Optical solitons under competing weakly nonlocal nonlinearity and cubic-quintic nonlinearities

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We present an analytic study on solitons in a nonlocal nonlinear medium. There are three types of competing nonlinearities that are taken into account in our model. They are weakly nonlocal nonlinearity, cubic nonlinearity and quintic nonlinearity. By means of the Lie symmetry analysis, we report the bright and dark solitons and their respective existence conditions.

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1. Introduction

Nonlocality of nonlinearity, including weak nonlocality, general nonlocality and strong nonlocality, exists in fluid mechanics, condensed matter physics, particle physics, nonlinear optics and several other fields [1-5]. In the past two decades, the nonlinear dynamics of nonlocal solitons were intensively investigated, and many integration tools were proposed to extract soliton solutions to the well-known nonlocal nonlinear Schrödinger equation (NNLSE), which models the propagation of solitons through nonlocal nonlinear systems [1-10].

Recently, soliton dynamics in an artificial synthetic nonlocal material with competing nonlinearities in which the nonlinear response of medium is the result of interaction between nonlocal nonlinearity and other nonlinear effects has attracted many attentions. The properties of dark solitons under competing generally nonlocal nonlinearity and local cubic nonlinearity or (and) quintic nonlinearity have been studied [11-41].

Under investigation in this work is the NNLSE with competing weakly nonlocal nonlinearity and parabolic law nonlinearity

$$i\frac{\partial u}{\partial z}+a\frac{\partial^2 u}{\partial x^2}+b\frac{\partial^2(|u|^2)}{\partial x^2}u + c|u|^2u+\mu|u|^4u=0 \tag{1}$$

where $u(x,t)$ is the slowly varying amplitude, while x and t are dimensionless transverse and propagation coordinates.

In Eq. (1), the first term gives the linear evolution, while the second term represents diffraction and finally the last three terms that are weakly nonlocal nonlinearity, cubic nonlinearity and quintic nonlinearity are due to competing nonlinearities. It should be noted that, in our previous studies, we performed four algorithms including traveling wave hypothesis, Jacobian elliptic equation expansion method, ansatz approach and Riccati equation expansion technique to integrate Eq. (1), and discussed the dynamic behaviors of solitons [16-18]. In present work, a different integration tool that is the Lie group analysis will be employed to extract solitons to Eq. (1).

2. Lie symmetry analysis

In this section, we will study Eq. (1) by Lie symmetry method.

In order to get our aim, we first use the following transformation:

$$u(x,t)=v(x,t)e^{i\varphi(x,t)} \tag{2}$$

Substituting Eq. (2) into Eq. (1), and then separating the real and imaginary parts, one gets

$$v_t+2av_x\varphi_x=0 \tag{3}$$

$$-v\varphi_t+2av_{xx}-av\varphi_x^2+2bvv_x^2+2bv^2v_{xx}+cv^3+\mu v^5=0 \tag{4}$$

Considering the following vector fields:

$$V=\xi(x,t,v,\varphi)\frac{\partial}{\partial x}+\tau(x,t,v,\varphi)\frac{\partial}{\partial t}+\eta(x,t,v,\varphi)\frac{\partial}{\partial v}+\phi(x,t,v,\varphi)\frac{\partial}{\partial \varphi} \tag{5}$$

and applying the second prolongation to Eqs. (3) and (4) yields

$$\tau=c_3 \tag{6}$$

$$\xi=c_1t+c_2 \tag{7}$$

$$\eta=0 \tag{8}$$

$$\phi=\frac{c_1x}{2a}+c_4 \tag{9}$$

Then, we get the Lie point symmetry generators

$$V_1=\partial_X \tag{10}$$

$$V_2=\partial_t \tag{11}$$

$$V_3=t\partial_x+\frac{x}{2a}\partial_\varphi \tag{12}$$

$$V_4=\partial_\varphi \tag{13}$$

and the corresponding groups

$$g_1:(x+\varepsilon,t,v,\varphi) \tag{14}$$

$$g_2:(x,t+\varepsilon,v,\varphi) \tag{15}$$

$$g_3:(t\varepsilon+x,t,v,\frac{x}{2a}\varphi+\varphi) \tag{16}$$

$$g_4:(x,t,v,\varphi+\varepsilon) \tag{17}$$

These imply that if $f(x,t)$ and $h(x,t)$ are the solutions to Eqs. (3) and (4), we have

$$v_1=f_1(x-\varepsilon,t) \tag{18-1}$$

$$\varphi_1=h_1(x-\varepsilon,t) \tag{18-2}$$

$$v_2=f_1(t-\varepsilon,x) \tag{19-1}$$

$$\varphi_2=h_1(t-\varepsilon,x) \tag{19-2}$$

$$v_3=f_3(x-t\varepsilon,t) \tag{20-1}$$

$$\varphi_3=\frac{x}{2a}\varepsilon h_3(x-t\varepsilon,t)+h_3(x-t\varepsilon,t) \tag{20-2}$$

$$v_4=f_4(t,x) \tag{21-1}$$

$$\varphi_4=h_4(t,x)-\varepsilon \tag{21-2}$$

Therefore, if we get the explicit solutions of $v(x,t)$ and $\varphi(x,t)$, then one can obtain the explicit solutions to Eq. (1) by using Eq. (2). Based on the Ref. [16], one can get many new solutions to Eq. (1).

Case 1: Ref. [16] gives the bright soliton to Eq. (1) in the form

$$u(x,t)=A\,\mathrm{sech}[B(x-mt)]e^{i(-\kappa x+\omega t+\theta)} \tag{22}$$

and then one can get new bright solitons using g_1

$$u(x,t)=A\,\mathrm{sech}[B(x-\varepsilon-mt)]e^{i[-\kappa(x-\varepsilon)+\omega t+\theta]} \tag{23}$$

Case 2: Ref. [16] gives the dark soliton to Eq. (1) in the form

$$u(x,t)=A\tanh[B(x-mt)]e^{i(-\kappa x+\omega t+\theta)} \tag{24}$$

and then one can get new dark soliton using g_1

$$u(x,t)=A\tanh[B(x-\varepsilon-mt)]e^{i[-\kappa(x-\varepsilon)+\omega t+\theta]} \tag{25}$$

Case 3: Ref. [16] gives the singular solitons to Eq. (1) in the form

$$u(x,t)=A\coth[B(x-mt)]e^{i(-\kappa x+\omega t+\theta)} \tag{26}$$

$$u(x,t)=A\,\mathrm{csch}[B(x-mt)]e^{i(-\kappa x+\omega t+\theta)} \tag{27}$$

and then one can get new singular solitons using g_1

$$u(x,t)=A\coth[B(x-\varepsilon-mt)]e^{i[-\kappa(x-\varepsilon)+\omega t+\theta]} \tag{28}$$

$$u(x,t)=A\,\mathrm{csch}[B(x-\varepsilon-mt)]e^{i[-\kappa(x-\varepsilon)+\omega t+\theta]} \tag{29}$$

where A, B and m are given in Ref. [16].

It is clear that if we choose any ε, one can get many new soliton solutions. Also, we can also get new solitons to Eq. (1) using other group via Ref. [16]. Here, we do not explain them. Consequently, we generalized the results in Ref. [16].

Finally, we discuss a special case, i.e. if we assume that $u(x,t)=v(x)e^{-i\lambda t}$, Eq. (1) becomes to

$$\begin{aligned}&\lambda v+av_{xx}+2bvv_x^2\\&+2bv^2v_{xx}+cv^3+\mu v^5=0\end{aligned} \tag{30}$$

Then, we can get the explicit solutions to Eq. (1) as follows:

Dark soliton

$$u(x,t)=C_4\,\tanh(C_2x+C_1)e^{-i(6C_2^2b-c)C_4^2t} \tag{31}$$

with $a=\dfrac{(8C_2^2b-c)C_4^2}{2C_2^2}$ and $d=-\dfrac{6C_2^2b}{C_4^2}$.

Singular periodic solution

$$u(x,t)=C_4\,\sec(C_2x+C_1)e^{-\frac{1}{2}i(4C_2^2b-c)C_4^2t} \tag{32}$$

with $a=\dfrac{(4C_2^2b-c)C_4^2}{2C_2^2}$ and $d=-\dfrac{6C_2^2b}{C_4^2}$

Jacobian sine elliptic periodic traveling wave solution

$$\begin{aligned}&u(x,t)=C_5\,\mathrm{sn}(C_3x+C_2,C_1)\\&\times e^{-i\frac{3C_1^4C_3^2a+3C_1^2C_3^2a+C_5^4d}{3C_1^2}t}\end{aligned} \tag{33}$$

with $b=-\dfrac{dC_5^2}{6C_1^2C_3^2}$ and

$c=-\dfrac{6C_2^4C_3^2a+2C_1^2C_5^2d+2C_5^4d}{3C_1^2C_5^2}$.

Remark 1: In Eqs. (28)-(30), C_i ($i=1,2,3,4,5$) are real constants. In particular, letting modulus $C_1=1$ in Eq. (30), one gets the explicit dark soliton to Eq. (1) in the form

$$\begin{aligned}&u(x,t)=C_5\,\tanh(C_3x+C_2)\\&\times e^{-i\frac{3C_3^2a+3C_3^2a+C_5^4d}{3}t}\end{aligned} \tag{34}$$

Remark 2: In this paper, we only consider the transformation $u(x,t)=v(x,t)e^{i\varphi(x,t)}$. In the future studies, we will consider other transformation in future work, such as $u=v+i\varphi$ and so on.

3. Conclusion

The nonlinear dynamical model (i.e. Eq. (1)) that describes the optical solitons propagate in a nonlinear medium with competing weakly nonlocal nonlinearity, cubic nonlinearity and quintic nonlinearity is investigated analytically. Via the Lie group analysis, some new bright, dark and singular solitons are derived. In addition, the explicit singular periodic solution and Jacobian sine elliptic periodic traveling wave solution are reported. It noted that if we take the modulus $C_1=1$, the sine periodic wave will degenerate to dark soliton.

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