Optical solitons in photonic crystal fibers with spatially inhomogeneous nonlinearities

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This paper studies the optical 1-soliton in the hollow-core photonic crystal fibers in the presence of space-dependent inter-modal dispersion, detuning and fiber loss. Two integration tools that are the Hirota's bilinear method and ansatz method are used. We report the explicit optical bright 1-solitons.

(Received July 17, 2014; accepted November 13, 2014)

Keywords: 1-soliton, Hollow-core photonic crystal fibers, Hirota's bilinear method, Ansatz method

1. Introduction

The concept of photonic crystal fibers (PCFs) was first introduced based on the theory of photonic crystal bandgap by Russell in 1992 [1] and then attracted much more attention [3-5]. PCFs have many unusual optical properties, including endless single mode characteristic, controllable dispersion properties and high birefringence, which make them be widely applied in the area of telecommunications and electromagnetics. According to the different structure of the fiber core, PCFs are classified as solid core photonic crystal fiber (SC-PCFs) and hollow-core photonic crystal fibers (HC-PCFs).

HC-PCFs possess excellent fundamental mode transmission characteristics and very low loss, which can greatly enhance the nonlinear interactions between light pulses and matter. By filling different gases or liquids in fiber core, many optical nonlinear effects, including the electromagnetically-induced transparency (EIT), saturated absorption and soliton self-frequency shift, had been studied in recent years [6-8].

The key idea of this paper is to study the nonlinear dynamics of HC-PCFs. In the presence of space-dependent inter-modal dispersion (IMD), detuning and fiber loss, the propagation of optical solitons through HC-PCFs is ruled by the following nonlinear Schrödinger equation:

$$iu_x + a(x)u_{tt} + b(x)|u|^2 u - i\lambda(x)u_t - \\ - \mu(x)u\int_{-\infty}^{t}|u(x,\tau)|^2 d\tau - k^2(x)u + i\gamma(x)u = 0 \tag{1}$$

where the dependent variable $u(x,t)$ is a complex wave function that represents the normalized electric-field envelope, while x and t are the independent variables that represent the distance along the HC-PCFs and time in a co-moving frame respectively. Here $a(x)$, $b(x)$, $\lambda(x)$, $k(x)$, and $\gamma(x)$ that are space modulated that represent the parameters of group velocity dispersion (GVD), Kerr law nonlinearity, IMD, detuning and fiber loss, and finally $\mu(x)$ is due to the Kerr nonlinearity of the gas.

Very recently, the optical solitons and breathers in homogeneous HC-PCFs had been studied [9-11]. However, to our knowledge, the optical solitons in inhomogeneous HC-PCFs have not reported in the existing papers. Hence the main work described in this paper is to construct exact solitons to Eq. (1), which will be investigated analytically by employing the Hirota's direct method and ansatz method. As a consequence, the explicit optical bright 1-soliton solutions to Eq. (1) are obtained.

2. Hirota's bilinear method

In this section, we will use the Hirota's bilinear method to get exact soliton solutions to Eq. (1).

2.1 Hirota's Bilinear forms to Eq. (1)

In order to solve Eq. (1), the starting point is the hypothesis [9-11]

$$u(x,t) = \frac{g(x,t)}{f(x,t)}, \tag{2}$$

where $f(x,t)$ is the real function while $g(x,t)$ is the complex function.

Introducing the Hirota's bilinear operators D_x and D_t, which are defined as [12]

$$D_x^m D_t^n (g \cdot f) = \\ = (\partial_x - \partial_\xi)^m (\partial_t - \partial_\tau)^n [g(x,t) f(\xi,\tau)]\Big|_{\xi=x,\tau=t} \tag{3}$$

From Eqs. (2) and (3), Eq. (1) can be rewritten as

$$i\frac{D_x(g \cdot f)}{f^2} + a(x)\frac{D_t^2(g \cdot f)}{f^2} - a(x)\frac{g}{f}\frac{D_t^2(f \cdot f)}{f^2} + \\ + b(x)\frac{g^2 g^*}{f^3} - i\lambda(x)\frac{D_t(g \cdot f)}{f^2} - \\ - \mu(x)\frac{g}{f}\int_{-\infty}^{t}\frac{gg^*}{f^2}d\tau - k^2(x)\frac{g}{f} + i\gamma(x)\frac{g}{f} = 0 \tag{4}$$

To begin with, balancing the parameter of f^{-3} to zero gives

$$D_t^2(f \cdot f) = \frac{b(x)}{a(x)} gg^* \tag{5}$$

Then substituting Eq. (5) into Eq. (4) yields

$$iD_x(g \cdot f) + a(x)D_t^2(g \cdot f) - i\lambda(x)D_t(g \cdot f) - \\ - \frac{2a(x)\mu(x) f_t g}{b(x)} - k^2(x) g \cdot f + i\gamma(x) g \cdot f = 0 \tag{6}$$

We use the integration $\int_{-\infty}^{t}[D_t^2(f \cdot f)/f^2]d\tau = 2 f_t/f$ [9]. Finally, the Hirota's bilinear forms to Eq. (1) are got that are given by Eqs. (5) and (6).

2.2 Analytical soliton solutions to Eq. (1)

The starting hypotheses are that $f(x,t)$ and $g(x,t)$ have the generalized power series expansions in the forms

$$f(x,t) = 1 + \sum_{n=1}^{\infty} \varepsilon^{2n} f_{2n}(x,t) = \\ = 1 + \varepsilon^2 f_2(x,t) + \varepsilon^4 f_4(x,t) + \cdots \tag{7}$$

$$g(x,t) = \sum_{n=1}^{\infty} \varepsilon^n g_n(x,t) = \\ = \varepsilon g_1(x,t) + \varepsilon^2 g_2(x,t) + +\varepsilon^3 g_3(x,t) + \cdots \tag{8}$$

where ε is the formal expansion coefficient.

Substituting Eqs (7) and (8) into the Hirota's bilinear forms (5) and (6), and then using the balancing principle gives a set of relations for $f_{2n}(x,t)$ and $g_n(x,t)$. Finally the analytical solutions to Eq. (1) can be got by solving those relations above.

In this work, we will focus on the 1-soliton. In this case, $g(x,t)$ takes the form [13]

$$g(x,t) = \varepsilon g_1(x,t) = \varepsilon \exp\{[a_{11}(x) + \\ + ia_{12}(x)]x + (b_{11} + ib_{12})t + k_{11} + ik_{12}\} \tag{9}$$

where a_{11} and a_{12} are the real functions while b_{11}, b_{12}, k_{11} and k_{12} are the real constants.

Substituting Eqs. (7) and (9) into the Hirota's bilinear form (5) yields

$$f_2(x,t) = \frac{b(x)}{4b_{11}^2 a(x)} \exp[2a_{11}(x)x + 2b_{11}t + 2k_{11}] \tag{10}$$

with $f_{2n}(x,t) = 0$ (n = 2, 3, 4,…).

Then, substituting Eqs. (7), (9) and (10) into the Hirota's bilinear form (6) yields

$$a_{11}(x) + a_{11}'(x)x - \lambda(x)b_{11} + 2a(x)b_{11}b_{12} + \gamma(x) = 0 \tag{11}$$

$$a_{12}(x) + a_{12}'(x)x - \lambda(x)b_{12} - a(x)(b_{11}^2 - b_{12}^2) + k^2(x) = 0 \tag{12}$$

$$\frac{a'(x)}{a(x)} - \frac{b'(x)}{b(x)} - 2[a_{11}(x) + a_{11}'(x)x] + \\ + 2\lambda(x)b_{11} - 4a(x)b_{11}b_{12} + 4ib_{11}\frac{a(x)\mu(x)}{b(x)} = 0 \tag{13}$$

Solving Eqs. (11)-(13), one obtains

$$a_{11}(x) = \frac{1}{x}\int[\lambda(x)b_{11} - 2a(x)b_{11}b_{12} - \gamma(x)]dx \tag{14}$$

$$a_{12}(x) = \frac{1}{x}\int[a(x)(b_{11}^2 - b_{12}^2) - \lambda(x)b_{12} - k^2(x)]dx \tag{15}$$

$$\mu(x) = \frac{1}{4ib_{11}a^2(x)}[a(x)b'(x) - 2\gamma(x)a(x)b(x) - \\ a'(x)b(x)] \tag{16}$$

Finally, without loss of generality, we take $\varepsilon = 1$, then the analytical bright soliton solution to Eq. (1) is given by

$$u(x,t) = \frac{g(x,t)}{f(x,t)} = \\ \frac{4b_{11}^2 a(x)\exp\{[a_{11}(x) + ia_{12}(x)]x + (b_{11} + ib_{12})t + k_{11} + ik_{12}\}}{4b_{11}^2 a(x) + b(x)\exp[2a_{11}(x)x + 2b_{11}t + 2k_{11}]} \tag{17}$$

where $a_{11}(x)$ and $a_{12}(x)$ are given by Eqs. (14) and (15), while the constraint condition for analytical solution to exist is given by Eq. (16).

3. Ansatz method

In this section, we will use the ansatz method [14-20] to get exact bright soliton solution to Eq. (1).

For bright soliton, the starting point is the hypothesis

$$u(x,t) = A\,\mathrm{sech}[a_{11}(x)x + b_{11}t + k_{11}]\exp\{i[a_{12}(x)x + b_{12} + k_{12}]\} \tag{18}$$

Substituting this hypothesis into Eq. (1) yields

$$
\begin{aligned}
& i\begin{Bmatrix} -[a'_{11}(x)x + a_{11}(x)]\operatorname{sech}[a_{11}(x)x + b_{11}t + k_{11}]\cdot \\ \cdot\tanh[a_{11}(x)x + b_{11}t + k_{11}] \\ + i[a'_{12}(x)x + a_{12}(x)]\operatorname{sech}[a_{11}(x)x + b_{11}t + k_{11}] \end{Bmatrix} + \\
& + a(x)\begin{Bmatrix} b_{11}^2\operatorname{sech}[a_{11}(x)x + b_{11}t + k_{11}] - \\ - 2b_{11}^2\operatorname{sech}^3[a_{11}(x)x + b_{11}t + k_{11}] - \\ - 2ib_{11}b_{12}\operatorname{sech}[a_{11}(x)x + b_{11}t + k_{11}]\cdot \\ \cdot\tanh[a_{11}(x)x + b_{11}t + k_{11}] - \\ - b_{12}^2\operatorname{sech}[a_{11}(x)x + b_{11}t + k_{11}] \end{Bmatrix} - \\
& - i\lambda(x)\begin{Bmatrix} -b_{11}\operatorname{sech}[a_{11}(x)x + b_{11}t + k_{11}]\cdot \\ \cdot\tanh[a_{11}(x)x + b_{11}t + k_{11}] + \\ + ib_{12}\operatorname{sech}[a_{11}(x)x + b_{11}t + k_{11}] \end{Bmatrix} - \\
& - \frac{A^2}{b_{11}}\mu(x)\operatorname{sech}[a_{11}(x)x + b_{11}t + k_{11}]\cdot \\
& \cdot\tanh[a_{11}(x)x + b_{11}t + k_{11}] + \\
& + i\gamma(x)\operatorname{sech}[a_{11}(x)x + b_{11}t + k_{11}] + \\
& + A^2 b(x)\operatorname{sech}^3[a_{11}(x)x + b_{11}t + k_{11}] - \\
& - k^2(x)\operatorname{sech}[a_{11}(x)x + b_{11}t + k_{11}] = 0 \qquad (19)
\end{aligned}
$$

Separating the real and imaginary parts, and then using the homogeneous balance principle, one obtains

$$a(x) = \frac{A^2}{2b_{11}^2} b(x) \qquad (20)$$

$$\gamma(x) = \rho(x)\tanh[a_{11}(x)x + b_{11}t + k_{11}] \qquad (21)$$

$$
\begin{aligned}
& a_{12}(x) + a'_{12}(x)x - \lambda(x)b_{12} - \\
& - a(x)(b_{11}^2 - b_{12}^2) + k^2(x) = 0
\end{aligned} \qquad (22)
$$

$$
\begin{aligned}
& a'_{11}(x)x + a_{11}(x) + 2b_{11}b_{12}a(x) - \\
& - b_{11}\lambda(x) - 2b_{11}\frac{a(x)\mu(x)}{b(x)} - \rho(x) = 0
\end{aligned} \qquad (23)
$$

where $\rho(x)$ is an auxiliary function.

Solving Eqs. (22) and (23), one obtains

$$a_{12}(x) = \frac{1}{x}\int[a(x)(b_{11}^2 - b_{12}^2) - \lambda(x)b_{12} - k^2(x)]dx \qquad (24)$$

$$a_{11}(x) = \frac{1}{x}\int[b_{11}\lambda(x) - 2b_{11}b_{12}a(x) + 2i\frac{b_{11}a(x)\mu(x)}{b(x)} + \rho(x)]dx \qquad (25)$$

Finally, the analytical bright soliton solution to Eq. (1) is obtained that is given by Eq. (18), $a_{11}(x)$ and $a_{12}(x)$ in which are given by Eqs. (24) and (25), while the constraint condition for analytical solution to exist are given by Eqs. (20) and (21).

4. Conclusion

This paper studied optical solitons, which are modeled by nonlinear Schrodinger's equation, in presence of inter-modal dispersion, detuning and attenuation which are all considered with spatially dependent coefficients. Hirota's bilinear method and ansatz approach revealed bright 1-soliton solution which comes with constraint conditions for its existence.

Acknowledgements

The work of the second author (Q.P. Zhu) was supported by the Scientific Research Fund of Hubei Provincial Education Department under Grant No. B2013193.

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