

Optical solitons in DWDM system with four-wave mixing

M. SAVESCU^a, A. A. ALSHAERY^b, E. M. HILAL^b, A. H. BHRAWY^{c,d}, QIN ZHOU^{e,f}, A. BISWAS^{g,c,*}

^aDepartment of Mathematics, Kutztown University of Pennsylvania, 15200 Kutztown Road, Kutztown, PA-19530, USA

^bDepartment of Mathematics, Faculty of Science for Girls, King Abdulaziz University, Jeddah, Saudi Arabia

^cDepartment of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah-21589, Saudi Arabia

^dDepartment of Mathematics, Faculty of Science, Beni-Suef University, Beni-Suef, Egypt

^eSchool of Electronics and Information Engineering, Wuhan Donghu University, Wuhan-430212, P. R. China

^fSchool of Physics and Technology, Wuhan University, Wuhan-430072, P.R. China

^gDepartment of Mathematical Sciences, Delaware State University, Dover, DE 19901-2277, USA

Soliton solutions in DWDM system is recovered in presence of four-wave mixing terms. The two laws of nonlinearity considered are Kerr law and parabolic law. Exact bright, dark and singular soliton solutions are retrieved by the aid of ansatz method. This integrability is achieved only with phase-matching condition for these components.

(Received October 5, 2014; accepted January 21, 2015)

Keywords: Solitons, DWDM, Four-wave mixing, Integrability

1. Introduction

Optical solitons provide cutting edge technology in fiber communications [1-20]. DWDM (Dense Wavelength Division Multiplexing) technology is engineered in the field of nonlinear fiber optics to achieve ultimate performance enhancement. It is imperative to address this technology further to provide bleeding edge results that is not only breathtaking but also provides an engineering marvel. This paper just exactly does that. Exact 1-soliton solution is obtained for DWDM systems that are studied in presence of four-wave mixing (4WM) terms with Kerr and parabolic laws of nonlinearity. Bright, dark and singular soliton solutions are retrieved with phase-matching conditions. There are a few constraint conditions that are listed. These conditions, also known as integrability criteria, must hold for the soliton solutions to exist.

The governing model is the nonlinear Schrödinger's equation (NLSE). In this paper, NLSE is considered with spatio-temporal dispersion (STD) in addition to group velocity dispersion (GVD). This additional form of dispersion serves as a well-posed model as proved during 2012 [6, 11]. The presence of STD can also reduce the effect of Internet bottleneck that is detrimental in fiber-optic communication system across trans-continental and trans-oceanic distances. The integration tool that is adopted in this paper is the ansatz method.

2. The model

The governing model is NLSE with GVD and STD. The two laws of nonlinearity that will be studied are Kerr law and parabolic law. The study will now be split

into the next couple of subsections where each of these laws will be considered in detail.

2.1 Kerr law

For Kerr law nonlinearity, DWDM model reads [12]

$$\begin{aligned}
 & i q_t^{(l)} + a_l q_{xx}^{(l)} + b_l q_{xt}^{(l)} \\
 & + \left\{ c_l |q^{(l)}|^2 + \sum_{n \neq l}^N \alpha_{l,n} |q^{(n)}|^2 \right\} q^{(l)} \\
 & + \sum_{n \neq l}^N \beta_{l,n} q^{(l)*} |q^{(n)}|^2 = 0
 \end{aligned} \quad (1)$$

Here, $1 \leq l \leq N$. The first term in (1) on left hand side is the linear evolution term, while a_l represents the coefficient of GVD; b_l represents the STD. Then, c_l is the coefficient of self-phase modulation (SPM) while $\alpha_{l,n}$ are the coefficients of cross-phase modulation (XPM), while $\beta_{l,n}$ accounts for 4WM. The independent variables are x and t that represents the spatial and temporal variables respectively. The dependent variable is $q^{(l)}(x, t)$ that represents soliton profile in every single channel for $1 \leq l \leq N$.

2.2 Parabolic law

This law is alternatively known as the cubic-quintic nonlinearity and arises in the nonlinear interaction between Langmuir waves and electrons. It describes the nonlinear interaction between the high frequency Langmuir waves and the ion acoustic waves by pondermotive forces [8]. For parabolic law nonlinearity, DWDM, with 4WM, is modeled as [12]:

$$\begin{aligned}
& i q_l^{(l)} + a_l q_{xx}^{(l)} + b_l q_{xt}^{(l)} \\
& + \left[c_l |q^{(l)}|^2 + \sum_{n \neq l}^N \alpha_{ln} |q^{(n)}|^2 \right] q^{(l)} \\
& + \left[d_l |q^{(l)}|^4 + \sum_{n \neq l}^N \left\{ |q^{(n)}|^2 \left(\beta_{ln} |q^{(n)}|^2 + \gamma_{ln} |q^{(l)}|^2 \right) \right\} \right] q^{(l)} \\
& + \delta_{ln} \sum_{n \neq l}^N q^{(l)*} \left(|q^{(n)}|^2 \right) \\
& + \lambda_{ln} \sum_{n \neq l}^N |q^{(l)}|^2 q^{(l)*} \left(q^{(n)} \right)^2 \\
& + \nu_{ln} \sum_{n \neq l}^N q^{(l)*} |q^{(n)}|^2 \left(q^{(n)} \right)^2 \\
& + \sigma_{ln} \sum_{n \neq l}^N \left(q^{(l)*} \right)^2 \left(q^{(n)} \right)^3 = 0
\end{aligned} \quad (2)$$

for $1 \leq l \leq N$. In (2), SPM terms are the coefficients of c_l and d_l , while XPM coefficients are α_{ln} , β_{ln} and γ_{ln} . Also, the terms with δ_{ln} , λ_{ln} , ν_{ln} , σ_{ln} are accounted for 4WM in parabolic law medium.

3. Ansatz method

This section will obtain exact bright dark and singular 1-soliton solutions to (1) and (2) by the ansatz approach. The special circumstance for which models (1) and (2) will be rendered integrable is the phase-matching condition. Therefore, the phase for solitons in all channels must remain the same. The starting hypothesis is

$$q^{(l)}(x, t) = P_l(x, t) e^{i(-\kappa x + \omega t + \theta)} \quad (3)$$

From the phase component, κ represents the frequency that stays the same in all channels, while ω is the same wave number, while θ is the phase constant

that also stays the same in all channels. The amplitude part is $P_l(x, t)$. The study will now be split into two sections.

3.1 Kerr law

Substituting the hypothesis (3) into (1) and splitting into real and imaginary parts leads to [12]

$$\begin{aligned}
& a_l \frac{\partial^2 P_l}{\partial x^2} + b_l \frac{\partial^2 P_l}{\partial x \partial t} + P_l (b_l \kappa \omega - \omega - a_l \kappa^2) \\
& + c_l P_l^3 + P_l \sum_{n \neq l}^N (\alpha_{ln} + \beta_{ln}) P_n^2 = 0
\end{aligned} \quad (4)$$

and

$$(1 - b_l \kappa) \frac{\partial P_l}{\partial t} + (b_l \omega - 2a_l \kappa) \frac{\partial P_l}{\partial x} = 0 \quad (5)$$

The imaginary part equation leads to the speed of the soliton that is given by

$$v = \frac{b_l \omega - 2a_l \kappa}{1 - b_l \kappa} \quad (6)$$

The speed of the soliton given by (6) remains valid for

$$b_l \kappa \neq 1 \quad (7)$$

This speed of the soliton stays the same irrespective of the type of nonlinearity that is being studied. The analysis of the real part equation will be conducted in the following three subsections that discuss bright, dark and singular solitons.

3.1.1 Bright solitons

For bright solitons, the starting hypothesis is given by [12-15]

$$P_l(x, t) = A_l \operatorname{sech}^{p_l} \tau \quad (8)$$

where

$$\tau = B(x - vt) \quad (9)$$

and A_l being the amplitude of each soliton with B being their width. The value of the unknown exponent p_l will naturally emerge during the course of derivation of the soliton solution. Substituting (8) into (4) leads to

$$\begin{aligned} & \left\{ (a_l - b_l v) p_l^2 B^2 + b_l \kappa \omega - \omega - a_l \kappa^2 \right\} \\ & - p_l (p_l + 1) B^2 (a_l - v b_l) \operatorname{sech}^{p_l+1} \tau \\ & + c_l A_l^2 \operatorname{sech}^{2p_l} \tau \\ & + \sum_{n \neq l}^N (\alpha_{l_n} + \beta_{l_n}) A_n^2 \operatorname{sech}^{2p_n} \tau = 0 \end{aligned} \quad (10)$$

By balancing principle, one recovers

$$p_l = p_n = 1 \quad (11)$$

for $1 \leq l \leq N$. Next, setting the coefficients of the linearly independent functions to zero yields

$$v = \frac{2a_l B^2 - c_l A_l^2 - \sum_{n \neq l}^N (\alpha_{l_n} + \beta_{l_n}) A_n^2}{2b_l B^2} \quad (12)$$

$$\omega = \frac{2a_l \kappa^2 - c_l A_l^2 - \sum_{n \neq l}^N (\alpha_{l_n} + \beta_{l_n}) A_n^2}{2(b_l \kappa - 1)} \quad (13)$$

Now, equating the speed of the solitons from real and imaginary parts given by (6) and (12) leads to the relation between the width and amplitude of the solitons

$$B = (b_l \kappa - 1) \times \sqrt{\frac{c_l A_l^2 + \sum_{n \neq l}^N \alpha_{l_n} A_n^2}{b_l^2 \left\{ 2a_l \kappa^2 - c_l A_l^2 - \sum_{n \neq l}^N (\alpha_{l_n} + \beta_{l_n}) A_n^2 \right\} - 2a_l (b_l^2 \kappa^2 - 1)}} \quad (14)$$

which imposes the constraint condition

$$\left(c_l A_l^2 + \sum_{n \neq l}^N \alpha_{l_n} A_n^2 \right) \times \left[\frac{b_l^2 \left\{ 2a_l \kappa^2 - c_l A_l^2 - \sum_{n \neq l}^N (\alpha_{l_n} + \beta_{l_n}) A_n^2 \right\}}{-2a_l (b_l^2 \kappa^2 - 1)} \right] > 0 \quad (15)$$

Thus, bright 1-soliton solution in DWDM system is given by

$$q^{(l)}(x, t) = A_l \operatorname{sech}^{p_l} [B(x - vt)] \times e^{i(-\kappa x + \omega t + \theta)} \quad (16)$$

where all parameter relations are discussed above.

3.1.2 Dark solitons

For dark soliton solutions, the starting point is the hypothesis given by [12-15]

$$P_l(x, t) = A_l \tanh^{p_l} \tau \quad (17)$$

where A_l and B are free parameters. Substituting this hypothesis into (4), the real part equation simplifies to

$$\begin{aligned} & p_l (p_l - 1) (a_l - b_l v) B^2 \tanh^{p_l-2} \tau \\ & - \left\{ \begin{array}{l} 2p_l (p_l - 1) (a_l - b_l v) B^2 \\ 2p_l B^2 (a_l - b_l v) \\ -b_l \kappa \omega - \omega - a_l \kappa^2 \end{array} \right\} \tanh^{p_l} \tau \\ & + p_l (p_l + 1) (a_l - b_l v) B^2 \tanh^{p_l+2} \tau \\ & + c_l A_l^2 \tanh^{3p_l} \tau \\ & + \sum_{n \neq l}^N (\alpha_{l_n} + \beta_{l_n}) A_n^2 A_l \tanh^{2p_n+p_l} \tau = 0 \end{aligned} \quad (18)$$

Balancing principle leads to (11). Similarly, the coefficients of the linearly independent functions give

$$v = \frac{2a_l B^2 + c_l A_l^2 + \sum_{n \neq l}^N (\alpha_{l_n} + \beta_{l_n}) A_n^2}{2b_l B^2} \quad (19)$$

$$\omega = \frac{2a_l \kappa^2 - c_l A_l^2 - \sum_{n \neq l}^N (\alpha_{l_n} + \beta_{l_n}) A_n^2}{2(b_l \kappa - 1)} \quad (20)$$

and finally the relation between the free parameters is:

$$B = (b_l \kappa - 1) \times \sqrt{\frac{c_l A_l^2 + \sum_{n \neq l}^N \alpha_{l_n} A_n^2}{2 \left[a_l (b_l \kappa^2 - 1) - b_l^2 \left\{ 2a_l \kappa^2 - c_l A_l^2 - \sum_{n \neq l}^N (\alpha_{l_n} + \beta_{l_n}) A_n^2 \right\} \right]}} \quad (21)$$

which imposes the constraint condition

$$\left(c_l A_l^2 + \sum_{n \neq l}^N \alpha_{l_n} A_n^2 \right) \times \left[\frac{b_l^2 \left\{ 2a_l \kappa^2 - c_l A_l^2 - \sum_{n \neq l}^N (\alpha_{l_n} + \beta_{l_n}) A_n^2 \right\}}{-a_l (b_l \kappa^2 - 1)} \right] < 0 \quad (22)$$

Thus, dark 1-soliton solution in DWDM system is given by

$$q^{(l)}(x, t) = A_l \tanh^{p_l} [B(x - vt)] \times e^{i(-\kappa x + \omega t + \theta)} \quad (23)$$

with the parameter dependences as discussed above.

3.1.3 Singular solitons

For singular solitons, one starts with the hypothesis [12-15]

$$P_l(x, t) = A_l \operatorname{csch}^{p_l} \tau \quad (24)$$

where A_l and B are all free parameters while p_l are unknown exponents whose value will be determined. Substituting (24) into (4) yields

$$\begin{aligned} & \left\{ (a_l - b_l v) p_l^2 B^2 + b_l \kappa \omega - \omega - a_l \kappa^2 \right\} \\ & + p_l (p_l + 1) B^2 (a_l - v b_l) \operatorname{csch}^{p_l+1} \tau \\ & + c_l A_l^2 \operatorname{csch}^{2p_l} \tau \\ & + \sum_{n \neq l}^N (\alpha_{ln} + \beta_{ln}) A_n^2 \operatorname{csch}^{2p_n} \tau = 0 \end{aligned} \quad (25)$$

From balancing principle, one recovers the same value of p_l for $1 \leq l \leq N$ as given by (11). Again from the coefficients of the linearly independent functions,

$$v = \frac{\left(2a_l B^2 + c_l A_l^2 + \sum_{n \neq l}^N (\alpha_{ln} + \beta_{ln}) A_n^2 \right)}{2b_l B^2} \quad (26)$$

$$\omega = \frac{2a_l \kappa^2 + c_l A_l^2 + \sum_{n \neq l}^N (\alpha_{ln} + \beta_{ln}) A_n^2}{2(b_l \kappa - 1)} \quad (27)$$

Next, equating the speed of the solitons from real and imaginary parts given by (6) and (26) leads to the relation between free parameters of the solitons

$$B = (b_l \kappa - 1) \times \sqrt{\frac{c_l A_l^2 + \sum_{n \neq l}^N \alpha_{ln} A_n^2}{2a_l (b_l^2 \kappa^2 - 1) - b_l^2 \left\{ 2a_l \kappa^2 - c_l A_l^2 - \sum_{n \neq l}^N (\alpha_{ln} + \beta_{ln}) A_n^2 \right\}}} \quad (28)$$

which imposes the constraint condition

$$\left(c_l A_l^2 + \sum_{n \neq l}^N \alpha_{ln} A_n^2 \right) \times \left[\frac{b_l^2 \left\{ 2a_l \kappa^2 - c_l A_l^2 - \sum_{n \neq l}^N (\alpha_{ln} + \beta_{ln}) A_n^2 \right\}}{-2a_l (b_l^2 \kappa^2 - 1)} \right] < 0 \quad (29)$$

Thus, singular 1-soliton solution in DWDM system is given by

$$q^{(l)}(x, t) = A_l \operatorname{csch}^{p_l} [B(x - vt)] \times e^{i(-\kappa x + \omega t + \theta)} \quad (30)$$

where the parameter relations are all given.

3.2 Parabolic law

In this case, substituting (3) into (2), leads to the same imaginary part as given by (5). Again, the speed will be the same as (6). The real part equation however from (2) is

$$\begin{aligned} & a_l \frac{\partial^2 P_l}{\partial x^2} + b_l \frac{\partial^2 P_l}{\partial x \partial t} + P_l (b_l \kappa \omega - \omega - a_l \kappa^2) \\ & + c_l P_l^3 + d_l P_l^5 + \sum_{n \neq l}^N (\alpha_{ln} + \delta_{ln}) P_n^2 P_l \\ & + (\beta_{ln} + v_{ln}) P_l P_n^4 + (\gamma_{ln} + \lambda_{ln}) P_l^3 P_n^2 \\ & + \sigma_{ln} P_l^2 P_n^3 = 0 \end{aligned} \quad (31)$$

It needs to be noted that, for parabolic law nonlinear medium, ansatz approach is only able to retrieve bright and singular solitons that are detailed in subsequent subsections. Thus discussions on dark solitons are not included.

3.2.1 Bright solitons

For bright soliton solution, with parabolic law, the starting hypothesis is given by [8, 12-17, 20]

$$P_l(x, t) = \frac{A_l}{(D + \cosh \tau)^{p_l}} \quad (32)$$

with unknown exponent p_l and τ is defined in (9). Substituting (32) into (31), simplifies to

$$\begin{aligned} & \frac{(a_l - b_l v) p_l^2 B^2 + b_l \kappa \omega - \omega - a_l \kappa^2}{(D + \cosh \tau)^{p_l}} \\ & - \frac{p_l (2p_l + 1) (a_l - b_l v) D B^2}{(D + \cosh \tau)^{p_l+1}} \\ & - \frac{p_l (p_l + 1) (a_l - b_l v) (D^2 - 1) B^2}{(D + \cosh \tau)^{p_l+2}} \\ & + \frac{c_l A_l^2}{(D + \cosh \tau)^{3p_l}} + \frac{d_l A_l^4}{(D + \cosh \tau)^{5p_l}} \\ & + \sum_{n \neq l}^N \left\{ \frac{(\alpha_{ln} + \delta_{ln}) A_n^2}{(D + \cosh \tau)^{2p_n + p_l}} \right. \\ & \left. + \frac{(\beta_{ln} + v_{ln}) A_n^4}{(D + \cosh \tau)^{4p_n + p_l}} \right. \\ & \left. + \frac{(\gamma_{ln} + \lambda_{ln}) A_l^2 A_n^2}{(D + \cosh \tau)^{2p_n + 3p_l}} \right. \\ & \left. + \frac{\sigma_{ln} A_l A_n^3}{(D + \cosh \tau)^{2p_l + 3p_n}} \right\} = 0 \end{aligned} \quad (33)$$

Now, balancing principle yields

$$p_l = p_n = \frac{1}{2} \quad (34)$$

for $1 \leq l \leq N$. Next, setting the coefficients of the linearly independent functions to zero yields

$$v = \frac{a_l B^2 D - c_l A_l^2 - \sum_{n \neq l}^N (\alpha_{l_n} + \beta_{l_n}) A_n^2}{b_l B^2 D} \quad (35)$$

$$v = \frac{\left[3a_l B^2 (D^2 - 1) + 4d_l A_l^4 + 4 \left\{ \sum_{n \neq l}^N (\beta_{l_n} + \nu_{l_n}) A_n^4 + \sum_{n \neq l}^N (\gamma_{l_n} + \lambda_{l_n}) A_l^2 A_n^2 + \sum_{n \neq l}^N \sigma_{l_n} A_l A_n^3 \right\} \right]}{3b_l B^2 (D^2 - 1)} \quad (36)$$

$$\omega = \frac{4a_l \kappa^2 D - c_l A_l^2 - \sum_{n \neq l}^N (\alpha_{l_n} + \beta_{l_n}) A_n^2}{4D(b_l \kappa - 1)} \quad (37)$$

Equating the speed v of the solitons from (35) and (36) gives

$$D = \frac{-2R + \sqrt{4R^2 + 9Q^2}}{3Q} \quad (38)$$

where

$$Q = c_l A_l^2 + \sum_{n \neq l}^N (\alpha_{l_n} + \delta_{l_n}) A_n^2 \quad (39)$$

$$R = d_l A_l^4 + (\beta_{l_n} + \nu_{l_n}) A_n^4 + (\gamma_{l_n} + \lambda_{l_n}) A_l^2 A_n^2 + \sigma_{l_n} A_l A_n^3 \quad (40)$$

Finally, equating the speed of the soliton between (6) and (36) gives the width of the solitons in the channels as

$$B = (b_l \kappa - 1) \times \sqrt{\frac{c_l A_l^2 + \sum_{n \neq l}^N (\alpha_{l_n} + \delta_{l_n}) A_n^2}{b_l^2 \left\{ c_l A_l^2 + \sum_{n \neq l}^N (\alpha_{l_n} + \delta_{l_n}) A_n^2 \right\} - 4a_l D}} \quad (41)$$

as long as

$$\left[c_l A_l^2 + \sum_{n \neq l}^N (\alpha_{l_n} + \delta_{l_n}) A_n^2 \right] \times \left[b_l^2 \left\{ c_l A_l^2 + \sum_{n \neq l}^N (\alpha_{l_n} + \delta_{l_n}) A_n^2 \right\} - 4a_l D \right] > 0 \quad (42)$$

Finally, the bright soliton solution in DWDM system with parabolic law nonlinearity is given by

$$q^{(l)}(x, t) = \frac{A_l}{\sqrt{D + \cosh[B(x - vt)]}} e^{i(-\kappa x + \omega t + \theta)} \quad (43)$$

where the parameters are defined in this subsection along with necessary restrictions that are needed for these solitons to exist.

3.2.2 Singular solitons

For singular solitons, the starting hypothesis is given by [12-17]

$$P_l(x, t) = \frac{A_l}{(D + \sinh \tau)^{p_l}} \quad (44)$$

with unknown exponent p_l and τ is defined in (9). Substituting (44) into (31), simplifies to

$$\begin{aligned} & \frac{(a_l - b_l v) p_l^2 B^2 + b_l \kappa \omega - \omega - a_l \kappa^2}{(D + \sinh \tau)^{p_l}} \\ & - \frac{p_l (2p_l + 1) (a_l - b_l v) D B^2}{(D + \sinh \tau)^{p_l + 1}} \\ & + \frac{p_l (p_l + 1) (a_l - b_l v) (D^2 + 1) B^2}{(D + \sinh \tau)^{p_l + 2}} \\ & + \frac{c_l A_l^2}{(D + \sinh \tau)^{3p_l}} + \frac{d_l A_l^4}{(D + \sinh \tau)^{5p_l}} \\ & = - \sum_{n \neq l}^N \left\{ \frac{(\alpha_{l_n} + \delta_{l_n}) A_n^2}{(D + \sinh \tau)^{2p_n + p_l}} + \frac{(\beta_{l_n} + \nu_{l_n}) A_n^4}{(D + \sinh \tau)^{4p_n + p_l}} \right. \\ & \left. + \frac{(\gamma_{l_n} + \lambda_{l_n}) A_l^2 A_n^2}{(D + \sinh \tau)^{2p_n + 3p_l}} + \frac{\sigma_{l_n} A_l A_n^3}{(D + \sinh \tau)^{2p_l + 3p_n}} \right\} \quad (45) \end{aligned}$$

Now, balancing principle yields (34). Similarly, as in the case of bright solitons, coefficients of linearly independent functions yield (35) and

$$v = \frac{\left(3a_l B^2 (D^2 + 1) + 4d_l A_l^4 + 4 \left\{ \sum_{n \neq l}^N (\beta_{l_n} + \nu_{l_n}) A_n^4 + \sum_{n \neq l}^N (\gamma_{l_n} + \lambda_{l_n}) A_l^2 A_n^2 + \sum_{n \neq l}^N \sigma_{l_n} A_l A_n^3 \right\} \right)}{3b_l B^2 (D^2 + 1)} \quad (46)$$

Equating the speed v of the solitons from (35) and (46) leads to

$$D = \frac{-2R + \sqrt{4R^2 - 9Q^2}}{3Q} \quad (47)$$

where Q and R are defined in (39) and (40). This relation for D introduces the restriction

$$\begin{aligned} & \left| 2 \left[d_l A_l^4 + (\beta_{l_n} + \nu_{l_n}) A_n^4 + (\gamma_{l_n} + \lambda_{l_n}) A_l^2 A_n^2 + \sigma_{l_n} A_l A_n^3 \right] \right| \\ & > 3 \left| c_l A_l^2 + \sum_{n \neq l}^N (\alpha_{l_n} + \delta_{l_n}) A_n^2 \right| \end{aligned} \quad (48)$$

Finally, equating the speed of the soliton between (6) and (46) gives the free parameter B as

$$B = 2(b_l \kappa - 1) \times \sqrt{\frac{c_l A_l^2 + \sum_{n \neq l}^N (\alpha_{l_n} + \delta_{l_n}) A_n^2}{b_l^2 \left\{ c_l A_l^2 + \sum_{n \neq l}^N (\alpha_{l_n} + \delta_{l_n}) A_n^2 \right\} - 4a_l D}} \quad (49)$$

with the same constraint as given by (42). Finally, singular 1-soliton solution in DWDM system with parabolic law nonlinearity is given by

$$q^{(l)}(x, t) = \frac{A_l}{\sqrt{D + \sinh[B(x - vt)]}} \times e^{i(-\kappa x + \alpha t + \theta)} \quad (50)$$

with the definition of the parameters in place.

4. Conclusions

This paper obtained 1-soliton solution to DWDM system that is considered with Kerr law and parabolic law nonlinearity. There are several constraint conditions that are listed. These constraints restrict the choice of free parameters that are available. For Kerr law medium, it is the bright, dark and singular 1-soliton solution that is retrieved; on the other hand for parabolic law medium, it is only bright and singular soliton solutions that are retrievable.

The results of this paper carry a lot of future prospects. Later additional integration tools will be

employed to recover these soliton solutions. Some of these integration architectures are Lie symmetry analysis, G²/G-expansion scheme, Kudryashov's method and several others. This is just a tip of the iceberg. That plethora of results will be reported later.

Acknowledgement

This work was funded by the Deanship of Scientific Research (DSR), King Abdulaziz University under grant number (64-130-35-HiCi). The authors therefore thankfully acknowledge this technical and financial support of KAU.

References

- [1] A. A. Alshaery, A. H. Bhrawy, E. M. Hilal, A. Biswas, *Journal of Electromagnetic Waves and Applications* **28**, 275 (2014).
- [2] A. H. Bhrawy, A. A. Alshaery, E. M. Hilal, Z. Jovanoski, A. Biswas, *Optik* **125**, 6162 (2014).
- [3] A. Biswas, *Journal of Nonlinear Optical Physics and Materials* **13**, 81 (2004).
- [4] A. Biswas, S. Konar, *Introduction to non-Kerr law optical solitons*, CRC Press Boca Raton, FL. (2006).
- [5] A. Biswas, K. R. Khan, A. Rahaman, A. Yildirim, T. Hayat, O. M. Aldossary, *J. Optoelectron. Adv. Mater.* **14**, 671 (2012).
- [6] X. Geng, Y. Lv, *Nonlinear Dynamics* **69**, 1621 (2012).
- [7] P. Green, D. Milovic, A. K. Sarma, D. A. Lott, A. Biswas, *Journal of Nonlinear Optical Physics and Materials* **19**, 339 (2010).
- [8] Z. Jovanoski, D. R. Rowland, *Journal of Modern Optics* **48**, 1179 (2001).
- [9] K. R. Khan, T. X. Wu, *IEEE Selected Topics in Quantum Electronics* **14**, 752 (2008).
- [10] R. Kohl, R. Tinaztepe, A. Chowdhury, *Optik* **125**, 1926 (2014).
- [11] S. Kumar, K. Singh, R. K. Gupta, *Pramana* **79**, 41 (2012).
- [12] M. Mirzazadeh, M. Eslami, M. Savescu, A. H. Bhrawy, A. A. Alshaery, E. M. Hilal, A. Biswas, Submitted.
- [13] M. Savescu, A. A. Alshaery, A. H. Bhrawy, E. M. Hilal, L. Moraru, A. Biswas, *Wulfenia* **21**, 366 (2014).
- [14] M. Savescu, A. H. Bhrawy, E. M. Hilal, A. A. Alshaery, A. Biswas, *Romanian Journal of Physics* **59**, 582 (2014).
- [15] M. Savescu, E. M. Hilal, A. A. Alshaery, A. H. Bhrawy, L. Moraru, A. Biswas, *J. Optoelectron. Adv. Mater.* **16**, 619 (2014).
- [16] Q. Zhou, D. Yao, X. Liu, F. Chen, S. Ding, Y. Zhang, F. Chen, *Optics and Laser Technology* **51**, 32 (2013).
- [17] Q. Zhou, D. Yao, F. Chen, *Journal of Modern Optics* **60**, 1652 (2013).
- [18] Q. Zhou, D. Yao, F. Chen, W. Li, *Journal of Modern Optics* **60**, 854 (2013).
- [19] Q. Zhou, *Journal of Modern Optics* **61**, 500-503. (2014).
- [20] Q. Zhou, *Optik* **125**, 3142 (2014).

*Corresponding author: biswas.anjan@gmail.com