

Optical solitons in birefringent fibers with four-wave mixing for parabolic law nonlinearity

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Optical solitons in birefringent fibers with parabolic law nonlinearity, with four-wave mixing, is addressed in this paper. Exact bright, dark and singular 1-soliton solution is obtained. There are several constraint conditions that naturally emerge from the structure of the soliton solution.

(Received September 12, 2014; accepted January 21, 2015)

Keywords: Solitons, Integrability, Birefringence, Parabolic law

1. Introduction

Optical solitons is one of the most fascinating areas of research in the field of nonlinear optics. Several results are reported in the past few decades [1-15]. In this area, the dynamics of solitons pulses propagating through optical fibers is investigated. This paper therefore investigates solitons that propagate through birefringent fibers. Birefringence is a common phenomenon that naturally occurs in optical fibers and it leads to the splitting of these solitons. Thus the solitons are polarized and thus it leads to differential group delay. The governing equation is the nonlinear Schrödinger's equation (NLSE). This paper studies NLSE with parabolic law nonlinearity, otherwise known as cubic-quintic law [6]. In addition to self-phase modulation (SPM) [1, 2] and cross-phase modulation (XPM) [1, 2, 3, 4], birefringence introduces the unwanted four-wave mixing (4WM) [9]. The case of birefringence in Kerr law, also known as cubic nonlinearity, in presence of 4WM is already reported during 2014 [9]. This paper will thus be an extension of earlier results from Kerr medium to parabolic law medium. As with Kerr law nonlinearity, ansatz approach will be implemented in this paper with parabolic law nonlinearity under phase-matching condition. For this integration scheme, constraint conditions will naturally emerge for these solitons to exist. After introducing the mathematical model in the following section, soliton solutions will be derived in subsequent subsections.

2. Mathematical model

Optical solitons in birefringent fibers with parabolic law nonlinearity is governed by the following coupled NLSE [1, 2, 4, 5, 9]:

$$\begin{aligned}
 & i q_t + a q_{xx} + (k_1 |q|^2 + 2k_1 |r|^2) q + \\
 & + (k_2 |q|^4 + 3k_2 |r|^4 + 6k_2 |q|^2 |r|^2) q + \\
 & + (k_1 + 3k_2 |q|^2 + 2k_2 |r|^2) r^2 q^* + k_2 r^3 (q^*)^2 = 0
 \end{aligned} \tag{1}$$

$$\begin{aligned}
 & i r_t + a r_{xx} + (k_1 |r|^2 + 2k_1 |q|^2) r + \\
 & + (k_2 |r|^4 + 3k_2 |q|^4 + 6k_2 |q|^2 |r|^2) r + \\
 & + (k_1 + 3k_2 |r|^2 + 2k_2 |q|^2) q^2 r^* + k_2 q^3 (r^*)^2 = 0
 \end{aligned} \tag{2}$$

Equations (1) and (2) represent the model for propagation of optical solitons through birefringent fibers that maintains parabolic law nonlinearity. In (1) and (2), a is the coefficient of group velocity dispersion (GVD), while k_l for $l = 1, 2$ are the coefficients of SPM and XPM terms respectively. The last two terms in (1) and (2) are accounted for 4WM. Here, 4WM is a nonlinear effect that stems out of third order nonlinearity. It occurs when at least two different frequency components co-propagate in some nonlinear medium. Also, x represents the spatial variable while t represents temporal variable. Finally, $q(x, t)$ and $r(x, t)$ are the complex-valued wave profiles for the polarized solitons [1, 2, 4, 5, 9].

The search for exact bright, dark and singular 1-soliton solutions to this model will be studied in the rest of this

paper. In order to integrate (1) and (2) for soliton solutions, the following assumptions are made for the wave profile:

$$q(x, t) = P_1(x, t) e^{i\phi(x, t)} \quad (3)$$

$$r(x, t) = P_2(x, t) e^{i\phi(x, t)} \quad (4)$$

where $P_l(x, t)$ ($l = 1, 2$) are the amplitude components of the soliton solution while the phase component $\phi(x, t)$ is given by

$$\phi(x, t) = -\kappa x + \omega t + \theta \quad (5)$$

Here, κ is the soliton frequency, while ω represents the wave number and θ is the phase constant. Substituting (3)-(5) into (1) and (2) and then decomposing into real and imaginary parts give

$$P_l(\omega + a_1\kappa^2) - a \frac{\partial^2 P_l}{\partial x^2} - k_1 P_l^3 - 3k_1 P_l P_l^2 - k_2 P_l^5 - 5k_2 P_l P_l^4 - 9k_2 P_l^3 P_l^2 - k_2 P_l^2 P_l^3 = 0 \quad (6)$$

and

$$\frac{\partial P_l}{\partial t} - 2a\kappa \frac{\partial P_l}{\partial x} = 0 \quad (7)$$

respectively, where $l = 1, 2$ and $\bar{l} = 3 - l$. From the imaginary part equation it is possible to obtain the speed (v) of the soliton as

$$v = 2a\kappa \quad (8)$$

This follows after assuming that the wave profile for P_l is given by the form $g(x - vt)$ where g is the wave profile with v being the speed of the wave. The real component given by (6) will be now further analysed for seeking bright, dark and singular solitons. The study will now be split into the following three subsections.

3. Bright solitons

In this case the assumption for P_l is taken to be

$$P_l(x, t) = \frac{A_l}{(D + \cosh \tau)^{p_l}} \quad (9)$$

where

$$\tau = B(x - vt) \quad (10)$$

The amplitude of the soliton is given by A_l and the inverse width is B . Finally, the speed of the soliton is v . Substituting (9) into (6) and simplifying leads to

$$\begin{aligned} & \frac{(\omega + a\kappa^2 - ap_l B^2)A_l}{(D + \cosh \tau)^{p_l}} + \frac{aDB^2 p_l(2p_l + 1)}{(D + \cosh \tau)^{p_l+1}} - \\ & - \frac{aA_l B^2 p_l(2p_l + 1)(D^2 - 1)}{(D + \cosh \tau)^{p_l+2}} - \frac{3k_1 A_l^3}{(D + \cosh \tau)^{3p_l}} - \\ & - \frac{3k_1 A_l A_l^2}{(D + \cosh \tau)^{p_l+2p_l}} - \frac{k_2 A_l^5}{(D + \cosh \tau)^{5p_l}} - \\ & - \frac{5k_2 A_l A_l^4}{(D + \cosh \tau)^{p_l+4p_l}} - \frac{9k_2 A_l^3 A_l^2}{(D + \cosh \tau)^{3p_l+2p_l}} - \\ & - \frac{k_2 A_l^2 A_l^3}{(D + \cosh \tau)^{2p_l+3p_l}} = 0 \end{aligned} \quad (11)$$

Balancing principle implies

$$p_l = \frac{1}{2} \quad (12)$$

for $l = 1, 2$. Next, from the real part equation (11) setting the coefficients of linearly independent functions to zero, leads to

$$\omega = -a\kappa^2 + \frac{aB^2}{4} \quad (13)$$

$$A_1 = A_2 = \frac{B}{2} \left(\frac{3a^2}{3k_1^2 + 4aB^2 k_2} \right)^{\frac{1}{2}} = A \quad (14)$$

and

$$D = k_1 \sqrt{\frac{3}{3k_1^2 + 4aB^2 k_2}} \quad (15)$$

Thus the relations (14) and (15) prompt the constraint

$$3k_1^2 + 4aB^2 k_2 > 0 \quad (16)$$

Hence bright 1-soliton solutions to (1) and (2) are

$$\begin{aligned} q(x, t) &= r(x, t) \\ &= \frac{A_l}{\sqrt{D + \cosh \tau}} e^{i(-\kappa x + \omega t + \theta)} \end{aligned} \quad (17)$$

where the definition of the parameters are in place.

3.1 Dark solitons

For dark solitons, the starting hypothesis will be

$$P_l(x, t) = (C + D \tanh \tau)^{p_l} \quad (18)$$

with the same definition of τ as in bright solitons. Substituting this hypothesis into (6) and simplifying, leads to

$$\begin{aligned} & D^2(\omega + a\kappa^2)(C + D \tanh \tau)^{p_l} + \\ & + aB^2 C p_l (4p_l + 2)(C + D \tanh \tau)^{p_l+1} \\ & - aB^2 p_l (p_l + 1)(C + D \tanh \tau)^{p_l+2} \\ & - 2aB^2 (3C^2 - D^2) p_l (4p_l - 2)(C + D \tanh \tau)^{p_l-1} \\ & + aB^2 C (C^2 - D^2) p_l (4p_l - 2)(C + D \tanh \tau)^{p_l-1} \\ & - aB^2 (C^2 - D^2) p_l (p_l - 1)(C + D \tanh \tau)^{p_l-2} \\ & - k_1 D^2 (C + D \tanh \tau)^{3p_l} - 3k_1 D^2 (C + D \tanh \tau)^{p_l+2p_l} \\ & - k_2 D^2 (C + D \tanh \tau)^{5p_l} - 5k_2 D^2 (C + D \tanh \tau)^{p_l+4p_l} \\ & - 9k_2 D^2 (C + D \tanh \tau)^{3p_l+2p_l} \\ & - k_2 D^2 (C + D \tanh \tau)^{2p_l+3p_l} = 0 \end{aligned} \quad (19)$$

Next, balancing principle yields p_l as given by (12). Subsequently, setting the coefficients of the linearly independent function to zero gives

$$B = \frac{k_1}{8} \sqrt{-\frac{3}{ak_2}} \quad (20)$$

$$C = D = -\frac{3k_1}{64k_2} \quad (21)$$

and

$$\omega = -\frac{3k_1^2 + 64a\kappa^2 k_2}{64k_2} \quad (22)$$

with the constraint

$$ak_2 < 0 \quad (23)$$

that stems out from (20). Thus, dark 1-soliton solution to birefringent fibers with FWM is given by

$$q(x, t) = r(x, t) = -\frac{3k_1}{64k_2} \sqrt{1 + \tanh \tau} e^{i(-\kappa x + \omega t + \theta)} \quad (24)$$

with the respective parameters and constraints as defined.

3.2 Singular solitons

For singular solitons, the starting hypothesis is

$$P_l(x, t) = \frac{A_l}{(D + \sinh \tau)^{p_l}} \quad (25)$$

where once again, the definition of τ stays the same as given by (10). In this case too, the parameters A and B are free parameters. Substitution of (25) into (6) yields $\xi_{\text{opt}} = 0$

$$\begin{aligned} & \frac{(\omega + a\kappa^2 - ap_l B^2) A_l}{(D + \sinh \tau)^{p_l}} + \frac{aDB^2 p_l (2p_l + 1)}{(D + \sinh \tau)^{p_l+1}} - \\ & - \frac{aA_l B^2 p_l (2p_l + 1)(D^2 - 1)}{(D + \sinh \tau)^{p_l+2}} - \frac{3k_1 A_l^3}{(D + \sinh \tau)^{3p_l}} \\ & - \frac{3k_1 A_l A_l^2}{(D + \sinh \tau)^{p_l+2p_l}} - \frac{k_2 A_l^5}{(D + \sinh \tau)^{5p_l}} - \\ & - \frac{5k_2 A_l A_l^4}{(D + \sinh \tau)^{p_l+4p_l}} - \frac{9k_2 A_l^3 A_l^2}{(D + \sinh \tau)^{3p_l+2p_l}} \\ & - \frac{k_2 A_l^2 A_l^3}{(D + \sinh \tau)^{2p_l+3p_l}} = 0 \end{aligned} \quad (26)$$

Balancing principle gives (12). Again, from the real part equation (26) setting the coefficients of linearly independent functions to zero leads to the wave number given by (13) and

$$A_1 = A_2 = \frac{B}{2} = \left(\frac{3a^2}{3k_1^2 + 4aB^2 k_2} \right)^{\frac{1}{4}} \quad (27)$$

as well as

$$D = k_1 \sqrt{-\frac{3}{3k_1^2 + 4aB^2 k_2}} \quad (28)$$

Therefore, the relations (27) and (28) imply

$$3k_1^2 + 4aB^2 k_2 < 0 \quad (29)$$

Hence singular 1-soliton solutions to (1) and (2) are

$$q(x, t) = r(x, t) = \frac{A}{\sqrt{D + \sinh \tau}} e^{i(-\kappa x + \omega t + \theta)} \quad (30)$$

where the parameters are defined and the constraints are all in place.

4. Conclusions

This paper obtained 1-soliton solution to birefringent fibers with parabolic law nonlinearity, in presence of 4WM terms. Bright, dark and singular soliton solutions were retrieved in this case. The constraint conditions were also

listed. These integrability criteria must be fulfilled with the soliton parameters for these solitons to exist. The results of this paper give a lot of hope for future. Later, in addition to 4WM terms, there are several perturbation terms that will be taken into account, such as self-steepening, nonlinear dispersion, inter-modal dispersion, higher order dispersion and several others [1]. These will lead to soliton solutions where the parameters will be furthermore involved. The results of those findings will be published later.

Acknowledgement

This work was funded by the Deanship of Scientific Research (DSR), King Abdulaziz University under grant number (64-130-35-HiCi). The authors therefore acknowledge technical and financial support of KAU.

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