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Realization of quantum logic gates based on coupled-resonator optical waveguide

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Quantum logic gates are basic blocks to construct quantum computing and quantum information processing systems. Quantum logic gates based on coupled-resonator optical waveguide (CROW) are analyzed with the advantage of integration with other devices. Photonic crystal CROW (PhC CROW) structure with different dimension and background medium exhibits electromagnetically induced transparency (EIT), which is used as single quantum bit (qubit) phase gate. Quantum Hadamard gate can also be realized by coupling element. Two qubit conditional quantum phase gate is achieved by concatenating two Hadamard gates and a phase gate doped with dopant. The simplicity and dynamic tunability of the device could be a useful solution for quantum information processing.

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1. Introduction

Quantum computation based on quantum logic gate is a candidate for high-speed and large capacity information processing, which can avoid the drawback of electronic bottleneck demonstrated in traditional technology [1~4]. Since quantum algorithm appeared, substantial efforts have been made to realize of the quantum information processing especially the quantum gate [5]. Quantum gates plus quantum bits are prepared to implement quantum computation, where these gates are basic components which apply unitary transformations to quantum bits [6~12].

Coupled-resonator optical waveguides (CROW) (also called coupled cavity waveguides (CCW)), consist of many optical resonators weakly coupled to each other. Transmission of electromagnetic waves in such a waveguide depends on the coupling property between different resonators, and the physical formalism can be described with tight binding approximation. Similarly, photon propagates in the waveguide by hopping from a resonator to another neighbor resonator [13, 14]. Photonic crystals (PCs) or photonic band gap materials arranged in periodic array, which give rise to waveguide due to chains of defects, can be applied to construct coupled-resonator optical waveguides [15,16]. In fact, these defects in photonic crystals constitute resonators.

Here, quantum gates based on CROW as shown in Fig.1 are discussed, which could realize single qubit quantum phase gate, Hadamard gate or two qubit conditional quantum phase gate. Furthermore, manipulation of phase and coupling characteristics in the device are discussed, and the dynamic quantum logic operations are theoretically demonstrated.

2. Realization and analysis of quantum logic gates based on CROW

CROW is made up of high dielectric constant rods embedded in low dielectric constant material with every fourth rod in a line removed, which is to say that it is composed of many defect cavities. As shown in Fig.1, the whole structure corresponds to Hadamard gate, phase gate and Hadamard gate from left to right, and two Hadamard gates are identical. To realize Hadamard gate, two linear waveguides and a coupling element are used. The two linear waveguides are two rows of CROW close to each other, and the coupling element includes two resonators. For example, defect cavities are disposed between the two rows. By introducing the coupling element and controlling the distance between the two rows of CROW, it is easy for the photon to hop in the waveguide. Two rows of CROW which are called the upper arm and the lower arm, can constitute a phase gate, which may be a single qubit or a part of the two qubit conditional quantum phase gate depending on different parameters. The phase of light propagated in the two arms can be controlled through different size parameters of the device, such as length difference of the two arms etc.

Light is input and then split into two parts after the first Hadamard gate, and the two parts acquire a relative phase after propagating through the phase gate, and finally get across the second Hadamard gate. The desired result of the second Hadamard gate can be obtained by controlling the relative phase of light in phase gate. The output of the second Hadamard gate is also the output of the whole device.

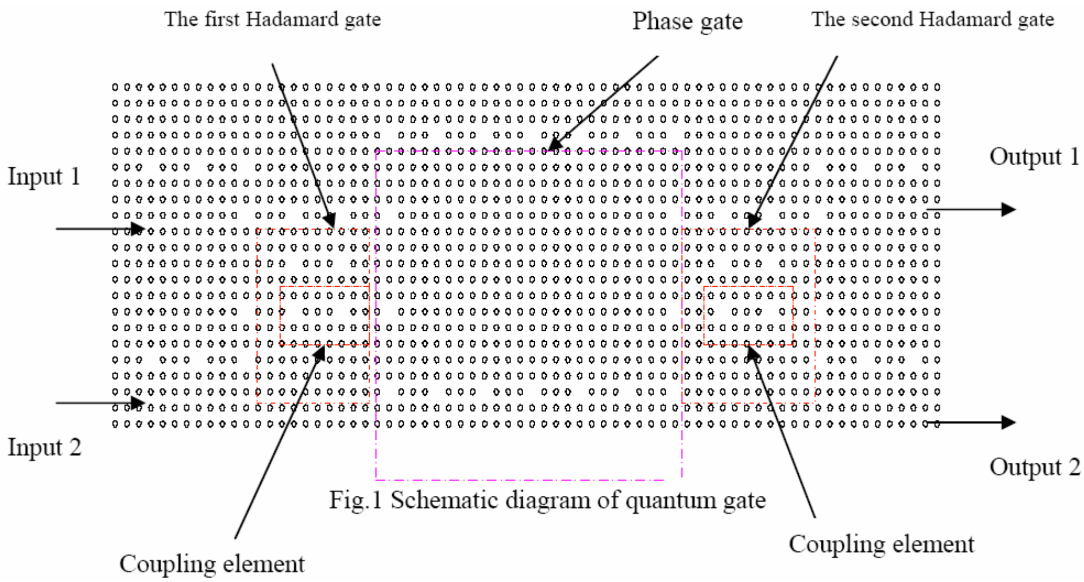

Fig.1. Schematic diagram of quantum gate.

2.1 Single qubit phase gate

The presented phase gate can be regarded as a MZ interferometer, and the light path-difference between the upper arm and lower arm can change the relative phase with different means.

2.1.1 Size and material effect

In the case of single qubit quantum phase gate, if we launch light into input 1 of the device, and then the accumulated phase difference between two arms can be expressed as [17]:

$$\Delta p(f) = \beta \Delta L \tag{1}$$

where β is propagation constant of the guided mode, ΔL is the length difference between the upper arm and lower arm. Since the CROW is composed of many defect cavities, and the period of these defect cavities is fixed, so ΔL depends on the difference of defect cavity number ΔN between two arms. The Δp can be written as:

$$\Delta p(f) = \Delta N \arccos\left(\frac{f - f_0}{k_1 f_0}\right) \tag{2}$$

where f_0 is eigenfrequency of an individual cavity, k_1 is the coupling factor which depends on the waveguide's bandwidth and hopping time constant. k_1 is related to the period of CROW device. When the distance between neighbor defect cavity increases, k_1 will decrease. Therefore, it is more difficult for photon to hop to the adjacent cavity. Fig. 2 shows accumulated phase difference versus the frequency of light, where ΔN is equal to 2 and 4 respectively. When the absolute value of coupling factor is decreased, a bigger Δp and a wider bandwidth are achieved.

The material effect in phase gate is analyzed as the following. Δp changes with different wave vector k in two arms, which is approximately given as [18]:

$$\Delta p = L \cdot \delta k \approx L \cdot \delta\omega / \left(d\omega / dk\right) \tag{3}$$

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he group velocity of light is v_G = dw/dk, and the optical frequency shift of a photonic band mode due to change in refractive index is $\delta\omega(k) = -\sigma\omega(\delta n / n)$ (σ is a proportional factor), so Eq. (3) can be written as:

$$\Delta p \approx -L\omega\sigma\delta n / \left(n\upsilon_G\right) \tag{4}$$

Group velocity is found to be $v_G = -f_0 R k_1 \sin(kR)$ (where R is the distance between neighbor defect cavity) in the CROW [13]. When the operating point is assumed to be at zero dispersion with k=(π/2)R, Eq. (4) can be written as:

$$\Delta p \approx \frac{\pi}{2} \frac{C_0\sigma}{f_0 k_1} \frac{\delta n}{n} \frac{1}{R^2} L \tag{5}$$

where C_0 is the velocity of light in vacuum. As shown in Fig.3, the accumulated phase difference increases with the increase in refractive index in one of the arms. In the other word, the refractive index difference δn in the two arms can achieve the desired phase difference.

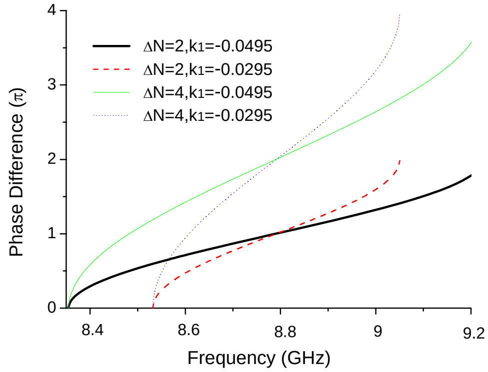


Fig. 2. Accumulated phase difference versus the frequency of light.

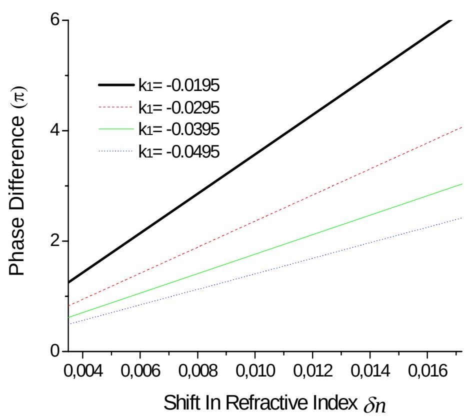


Fig. 3. Accumulated phase difference versus the shift in refractive index.

2.1.2 Electromagnetically induced transparency effect (EIT)

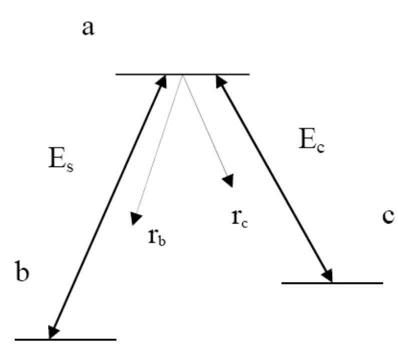
EIT effect may also be used to realize the single qubit quantum phase gate. When the dispersion of the CROW is very small, the group velocity of the light is close to its phase velocity. The group velocity can be used to represent the phase in the arm. Furthermore, the group velocity in the upper arm of the device is different from that in the lower arm, and the phase difference between two arms can be controlled with different group velocities. If one of the two arms could be placed in EIT background and the other arm is placed in an ordinary background, or, both arms are positioned in EIT background, the group velocity of each arm can be changed with the different parameters of the EIT background.

If impurity atoms or quantum dots, which could exhibit electromagnetically induced transparency (EIT) effect, are doped into the low dielectric constant material, the EIT background mentioned above can be realized. For example, as shown in Fig. 4(a), the three-energy-level atoms are doped into the low dielectric constant material [19]. These energy levels (called a, b, c) are arranged in Λ -scheme. The control field E_c is resonant with the ac transition, and the signal light filed E_s is resonant with the ab transition. Decay rate r_b and r_c are the probabilities per unit time that quantum state will perform a quantum jump from |a> to |b> and |a> to |c>, respectively [20]. We introduce a notation $r= r_b+r_c$ to indicate the entire decay rate.

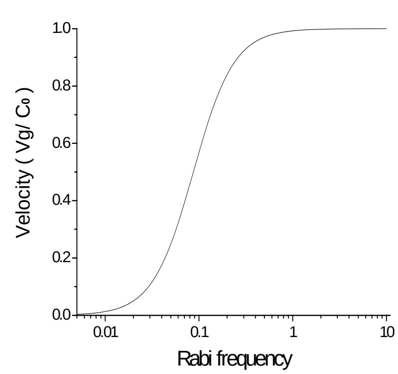
If one arm is in EIT background, group velocity of light in this arm would depend on the control field of this background, as follow [21]:

$$\upsilon_G = \frac{C_0 \alpha \Omega_c^2}{1+\alpha\Omega_c^2} \tag{6}$$

where Ω_c is the Rabi frequency of control field(the unit of the Rabi frequency is normalized with the r), $\alpha=1/(2C\omega_{ab})$, ω_{ab} is the resonate frequency of ab transition and C is a material constant related to the density of excitation centers and the dipole moment. Fig. 4(b) shows the group velocity increases with Rabi frequency. It is considered that the coupling factor k_1 also increases with Rabi frequency, which results in a greater velocity. Finally, the desired phase in this arm can be obtained by controlling the group velocity, and the different phase difference can be achieved.



(a)



(b)

Fig. 4. (a) Three-energy-level Λ-scheme of the atoms; (b) Group velocity versus increasing Rabi frequency.

2.2 Hadamard gate

Quantum state could be transfered from ground state 0 and 1 to intermediate state with the Hadamard gate, which is equivalent to optical coupler except for a phase factor [22]. As shown in Fig.1, two linear waveguides and a coupling element constitute Hadamard gate, and the relationship between input and output light of this gate can be expressed as [18]:

$$A_{OUT1} = A_{IN1}\left(1-\frac{ib}{\omega-\omega_0+ib}\right)-A_{IN2}\left(\frac{ib}{\omega-\omega_0+ib}\right) \tag{7}$$

$$A_{OUT2} = A_{IN2}\left(1-\frac{ib}{\omega-\omega_0+ib}\right)-A_{IN1}\left(\frac{ib}{\omega-\omega_0+ib}\right) \tag{8}$$

where A_{in1}, $A_{in1,}$ A_{out1}, A_{out2} are amplitudes of lights in these input and output ports respectively.

If the light is imported into Hadamard gate with a previous relative phase difference between two input ports of the gate (e.g. the second Hadamard gate), the probability of photon hoping on either output port will depend on the relative phase. Fig. 5 shows the output light pulse intensity of this Hadamard gate versus previous phase difference, wherein the intensity will change through a whole period with phase difference △p varying from –π to π. The light leave the Hadamard gate with equal probability 1/2 at each output port when △p is -π,0 and π. The probability at either output port will become 1 or 0 with △p at ±π/2.

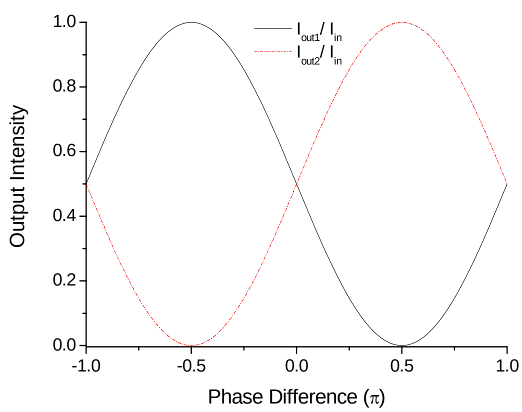


Fig. 5. Output pulse intensity versus phase difference. I_{out1} and I_{out2} are the two outputs of Hadamard gate, I_{in} is the total intensity of input pulse.

2.3 Two qubit conditional quantum phase gate

The present device can also realize the two qubit conditional phase gate with atom or quantum dots doped into the defect cavities in phase gate area. The dopant will resonate with the propagating light when the external electric field is applied. As mentioned above, when the light is launched from the left input port 1 and 2, so the quantum state of the light propagating through the first Hadamard gate could be expressed as [22]:

$$|11\rangle\rightarrow\frac{|20\rangle-|02\rangle}{\sqrt{2}},\ |00\rangle\rightarrow|00\rangle,\ |01\rangle\rightarrow\frac{|01\rangle-|10\rangle}{\sqrt{2}},\ |10\rangle\rightarrow\frac{|01\rangle+|10\rangle}{\sqrt{2}} \tag{9}$$

where 0, 1 and 2 are the photon number in the guided mode, and |11>, |00>, |01>, |10> are the different quantum state of the two qubit in input port 1 and 2. Arrows represent the Hadamard transform. If both the light in input 1 and input 2 of the first Hadamard gate are at one-photon states, each arm of phase gate will generate two-photon state behind the first Hadamard gate. Quantum state in the phase gate will remain at one-photon state or vacuum state in the rest input cases. Here, the defect cavities are doped with three-level atoms, which have three energy levels and resonate with only the two-photon state light in phase gate area, and the light in two-photon state gets an extra π phase change after through the phase gate. Therefore, the two-photon input state gets a minus transform after the second Hadamard gate while the rest input cases remain unchanged [22]:

$$\frac{|20\rangle-|02\rangle}{\sqrt{2}}\xrightarrow{\Delta p}\frac{|02\rangle-|20\rangle}{\sqrt{2}}\rightarrow-|11\rangle,\ |00\rangle\rightarrow|00\rangle,\ \frac{|01\rangle-|10\rangle}{\sqrt{2}}\rightarrow|01\rangle,\ \frac{|01\rangle+|10\rangle}{\sqrt{2}}\rightarrow|10\rangle \tag{10}$$

The two qubit conditional quantum phase gate is achieved by concatenating two Hadamard gate and a phase gate doped with dopant. Here, the physical formalism of the phase gate is different from that of the single qubit quantum phase gate case. Whether the resonance would occur or not depends on the different types of input quantum states.

3. Conclusions

We have presented an analysis of realization of quantum logic gates such as single qubit quantum phase gate, Hadamard gate and two qubit conditional quantum phase gate based on coupled-resonator optical waveguides (CROW). Manipulation of phase and coupling characteristics in the device are discussed and the dynamic quantum logic operations are numerically demonstrated. Results show that the photonic crystal CROW structure with different sizes or background media exhibiting electromagnetically induced transparency (EIT) can give desired phase difference and has the advantage of integration with other devices. In addition, the Hadamard gate including a coupling element can realize different output with different phase differences. The simplicity and dynamic tunability of the present device could be a useful solution for quantum information processing.

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References

[1] L. J. Schulman, Nature **438**(7067), 431 (2005).
[2] A. M. Steane, Quantum Information& Computation **7**(3), 171 (2007).
[3] M. R. Geller, E. J. Pritchett, A. Galiautdinov, Physica Status Solidi B-Basic Solid State Physics **246**(5), 972 (2009).
[4] A. Barenco, Proceedings Of The Royal Society Of London Series A-Mathematical And Physical Sciences **449**(1937), 679 (1995).
[5] P. W. Shor, Society for Industrial and Applied Mathematics Journal on Computing **26**(5), 1484 (1997).
[6] S. J. van Enk, J. I. Cirac, P. Zoller, Physical Review Letters **79**(25), 5178 (1997).
[7] D. Ahn, J. H. Oh, K. Kimm, Physical Review A **61**(5), 052310 (2000).
[8] D. Leung, L. Vandersypen, X. L. Zhou, Physical Review A **60**(3), 1924 (1999).
[9] K. Kim, C. F. Roos, L. Aolita, Physical Review A **77**(5), 050303(2008).
[10] O. Zilberberg, B. Braunecker, D. Loss, Physical Review A **77**(1), 012327 (2008).
[11] L. M. Duan, B. Wang, H. J. Kimble, Physical Review A **72**(3), 032333 (2005).
[12] D. G. Angelakis, A. Kay, arXiv: 0707.3946v1 [quant-ph] 26, Jul 2007.
[13] A. Yariv, Y. Xu, R. K. Lee, A. Scherer, Optics Letters **24**(11), 711 (1999).
[14] S. Mookherjea, A. Yariv, IEEE Journal Of Selected Topics In Quantum Electronics **8**(3), 448 (2002).
[15] S. Lin, E. Chow, V. Hietala, P. R. Villeneuve, J. D. Joannopoulos, Science **282**, 274 (1998).
[16] A. L. Reynolds, U. Peschel, F. Lederer, P. J. Roberts, T. F. Krauss, P. J. I. de Maagt, IEEE Transactions On Microwave Theory And Techniques **49**(10), 1860 (2001).
[17] A. Martinez, A. Griol, P. Sanchis, J. Marti, Optics Letters **28**(6), 405 (2003).
[18] M. Soljacic, S. G. Johnson, S. Fan, Journal of the Optical Society of America B **19**(9), 2052 (2002).
[19] P. Janes, J. Tidstrom, L. Thylen, Journal of Lightwave Technology **23**(11), 3893 (2007).
[20] J. Tidstrom, P. Janes, L. Mauritz Andersson, Physical Review A **75**(5), 053803 (2005).
[21] C. W. Neff, L. M. Andersson, M. Qiu, Optics Express **15**(16), 10362 (2007).
[22] D. G. Angelakis, M. F. Santos, V. Yannopapas, Physics Letters A **362**(5-6), 377 (2007).

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