was deduced from Maxwell's equation.

The NLSE arises in the literature of optical solitons, with various forms of nonlinearity that depends on the studied context. The most known mathematical modeling of optical systems generally is expressed by types of NLSE. The details of NLSE are given in the studies on nonlinear optics [1]-[8].

It is fundamental to obtain the general solutions of these corresponding nonlinear equations. As a result, the general solutions of these equations give us an important information regarding the character and the structure of the equations. Many effective methods were utilized to provide much information for physicians and engineers. Some of these methods are Tanh [9], G'/G -expansion [10], Jacobi elliptic function [11], functional variable [12], Hirota bilinear [13], exp-function [14], and first integral methods [15]. We recall that all these methods are very effective for getting the traveling wave solutions NPDE.

FIM was presented initially to the literature by finding the solutions of the Burgers-KdV equation [15]. This method was successfully utilized to NPDE and some fractional differential equations. Recently, various investigations on this method were reporting interesting results. Thus, Raslan [16] utilized this method for the Fisher equation. Tascan and Bekir [17] utilized this method to solve the Cahn-Allen equation. Abbasbandy and Shirzadi [18] have discussed Benjamin Bona-Mohany equation within method. We recall that Jafari et al. [19] and K. Hosseini et al. [20] has researched w.r.t for Biswas-Milovic equation and KP equation so on [21]-[33].

In this manuscript, we present the governing equation NNLSE in section 2. The FIM is described and applied in section 3. The conclusions are reported in the last section.

2. The governing equation

One of the type of the NLSE is NNLSE [34]-[36] that is going to be investigated in here. The dimensionles form of the NNLSE [37] with time dependent coefficients is

$$\begin{aligned} & ih_t + a(t)h_{xx} + b(t)|h|^2 h + \\ & c(t)|h|^4 h + d(t)\left(|h|^2\right)_{xx} h = 0, \end{aligned} \tag{1}$$

where the terms $a(t), b(t), c(t)$ and $d(t)$ symbolize group velocity dipersion, weakly non-local nonlinearity (*a* and *d*),parabolic law nonlinearity (*b* and *c*). We will discuss the equation (1) in the non-local nonlinearity, which seems in the medium at a point determines on optical pulse intensityat that point.

3. The method

FMI is described briefly below:

1. Consider an usual NPDE in:

$$W(h, h_t, h_x, h_{xt}, h_{tt}, h_{xx}, \ldots) = 0, \tag{2}$$

then Eq. (2) becomes an ODE, namely

$$L(H, H', H'', H''', \ldots) = 0 \tag{3}$$

such that $\xi = x \mp ct$ and $H' = \partial H(\xi) / \partial \xi.$

2. It could be taken of ODE (3) as

$$h(x,t)=H(\xi). \quad (4)$$

3. A new independent variable is considered as

$$H(\xi)=Q(\xi), G(\xi)=\partial Q(\xi)/\partial\xi \quad (5)$$

That gives a new system of ODEs, namely

$$\begin{aligned}\partial Q(\xi)/\partial\xi &= G(\xi)\\ \partial F(\xi)/\partial\xi &= P(Q(\xi),G(\xi))\end{aligned} \quad (6)$$

4. Following the qualitative theory of ODE [38], if it will be possible to obtain the integrals for (6), then we obtaine immediately the solutions of (6). The Division Theorem (DT) [39] shows the way how to obtain the first integrals.

3.1 Application

The Eq. (1) turns to the following ODE system in real and imaginary parts by using the following transformation $h=H(\xi)e^{i[-\alpha x+vt]}, \xi=\beta(x-wt).$

$$w-2a\alpha=0, \quad (7)$$

$$\begin{aligned}&-\left(v+a\alpha^2\right)H+bH^3+cH^5+\\ &2d\beta^2 HH_\xi^2+\left(2dH^2+a\right)\beta^2 H_{\xi\xi}=0,\end{aligned} \quad (8)$$

Then, by using (7) we have

$$w=2a\alpha \quad (9)$$

and it seems from the (7), the veolcity of the soliton $w(t)$ is related with the group velocity term $a(t)$.

By using another transformation (5) we have

$$\begin{aligned}Q_\xi &= G,\\ G_\xi &= \left(\frac{1}{\left(2dQ^2+a\right)\beta^2}\right)\\ &\left[\left(v+a\alpha^2\right)Q-bQ^3-cQ^5-2d\beta^2 QG^2\right],\end{aligned} \quad (10)$$

Suppose that $d\tau=\dfrac{d\xi}{\left(2dQ^2+a\right)\beta^2}$ then (10) becomes

$$\begin{aligned}Q_\tau &= \left(2dQ^2+a\right)\beta^2 G,\\ G_\tau &= \left(v+a\alpha^2\right)Q-bQ^3-cQ^5-2d\beta^2 QG^2.\end{aligned} \quad (11)$$

According to FIM, it is supposed that $Q(\tau)$ and $G(\tau)$ are non-trivial solutions of Eq. (10) and $F(Q,G)=\sum_{i=0}^{r} a_i(Q)G^i$ is an irreducible function in the domain $C[Q,G]$ such that

$$F(Q(\tau),G(\tau))=\sum_{i=0}^{r} a_i(Q)G^i=0, \quad (12)$$

where $a_i(Q)$, $(i=0,1,2,\ldots,r)$ are polynomials of Q and $a_r(Q)\neq 0$. Eq.(12) is the first integral for system (11), owing to the DT, there exists $g(Q)+h(Q)G$ in $C[Q,G]$ as:

$$\begin{aligned}dF/d\tau &= \frac{dF}{dQ}\frac{dQ}{d\tau}+\frac{dF}{dG}\frac{dG}{d\tau}\\ &=[g(Q)+h(Q)G]\sum_{i=0}^{r} a_i(Q)G^i\end{aligned} \quad (13)$$

In here, we only consider $r=1$ in Eq. (13).
If we equate the coefficients of $G^i(i=0,1,2,\ldots,r)$ of Eq. (12) for $r=1$, we have

$$\dot{a}_1(Q)\left(2dQ^2+a\right)\beta^2=a_1(Q)\left(h(Q)+2d\beta^2 Q\right) \quad (14)$$

$$\dot{a}_0(Q)\left(2dQ^2+a\right)\beta^2=a_1(Q)g(Q)+h(Q)a_0(Q) \quad (15)$$

$$\begin{aligned}&a_0(Q)g(Q)=a_1(Q)\\ &\left[\left(v+a\alpha^2\right)Q-bQ^3-cQ^5\right]\end{aligned} \quad (16)$$

Since $a_1(Q)(i=0,1)$ is polynomial of $Q, a_1(Q)$ is a constant and $h(Q)=-2d\beta^2 Q$ from (14). For convenience, it is obtained $a_1(Q)=1.$ Byequalization the degrees of $g(Q)$ and $a_0(Q)$ we firstly conclude the degree of $g(Q)$ is equal to two. Then, we assume that $g(Q)=G_0+G_1Q+G_2Q^2$, we obtain from Eq. (16) as follows

$$a_0(Q)=A_0+A_1Q+A_2Q^2+A_3Q^3 \quad (17)$$

Replacing $a_0(Q)$, $a_1(Q)$ and $g(Q)$ in Eq. (16) to separate the common factor of the same terms, then equating the coefficients of Q^i to zero, we have following case:

$$v=\frac{cG_0{}^2-G_2(bG_0+aG_2\alpha^2)}{G_2{}^2}, A_0=0$$
$$A_2=G_1=0, A_1=-\frac{cG_0-bG_2}{G_2{}^2}, A_3=-\frac{c}{G_2}. \tag{18}$$

setting (18) into (12), we have

$$Q_\xi=\frac{bG_2-cG_0}{G_2{}^2}Q(\xi)+\frac{c}{G_2}Q^3(\xi) \tag{19}$$

If we seek the solution of the Eq.(1) with the equation (19), we have the following constraints

$$v=-a\alpha^2,\ a=\frac{3bd}{c},$$
$$G_2=\pm\sqrt{-24d\beta^2c}, G_0=\frac{bG_2}{c}. \tag{20}$$

Then the solution of the Eq. (1) is obtained with the aid of the Eq. (19), the solution is as follows

$$h(x,t)=\frac{\left(cG_0-bG_2\right)^2 e^{\frac{2(bG_2-cG_0)(\beta(x-wt)+G_2{}^2C_0)}{G_2{}^2}}}{1+cG_2e^{\frac{2(bG_2-cG_0)(\beta(x-wt)+G_2{}^2C_0)}{G_2{}^2}}}e^{i[-\alpha x+vt]}, \tag{21}$$

where C_0 arbitrary is constant.

Secondly, we conclude the degree of $g(Q)$ is equal to three. Then, we assume that $g(Q)=G_0+G_1Q+G_2Q^2+G_3Q^3$, we obtain from Eq. (16) as follows

$$a_0(Q)=A_0+A_1Q+A_2Q^2 \tag{22}$$

Replacing $a_0(Q)$, $a_1(Q)$ and $g(Q)$ in Eq. (16) to separate the common factor of the same terms, then equating the coefficients of Q^i to zero, we have following case

$$A_1=G_1=G_2=0, G_3\alpha\neq 0, A_0=\frac{cG_1-bG_3}{G_3{}^2}$$
$$v=\frac{cG_1{}^2-G_3\left(aG_3\alpha^2+bG_1\right)}{G_3{}^2}, A_2=\frac{-c}{G_3}. \tag{23}$$

Setting (23) into (12), we have

$$Q_\xi=\frac{bG_3-cG_1}{G_3{}^2}+\frac{c}{G_3}Q^2(\xi) \tag{24}$$

So we have the following solution families depending on the Eq. (24)

Family I.

The dark optical soliton of the Eq. (1) is as following

$$h(x,t)=\sqrt{\frac{cG_1-bG_3}{cG_3}}\tanh\left[-\sqrt{\frac{c\left(cG_1-bG_3\right)}{G_3{}^3}}\beta\left(x-wt\right)\right]e^{i[-\alpha x+vt]} \tag{25}$$

where $w(t)=2a(t)\alpha\beta, G_1=\frac{i}{2}\sqrt{\frac{3}{cd}}(ac+bd)\beta$ and $G_3=i\sqrt{6cd}\beta$.

Family II.

Chen and Zhang [40] studied the following equation

$$Q_\xi=A+BQ(\xi)+CQ^2(\xi) \tag{26}$$

and they presented the solution of the Eq. (26) is

$$Q(\xi)=\tanh\left(\xi\right)\pm i\operatorname{sech}\left(\xi\right) \tag{27}$$

where $A=-C=\frac{1}{2}$ and $B=0$. So we could seek the dark-bright combo optical soliton solution with the aid of the (26). If we choose $G_3=-2c$ and $G_1=2\left(\frac{b}{c}-1\right)$, then the Eq. (24) is reduced to (26) with the constraint $A=-C=\frac{1}{2}$ and $B=0$. Consequently we have the following dark-bright combo optical soliton for the Eq. (1)

$$h(x,t)=\begin{pmatrix}\tanh\left[\beta\left(x-wt\right)\right]\pm\\ i\mathrm{sech}\left[\beta\left(x-wt\right)\right]\end{pmatrix}e^{i[-\alpha x+vt]} \quad (28)$$

Where

$v=\frac{1}{2}\left(-2a\alpha^2-a\beta^2+d\beta^2\right), b=\frac{\beta^2}{2}\left(4d-a\right)$ and $c=-\frac{3d\beta^2}{2}$.

4. Conclusion

FIM was utilized for acquiring new exact solutions of NNLSE with power law nonlinearity and time dependent coefficients. We have acquired new dark and dark-brigth optical combo exact solutions of NNLSE. It is illustrated veolcity functions $w(t)$ and $v(t)$ is related with the group velocity term $a(t)$ and $d(t)$. In the other hand it is seen that group velocity dipersion, weakly non-local nonlinearity terms $a(t)$ and $d(t)$ are effected the parabolic law nonlinearity terms $b(t)$ and $c(t)$ for the dark-brigth optical combo exact solution of NNLSE. For the next studies we will discuss the Eq. (1) with the snoidal and cnoidal soliton solutions.

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