Optical solitons of generalized resonant dispersive nonlinear Schrödinger's equation with power law nonlinearity

MOHAMMAD MIRZAZADEH[a], QIN ZHOU[b], ANJAN BISWAS[c,d,*], MILIVOJ BELIC[e]
[a]*Department of Mathematics, Faculty of Mathematical Sciences,University of Guilan, Rasht, Iran*
[b]*School of Electronics and Information Engineering, Wuhan Donghu University, Wuhan-430212, P. R. China*
[c]*Department of Mathematical Sciences, Delaware State University, Dover, DE 19901-2277, USA,*
[d]*Faculty of Science, Department of Mathematics, King Abdulaziz University, Jeddah-21589, Saudi Arabia,*
[e]*Science Program, Texas A & M University at Qatar, PO Box 23874, Doha, Qatar*

This paper obtains soliton and other solutions to the generalized resonant dispersive nonlinear Schrödinger's equation with power law nonlinearity by the aid of trial solution method. Bright and singular soliton solutions are obtained. In addition, as a byproduct singular periodic solutions are also reported that naturall fall out of the integration scheme that is implemented in this paper.

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1. Introduction

Optical solitons is a very fascinating area of study in nonlinear optics and optoelectronics. There are several aspects of these solitons that are addressed in this context. These are numerical simulations, integrability aspects, conservation laws, adiabatic dynamics, multiple-scale analysis and several others [1-20]. However, one of the most challenging issues, as always, is to obtain soliton solutions to the governing model by analytical method. A closed form analytical solution is always welcome in this field. These solutions give a complete picture of the model and thus provides an overall exposure to the underlying scientific features to the governing model equation. There are several integration tools available to carry out this integration to reveal solitons and other solutions. This paper will implement trial solution technique to generalized resonant dispersive nonlinear Schrödinger's equation (GRD-NLSE), with power law nonlinearity to extract optical soliton solution. In the past, Lie symmetry analysis, ansatz approach and other algorithms were implemented [1, 3, 11, 12, 14, 15]. However, with power law nonlinear medium, this paper provides a generalized flavor to the results that are reported earlier.

2. Mathematical model

The dimensionless form of GRD-NLSE that will be analyzed in this paper is given by

$$i(|\psi|^{n-1}\psi)_t + \alpha(t)(|\psi|^{n-1}\psi)_{xx} + \beta(t)|\psi|^m\psi + \gamma(t)\left\{\frac{(|\psi|^n)_{xx}}{|\psi|}\right\}\psi = 0$$

For this model, $\psi(x,t)$ is the wave profile and is a complex valued function. $\alpha(t)$ and $\beta(t)$ respectively represent the coefficients of the generalized group velocity dispersion (GVD) and power law nonlinearity. Then $\gamma(t)$ represent the coefficient of the resonant term that appears in the study of Madelung fluids. All of these coefficients are taken to be time-dependent. The parameter m dictates power law nonlinearity. When $m = 2$, this model equation collapses to Kerr law that is occasionally referred to cubic NLSE. Finally, the parameter n governs generalized GVD. For $n = 1$, this model equation condenses to regular NLSE. This parameter n thus maintains GVD on a generalized setting. During long distance soliton propagation, the evolution and the GVD gets distorted and thus modified. Therefore, it is necessary to consider NLSE where the evolution and GVD are modified to maintain the dynamics of soliton propagation from a realistic perspective.

3. Trial equation method

In this section we outline the main steps of the trial equation method [9, 12, 13] as follows:

Step-1: Suppose a nonlinear evolution equation with time-dependent coefficients is given in the form

$$P(u,u_t,u_x,u_{tt},u_{xt},u_{xx},...)=0 \tag{1}$$

can be converted to an ordinary differential equation (ODE)

$$Q(U,U',U'',U''',...)=0 \tag{2}$$

using a traveling wave variable $u(x,t)=U(\xi)$, $\xi=x-v(t)t$, where $U=U(\xi)$ is an unknown function, Q is a polynomial in the variable U and its derivatives. If all terms contain derivatives, then Eq. (2) is integrated where integration constants are considered zeros.

Step-2: Choose a trial equation

$$(U')^2=F(U)=\sum_{l=0}^{s}a_lU^l \tag{3}$$

where $a_l,(l=0,1,...,s)$ are constants to be determined. Substituting Eq. (3) and other derivative terms such as U'' or U''' and so on into Eq. (2) yields a polynomial $G(U)$ of U. According to the balance principle we can determine the value of s. Setting the coefficients of $G(U)$ to zero, we get a system of algebraic equations. Solving this system, we shall determine $v(t)$ and values of $a_0,a_1,...,a_s$.

Step-3: Rewrite Eq. (3) by the integral form

$$\pm(\xi-\xi_0)=\int\frac{1}{\sqrt{F(U)}}dU \tag{4}$$

Based on common discriminant system of a polynomial, we classify the roots of $F(U)$, and integrate (4). This leads to exact solutions to the model.

3.1 Application to GRD-NLSE

In this subsection, trial equation method will obtain exact solutions GRD-NLSE with time-dependent coefficients and power law nonlinearity [1, 3, 12]

$$\begin{aligned} &i(|\psi|^{n-1}\psi)_t+\alpha(t)(|\psi|^{n-1}\psi)_{xx}+\beta(t)|\psi|^m\psi \\ &+\gamma(t)\left\{\frac{(|\psi|^n)_{xx}}{|\psi|}\right\}\psi=0 \end{aligned} \tag{5}$$

Under the traveling wave transformation

$$\psi(x,t)=U(\xi)e^{i(-\kappa x+\omega(t)t)} \tag{6-1}$$

$$\xi=x+2\kappa\int_0^t\alpha(t')dt' \tag{6-2}$$

we have

$$\begin{aligned} &(\alpha(t)+\gamma(t))(U^n)'' \\ &-\left(t\frac{d\omega(t)}{dt}+\omega(t)+\kappa^2\alpha(t)\right)U^n \\ &-\beta(t)U^{n+1}=0 \end{aligned} \tag{7}$$

In order to obtain closed form solutions, the transformation

$$U(\xi)=V^{\frac{1}{m+1-n}} \tag{8}$$

is applied that will reduce Eq. (5) into the ODE

$$\begin{aligned} &(\alpha(t)+\gamma(t))n(2n-m-1)(V')^2 \\ &+(\alpha(t)+\gamma(t))n(m+1-n)VV'' \\ &-\left(t\frac{d\omega(t)}{dt}+\omega(t)+\kappa^2\alpha(t)\right)(m+1-n)^2V^2 \\ &+\beta(t)(m+1-n)^2V^3=0 \end{aligned} \tag{9}$$

Balancing VV'' with V^3 in Eq. (9), then we get $s=3$. Using the solution procedure of the trial equation method, we obtain the system of algebraic equations as follows:

V^3 Coeff.:

$$\begin{aligned} &\frac{3}{2}n(m+1-n)(\alpha(t)+\gamma(t))a_3 \\ &+(\alpha(t)+\gamma(t))n(2n-m-1)a_3 \\ &+(m+1-n)^2\beta(t)=0 \end{aligned} \tag{10-1}$$

V^2 Coeff.:

$$\begin{aligned} &(\alpha(t)+\gamma(t))n(2n-m-1)a_2 \\ &-(m+1-n)^2\left(t\frac{d\omega(t)}{dt}+\omega(t)+\kappa^2\alpha(t)\right) \\ &+n(m+1-n)(\alpha(t)+\gamma(t))a_2=0 \end{aligned} \tag{10-2}$$

V^1 Coeff.:

$$\begin{aligned} &(\alpha(t)+\gamma(t))n(2n-m-1)a_1 \\ &+\frac{n}{2}(m+1-n)(\alpha(t)+\gamma(t))a_1=0 \end{aligned} \tag{10-3}$$

V^0 Coeff.:

$$\left(\alpha(t)+\gamma(t)\right)n(2n-m-1)a_0=0 \qquad (10\text{-}4)$$

Solving the above system of algebraic equations, we obtain the following:

$$a_0=0,\ a_1=0,$$

$$a_3=-\frac{2\beta(t)(m+1-n)^2}{n(m+1+n)\left(\alpha(t)+\gamma(t)\right)}$$

$$\omega(t)=-\frac{1}{t}\int_0^t\left\{\begin{array}{l}\kappa^2\alpha(t') \\ -\dfrac{n^2a_2\left(\alpha(t')+\gamma(t')\right)}{(m+1-n)^2}\end{array}\right\}dt'$$

where κ and a_2 are arbitrary constants.

Substituting these results into Eqs. (3) and (4), gives

$$\pm(\xi-\xi_0)=$$
$$\int\frac{1}{\sqrt{a_2V^2-\dfrac{2\beta(t)(m+1-n)^2}{n(m+1+n)}\begin{pmatrix}\alpha(t)\\+\gamma(t)\end{pmatrix}V^3}}dV \qquad (11)$$

Integrating Eq. (11), exact 1-soliton solutions of Eq. (5) are obtained as

$$\psi_1(x,t)=\left[\begin{array}{l}\dfrac{a_2n(m+1+n)\left(\alpha(t)+\gamma(t)\right)}{2\beta(t)(m+1-n)^2} \\ \times\operatorname{sech}\left(\begin{array}{l}\dfrac{\sqrt{a_2}}{2} \\ \times\left(\begin{array}{l}x \\ +2\kappa\int_0^t\alpha(t)dt-\xi_0\end{array}\right)\end{array}\right)\end{array}\right]^{\frac{1}{m+1-n}} \qquad (12)$$
$$\times e^{i\left(-\kappa x-\int_0^t\left\{\kappa^2\alpha(t')-\frac{n^2a_2\left(\alpha(t')+\gamma(t')\right)}{(m+1-n)^2}\right\}dt'\right)}$$

and

$$\psi_2(x,t)=\left[\begin{array}{l}-\dfrac{a_2n(m+1+n)\left(\alpha(t)+\gamma(t)\right)}{2\beta(t)(m+1-n)^2} \\ \times\operatorname{csch}^2\left(\dfrac{\sqrt{a_2}}{2}\left(\begin{array}{l}x \\ +2\kappa\int_0^t\alpha(t)dt-\xi_0\end{array}\right)\right)\end{array}\right]^{\frac{1}{m+1-n}} \qquad (13)$$
$$\times e^{i\left(-\kappa x-\int_0^t\left\{\kappa^2\alpha(t')-\frac{n^2a_2\left(\alpha(t')+\gamma(t')\right)}{(m+1-n)^2}\right\}dt'\right)}$$

which are bright and singular solitons respectively and these exist for $a_2>0$.

However, for $a_2<0$, singular periodic solutions are given by

$$\psi_3(x,t)=\left[\begin{array}{l}\dfrac{a_2n(m+1+n)\left(\alpha(t)+\gamma(t)\right)}{2\beta(t)(m+1-n)^2} \\ \times\sec^2\left(\dfrac{\sqrt{-a_2}}{2}\left(\begin{array}{l}x \\ +2\kappa\int_0^t\alpha(t)dt-\xi_0\end{array}\right)\right)\end{array}\right]^{\frac{1}{m+1-n}} \qquad (14)$$
$$\times e^{i\left(-\kappa x-\int_0^t\left\{\kappa^2\alpha(t')-\frac{n^2a_2\left(\alpha(t')+\gamma(t')\right)}{(m+1-n)^2}\right\}dt'\right)}$$

and

$$\psi_4(x,t)=\left[\begin{array}{l}\dfrac{a_2n(m+1+n)\left(\alpha(t)+\gamma(t)\right)}{2\beta(t)(m+1-n)^2} \\ \times\csc^2\left(\dfrac{\sqrt{-a_2}}{2}\left(\begin{array}{l}x \\ +2\kappa\int_0^t\alpha(t)dt-\xi_0\end{array}\right)\right)\end{array}\right]^{\frac{1}{m+1-n}} \qquad (15)$$
$$\times e^{i\left(-\kappa x-\int_0^t\left\{\kappa^2\alpha(t')-\frac{n^2a_2\left(\alpha(t')+\gamma(t')\right)}{(m+1-n)^2}\right\}dt'\right)}$$

These singular periodic solutions are not studied in optics. However, they are listed here for a complete spectrum.

4. Conclusion

This paper successfully demonstrated the application of trial solution method to secure soliton solutions to GRD-NLSE with power law nonlinearity. Two forms of optical soliton solutions are retrieved. They are bright and singular. Additionally, as a byproduct, singular periodic solutions naturally emerged from this scheme. These soliton solutions will be of great asset in the field of optoelectronics. Later, further generalized form of GRD-NLSE will be studied. The results of those projects will soon be reported. Furthermore, Lie symmetry approach will also reveal conservation laws. These results are all under way.

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*Corresponding author: biswas.anjan@gmail.com