

Study of vibration characteristic of phononic crystals flexible mechanical arms

XUE-WEI KANG, YAN ZHANG*, LIN-HUA JIANG, LIN HAN, HONG-QIANG CHU, QIAO ZHU
College of Mechanics and Materials, Hohai University, Nanjing 210098, Nanjing, People's Republic of China

Phononic crystals (PCs) flexible mechanical arms are introduced to study the vibration characteristic. Two research models, single PCs arm structure and two-link PCs arm structure are considered. The finite periodic structures are established to describe the ideal PCs structures. The finite element analysis shows that the angle doesn't influence the range of the band gaps (BGs) while it influences the attenuation value of each BG. The more close to 90 degree gives more distinct BGs. For axial vibrations, the two-link PCs arm structure has the advantage to enhance the attenuation effect.

(Received March 1, 2014; accepted September 11, 2014)

Keywords: Flexible mechanical arms, Phononic crystal, Band gap, Vibration control, Finite element analysis

1. Introduction

With the development of the engineering technology, the flexible mechanical arms are paid more attention because of its advantages such as low energy consuming, wide application range, fast response speed and high efficiency. Therefore, it is widely used in the field of aerospace, manufactory and national defense, etc [1]. However, this "long and slender" arm structure has still a obvious disadvantage: under the complex loading and moving, the arms may be affected by shear, axial transformation and vibration. In the case of some "fast-speed moving" and "high precision" working conditions, the general flexible mechanical arms may spend more time to complete their missions [2-4]. For example, the distance control mechanical arms should work continuously 47 hours to assemble the space station while 20~30 % time is wasted to wait for the elimination of some residual vibration [5]. In some working condition of high-speed laser cutting machine, the maximum acceleration of mechanical arms is up to 3~5 G while the precision requirement of the cutting is within 0.2 mm [6, 7]. The effective method to control, reduce and eliminate the vibration of the flexible mechanical arm is important and necessary.

In the recent two decade years, a kind of periodic composite materials is received much attention. It is observed that certain frequency ranges of elastic wave cannot propagate in it. This phenomenon is similar to the photonic band gap in photonic crystals so that these periodic composite materials are so called phononic crystals (PCs) by analogy [8, 9]. And also, these certain frequency ranges are defined as phononic band gaps (BGs). With some advisable selection of composites and

their geometric forms, adjustable band gaps can be obtained. These PCs materials provide a new way to reduce and isolate vibrations in engineering. Some applications of PCs like: PCs beam structure, PCs beam-foundation structure, PCs plate structure, etc, have been proposed and relevant study shows these PCs structures have the evident effect on vibration attenuations [10-15]. Meanwhile, some theoretical methods have been found to calculate the band gap such as the plane wave expansion (PWE) method [16, 17], the transfer matrix (TM) method [18-20], the finite difference time domain (FDTD) method [21, 22], the multiple scattering theory (MST) [23], and the lumped-mass (LM) method [24]. The evolution of the methods accelerates the applications of PCs in engineering. However, the PCs structure is not taken into account about the flexible mechanical arms.

Inspired by the concept of PCs, the proposition of the flexible mechanical arm with periodic composite materials provides a new way to control the residual vibrations caused by loading and moving. In this paper, two preliminary models, single PCs arm structure and two-link PCs arm structure, are proposed first. Then, the corresponding frequency responses of two models are calculated simultaneously by Abaqus, and their vibration characteristic is analyzed. Finally, we draw the conclusion.

2. Theory

2.1 Research model

Fig. 1 shows the two research models of

two-dimensional articulated flexible mechanical arms. These long and slender arms can be regarded as Euler beam. The connection points, composed by driving motor and reduction gears, are the joints of the mechanical arms. They can be simplified as rigid joints. In the case of Model A (Fig. 1(a)), OB arm is replaced by PCs and the other arms keep normal. In Model B (Fig. 1(b)), OA arm and AB arm are replaced by PCs to reduce the vibrations, while others keep normal. In this paper, they are called single PC arm model and two-link PC arm model respectively. In Model B, Point A is located between OA arm and AB arm while the included angle is defined as θ . It is changeable in the general working condition. In this preliminary research, the PCs structure is simplified as a two-component periodic material with material X and Y. The corresponding length of components are l_x and l_y . Material X and Y have the same cross-section sizes. The axial loading impulses and flexural loading impulses with certain frequency range are loaded to Point O while the output responses can be observed at the other side (Point B) of the models.

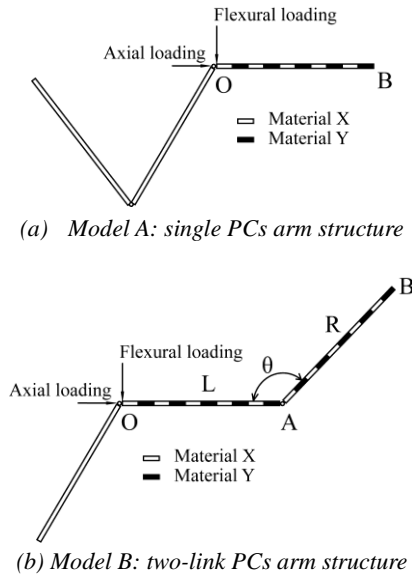


Fig. 1. Research models of flexible mechanical arms.

2.2 Vibration equation

The joint A in the Model B can be simplified as a rigid point. When a vibration propagates through this point, force and displacement should be decomposed according to the coordinating conditions. This means whatever this model is loaded by axial or flexural loading impulses individually, the vibration wave will be decomposed to axial and flexural vibration simultaneously on the joint A.

Considering the homogeneity thin rod, the governing equation of the axial-free vibration becomes

$$EA \frac{\partial^2 u(x,t)}{\partial x^2} - \rho A \frac{\partial^2 u(x,t)}{\partial t^2} = 0 \quad (1)$$

where $u(x,t)$ is the axial displacement; E is the elastic modulus; ρ and A are the density and the cross-sectional area. For the rectangular beam $A = b \cdot h$, in which b and h , respectively, are the width and the height of beam.

Neglecting the effects of the shear deformation and rotary inertia, the governing equation of the flexural-free vibration could be written as

$$EI \frac{\partial^4 y(x,t)}{\partial x^4} + \rho A \frac{\partial^2 y(x,t)}{\partial t^2} = 0 \quad (2)$$

where $y(x,t)$ is the lateral displacement; I is the area moment of inertia with respect to the axis perpendicular to the beam axis.

From the equilibrium of forces, bending moment, continuity of arms displacement and rotation at the joint A, we can obtain

$$\begin{aligned} Q^L &= -F^R \sin \theta - Q^R \cos \theta \\ F^L &= -F^R \cos \theta + Q^R \sin \theta \\ Y^L &= -U^R \sin \theta - Y^R \cos \theta \\ M^L &= M^R \\ \alpha^L &= \alpha^R \end{aligned} \quad (3)$$

where Q and F are axial and lateral force; U and Y are axial and lateral displacement; M and α are bending moment and rotation; Superior L and R represent the left and right arm of the two-link structure. That is to say, Q^L represent the axial force on the joint A of the left arm, and by this analogy.

3. Results and discussion

3.1 Calibration

The ideal PCs structure is composed by infinite repetition of cells and it doesn't exist really. In reality, the vibration characteristic of the finite periodic structure with the same periodicity could reflect the BGs of the ideal PCs.

In this article, finite period structures are structured to describe the system of PCs flexible mechanical arms. The software Abaqus is used to calculate the frequency response of the axial and flexural loading impulses. The progress can be summarized as follows. First, two materials, aluminum (material X) and wolfram (material Y) are chosen. The parameters are respectively $\rho_x = 2730 \text{ kg/m}^3$, $E_x = 77.6 \text{ GPa}$, $G_x = 28.8 \text{ GPa}$, $\rho_y = 19100 \text{ kg/m}^3$, $E_y = 354.1 \text{ GPa}$, $G_y = 131.1 \text{ GPa}$. Second, the geometric parameters are given as $l_x = l_y = 0.5 \text{ m}$, $b_x = b_y = 0.05 \text{ m}$, $h_x = h_y = 0.05 \text{ m}$. Third, establish finite element model with a given periodic number (see in Fig. 1) in Abaqus, meshing by B21 plane beam elements. Finally, the frequency responses can be received at the end (point B) of the PCs arm structures while the harmonic axial and flexural displacement impulses are applying on the point O separately from 0

Hz to 1000 Hz. The BGs can be easily found and analyzed from the distinct attenuation frequency ranges.

Fig. 2 shows the frequency response of the single PCs arm structure (Model A) with 5-cell. One axial vibration BG and five flexural vibration BGs exist in 0-1000Hz. Fig. 3 shows the frequency response of the two-link PCs arm structure (Model B) with 10-cell and $\theta=120^\circ$. Two axial vibration BGs and five flexural vibration BGs exist in 0-1000Hz. The dash area in the Fig. 2 and Fig. 3 is the intersection of the axial and flexural BGs. Both axial and flexural vibration in these frequency ranges can be attenuated.

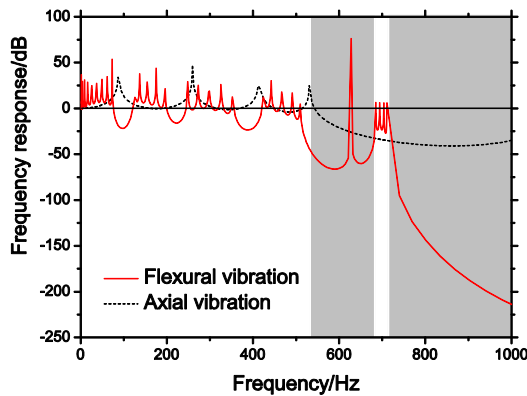


Fig. 2. Frequency response of single PCs arm structure.

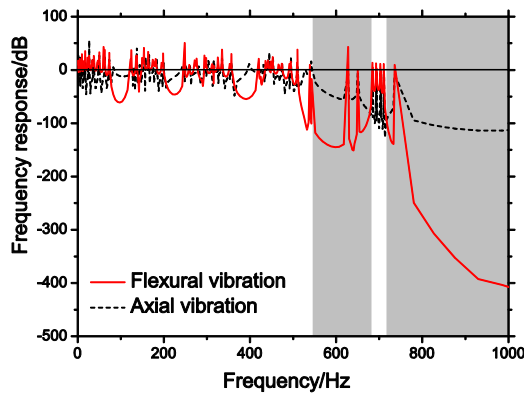


Fig. 3. Frequency response of two-link PCs arm structure with angle of 120 degree.

3.2 Influence of the angle

In general working conditions, the angle between two arms will be changed now and then. In the two-link PCs arm structure (Model B), we study the influence of the angle. We choose some values of angle from 60 to 120.

Fig. 4 shows the axial vibration frequency response of Model B with 60 degree, 90 degree and 120 degree. For comparison and analysis, the range and maximum attenuation value of first two BGs are organized in Table 1. We can find that the frequency range of BGs doesn't change much with the evolution of

the angle. With the increasing of the angle, the maximum attenuation value of each BG first increases and then decreases. The max attenuation value is observed when the angle is 90 degree.

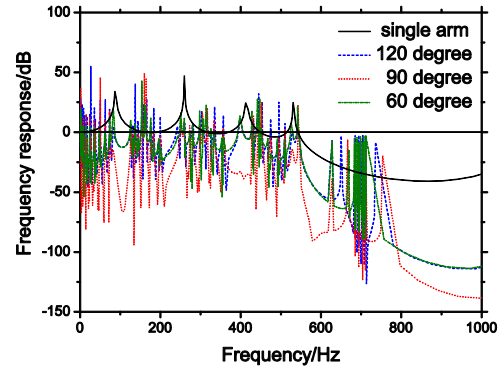


Fig. 4. Frequency response of axial vibration of Model A (single arm) and Model B with 60 degree, 90 degree and 120 degree.

Table 1. Range (Hz) and maximum attenuation value (dB) of BGs at axial vibration.

	Two-link arms			Single arm	
	Angle(degree)	60	90		120
1 st BG	Beginning(Hz)	542.65	542.51	542.62	542.81
	End(Hz)	683.23	685.06	684.69	1000
	Max attenuation(dB)	77.54	90.96	74.43	40.83
2 nd BG	Beginning(Hz)	711.60	710.47	710.23	
	End(Hz)	1000	1000	1000	
	Max attenuation(dB)	113.74	138.67	112.67	

Fig. 5 shows the flexural vibration frequency response of Model B with 60 degree, 90 degree and 120 degree. The data of first three BGs is organized in Table 2. We can get the same conclusions as axial vibration before.

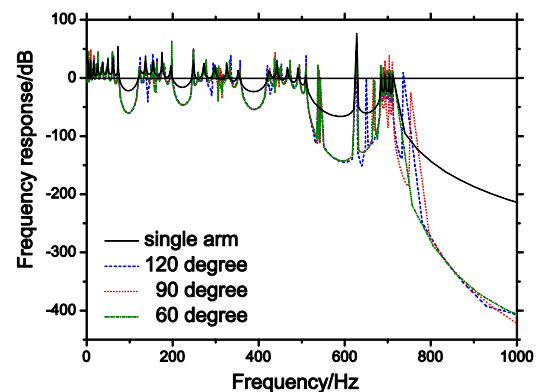


Fig. 5. Frequency response of flexural vibration of Model A (single arm) and Model B with 60 degree, 90 degree and 120 degree.

Table 2. Range (Hz) and maximum attenuation value (dB) of BGs at flexural vibration.

	Two-link arms			Single arm	
	Angle(degree)	60	90		120
1 st BG	Beginning(Hz)	79.93	73.93	73.96	77.01
	End(Hz)	115.62	123.50	135.26	128.84
	Max attenuation(dB)	60.29	61.62	60.16	21.91
2 nd BG	Beginning(Hz)	198.20	199.03	199.29	198.69
	End(Hz)	248.55	248.55	248.55	248.78
	Max attenuation(dB)	46.89	48.63	46.13	16.05
3 rd BG	Beginning(Hz)	356.50	356.70	356.25	353.77
	End(Hz)	413.10	426.70	433.88	424.11
	Max attenuation(dB)	54.10	58.12	54.50	23.51

3.3 Discussion of two models

According to the axial and flexural vibration frequency response of Model A on Fig. 4 and Fig. 5, we present the results of Model A in Table 1 and 2, as well as Model B. Compared with Model A, the ranges of BGs of Model B are slightly narrow. However, it is clear that both attenuation values with different degrees of Model B are both greater than the value of Model A. We draw the conclusion that the periodic number affects the attenuation effect. The more periodic number given, the stronger attenuation BGs are.

4. Conclusion

In conclusion, we study the vibration characteristic of phononic crystals mechanical arms by the frequency response analysis of two models (single PCs arm structure and two-link PCs arm structure) based on the finite element method. The influence of the angle and comparison between two models are discussed. We draw the following conclusions:

1. Single PCs arm structure (Model A) and two-link PCs arm structure (Model B) are both efficient in reducing and isolating vibration.
 2. For Model B, the range of BGs doesn't change much with the angle while the attenuation value of each BG first increases and then decreases with the increasing of angle. We obtain the maxim attenuation value in the case of 90 degree.
 3. Model B has the advantage of enhancing the efficiency of axial vibration BGs compared with Model A.
 4. Periodic number of PCs structure affects the BGs.
- As we know, the frequencies of flexural vibration BGs are lower and more efficient than axial vibration BGs. With the angle close to 90 degree, more axial vibration transfer to flexural vibration, more efficient the BGs are.

Acknowledgement

This work is supported by the National Natural Science Foundation of China (No. 51278167, 51479089 and 51479051) and the Natural Science Foundation of Jiangsu province (No. BK20131374).

References

- [1] W. J. Book, M. Majette, J. Dyn. Syst.-T. ASME **105**, 245 (1983).
- [2] E. H. K. Fung, D. T. W. Yau, J. Sound Vib. **241**, 857 (2001).
- [3] T. Fukuda, Bulletin of JSME **29**, 1269 (1986).
- [4] Y. Sakawa, F. Matsuno, S. Fukushima, J. Robotic Syst. **2**, 453 (1985).
- [5] T. E. Alberts, H. Xia, Y. Chen, J. Dyn. Syst.-T. ASME **3**, 468 (1992).
- [6] R. L. Kosut, J. of Guidance **2**, 104 (1983).
- [7] K. Krishnamurthy, J. Sound Vib. **2**, 205 (1992).
- [8] M. M. Sigalas, E. N. Economou, J. Sound Vib. **158**, 377 (1992).
- [9] R. Martínez-Sala, J. Sancho, J. V. Sánchez, V. Gómez, J. Llinares, F. Meseguer, Nature **378**, 241 (1995).
- [10] Y. Zhang, J. He, L. Jiang, Adv. Mech. Eng. **2013**, 7 (2013).
- [11] L. Han, Y. Zhang, L. H. Jiang, Z. M. Zhang, Phys. Status Solidi-R. **7**, 514 (2013).
- [12] Y. Zhang, L. Han, L. H. Jiang, Phys. Status Solidi B. **250**, 1439 (2013).
- [13] L. Han, Y. Zhang, L. Jiang, Z. Ni, Optoelectron. Adv. Mater.-Rapid Comm. **7**, 631 (2013).
- [14] H. S. Shu, W. Liu, S. D. Li, J. Vib. Control (2014).
- [15] H. S. Shu, L. Q. Dong, S. D. Li, W. Liu, S. G. Liu, W. Y. Wang, D. Y. Shi, D. Zhao, J. Phys. D Appl. Phys. **47**, 295501 (2014).
- [16] Y. Zhang, Z. Ni, L. Han, Z. M. Zhang, H. Y. Chen, Optoelectron. Adv. Mater.-Rapid Comm. **5**, 870 (2011).
- [17] Y. Zhang, Z. Ni, L. Han, Z. Zhang, L. Jiang, Optoelectron. Adv. Mater.-Rapid Comm. **6**, 1029 (2012).
- [18] M. M. Sigalas, C. M. Soukoulis, Phys. Rev. B **51**, 2780 (1995).
- [19] L. Han, Y. Zhang, Z. Ni, Z. Zhang, L. Jiang, Physica B **407**, 4579 (2012).
- [20] Z. Q. Ni, Y. Zhang, L. H. Jiang, L. Han, Int. J. Mod. Phys. B **6**, 1450093 (2014).
- [21] M. S. Kushwaha, P. Halevi, L. Dobrzynski, B. Djafari-Rouhani, Phys. Rev. Lett. **71**, 2022 (1993).
- [22] M. S. Kushwaha, P. Halevi, G. Martinez, L. Dobrzynski, B. Djafari-Rouhani, Phys. Rev. B **49**, 2313 (1994).
- [23] I. E. Psarobas, N. Stefanou, A. Modinos, Phys. Rev. B **62**, 5536 (2000).
- [24] G. Wang, J. Wen, X. Wen, Phys. Rev. B **71**, 104302 (2005).

*Corresponding author: yan.zhang@hhu.edu.cn