

# Study the effect of temperature on the optimum length of Er<sup>3+</sup> doped almino-germanosilicate, aluminum-oxide and yttria-silicate glass

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We study the effect of temperature on the optimum fiber length for maximum gain of Erbium doped almino-germanosilicate, aluminum-oxide and yttria-silicate glass, with fixed pump power. The optimum length depends strongly on the temperature and increases as the temperature increases. The maximum gain also depends on the wavelength and power of the signal which validate our findings through distributed gain measurements and so optimum length too.

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## 1. Introduction

In order to increase the transmission capacity of wavelength division multiplexing (WDM) systems, optical amplification outside the conventional band (C-band, 1540–1560 nm) and L-band (1560–1610 nm) is required and the various parameters which affected the gain such optimum length of the fiber and the temperature must be studied. The short wavelength band (S-band, 1480–1520 nm) is particularly attractive, being characterized by low loss in silica optical fiber. Many techniques have been developed to realize S-band amplification such as thulium-doped fluoride fiber amplifier (TDFFA) and a fiber Raman amplifier [1,2].

While these amplifiers have enabled impressive gain, noise figure and system performance, they have not matched conventional erbium-doped fiber amplifiers (EDFAs) in terms of efficiency, simplicity, reliability and cost.

To calculate the maximum gain at different values of temperature the approximate McCumber procedure is often used to predict the emission cross-section spectrum of the 1.5-  $\mu\text{m}$  transition of Er-doped glass fibers from the transition's measured absorption spectrum at different values of temperature [3].

A transcendental equation for the Maximum Gain ( $G_{\text{max}}$ ) at Optimum Length ( $L_{\text{opt}}$ ) of single-channel Erbium Doped Fiber Amplifiers (EDFAs) with fixed pump power was arrived by Ruhl [4] and, independently, Lin and Chi [5]. Desurvire also arrived to the same equations in his first book [6]. The equivalent equations for  $G_{\text{max}}$  and  $L_{\text{opt}}$  are Eqs. (10) and (11) in [4], (7) and (10) in [5], and (1.139) and (1.157) in [6], respectively. Importantly, it was shown in [7] that these equations, deduced in principle for single-channel amplifiers, can be easily extended to the case of EDFAs operating under Wavelength Division Multiplexing (WDM) conditions.

In this article, we study the effect of temperature on the optimum fiber length for maximum gain of Erbium doped almino-germanosilicate, aluminum-oxide and yttria-silicate glass, with fixed pump power.

## 2. Model

### a. Calculation of maximum gain

The McCumber relation states that the absorption cross section  $\sigma_a(\nu)$  and the emission cross section  $\sigma_e(\nu)$  spectra between a ground state (manifold of eight sublevels of energy  $E_{1j}$ ) and the excited state (is a manifold of seven sublevels of energy  $E_{2j}$ ) are related by [8]

$$\sigma_e(\nu) = \sigma_a(\nu) \exp\left(\varepsilon - \frac{h\nu}{kT}\right) \quad (1)$$

where  $k$  is the Boltzmann constant,  $T$  the absolute temperature, and  $\nu$  the optical frequency. The parameter  $\varepsilon$  [8] is defined as:

$$\exp\left(\frac{\varepsilon}{kT}\right) = \frac{\sum_{j=1}^8 \exp\left(-\frac{E_{1j}}{kT}\right)}{\sum_{j=1}^7 \exp\left(-\frac{E_{2j}}{kT}\right)} \exp\left(\frac{E_o}{kT}\right) = R \exp\left(\frac{E_o}{kT}\right) \quad (2)$$

where  $E_o = E_{21} - E_{11}$  is the energy difference between the lowest energy levels of the two manifolds [9] and Table 1.

If  $\eta_p$  and  $\eta_k$  are the ratio of the emission to the absorption cross section for the pump and the signal, respectively,

where

$$\eta_k = \frac{\sigma_e(\nu)}{\sigma_a(\nu)} \quad (3)$$

We can neglect ESA also because it is absent for a pump at 980 nm. For simplicity, we assume that the gain spectrum is homogeneously broadened. Within these

limits, the most fundamental limitation is due to the energy conservation,

$$G \leq 1 + \frac{\lambda_p P_p^{in}}{\lambda_k P_k^{in}} \quad (4)$$

where  $P_p^{in}$  and  $P_k^{in}$  is the input power for the pump and signal respectively, and  $\lambda_p$ ,  $\lambda_k$  the pump and signal wavelength.

In actual amplifiers, the absorption of pump photons (and therefore gain) is limited by the finite number of rare earth ions existing in the medium, the maximum signal gain corresponding to a three-level laser medium of length  $L$  is given by:

$$G = \frac{P_k^{out}}{P_k^{in}} = e^{(\rho \sigma_e(\lambda_k)L)} \quad (5)$$

In addition if we consider how many active ions are available in the medium for certain pump, we have the following expression:

$$G < \exp\left(\frac{\eta_k - \eta_p}{1 + \eta_p} \alpha_k L\right) \quad (6)$$

where  $\alpha_k$  is the absorption coefficient for the signal, (for pump at 980 nm  $\eta_p = 0$ ). [10].

### b. Calculation of optimum length at maximum gain

The expression for  $G_{max}$  derived from the TPE model is a transcendental equation, while  $L_{opt}$  is explicitly given in terms of the fibers intrinsic parameter and  $G_{max}$ . We stress that the advantages in using the TPE model is not just to have a simple transcendental equation for the gain as a function of the input powers and fiber parameters (avoiding the tedious and time-consuming numerical integrations of the SCD model), but also that these fiber parameters are combined into just two easily-measurable constants per wavelength: the absorption constant  $\alpha_k$  and the saturation power  $P_k^{sat}$ , at wavelength  $\lambda_k$ . To stress this fact, we write the equations for  $G_{max}$  and  $L_{opt}$  in terms of these two spectral constants [11]:

$$G_{max} \exp\{AP_k^{in}(G_{max} - 1)\} = \frac{1}{BP_p^{in}} \exp\left\{B \frac{\alpha_k}{\alpha_p} (P_p^{in} - B)\right\} \quad (7)$$

and

$$L_{opt} = \frac{1}{\alpha_p} \left\{ P_p^{in} - B + \ln(BP_p^{in}) + P_k^{in} \frac{P_k^{sat}}{P_p^{sat}} \frac{\alpha_p}{\alpha_k} (G_{max} - 1) \right\}, \quad (8)$$

Where

$$A = \frac{\alpha_k P_k^{sat} - \alpha_p P_p^{sat}}{\alpha_p P_p^{sat}}, \quad B = \frac{\alpha_k P_k^{sat} - \alpha_p P_p^{sat}}{\alpha_k P_k^{sat}}, \quad (9)$$

with

$$P_{p,k}^{in} = \frac{P_{p,k}^{in}}{P_{p,k}^{sat}}$$

and  $\alpha_k, \alpha_p$  are the absorption and pump coefficients respectively,  $P_p^{in}$  is input pump power and  $P_k^{in}$  is the input signal power,  $P_{p,k}^{sat}$  is the saturation power of the pump and signal respectively.

## 3. Results

The parameters used in calculations Mc-cumber cross section is given in Table 1 [8] and the parameters used in calculations the maximum gain and optimum amplifier length also given in Tables 2 and Table 3 [12].

Table 1. Parameters for Mc-cumber cross section calculations.

Fiber	Core glass	R	$\Delta E_1=7E_1$ (cm <sup>-1</sup> )	$\Delta E_2=6E_2$ (cm <sup>-1</sup> )
1	Ge-Sio <sub>2</sub>	1.35	361	450
2	NA	1.54	264	459
3	Ge/P-Sio <sub>2</sub>	1.27	304	374
4	Al/P-Sio <sub>2</sub>	1.27	393	295
5	Fluorophosphates	0.99	415	325
6	Al-Sio <sub>3</sub>	1.08	231	467
7	Al-Sio <sub>2</sub>	1.65	554	458
8	Al-P-Sio <sub>2</sub>	1.09	432	413

Table 2. The optical parameters of Er-doped fiber glass.

Physical meaning	Symbol	Value
pump wavelength	$\lambda_p$	980 nm
pump input power	$P_p^{in}$	18.4 dBm
signal wavelength	$\lambda_k$	1550 nm
length of EDFA	$L$	27 m
core radius of EDFA	$r$	1.277 $\mu$ m
core area of EDFA	$A$	$5.1 \times 10^{-12}$ m <sup>2</sup>
overlap factor of EDFA	$\Gamma$	0.5
fluorescence time of EDFA	$\tau$	10.5 ms
ion density of EDFA	$\rho$	$1.01 \times 10^{25}$ ions/m <sup>3</sup>

Table 3. The parameters used in this calculation of Optimum length.

Parameter	value
$\alpha_s$	$0.2 \text{ m}^{-1}$
$\alpha_p$	$0.1 \text{ m}^{-1}$
$P_s^{sat}$	$0.02 \text{ mW}$
$P_p^{sat}$	$0.01 \text{ mW}$
$P_p^{in}$	$64 \text{ mW}$
$P_s^{in}$	$20 \text{ mW}$

The normalized experimental values of the absorption cross section and the McCumber theory calculation for the erbium doped alumino-germanosilicate glass fiber amplifier at temperature range 290-310 °K is represented in Fig. 1. This figure shows that as the temperature slightly increase the emission cross section increases but the wavelength at maximum gain not shifted with temperature changes. So that the window at which the signal is amplified not affect by slightly temperature changes.

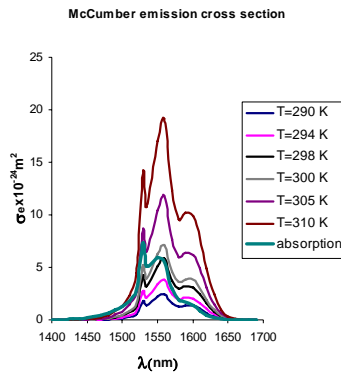


Fig. 1. Emission spectrum with the experimental absorption cross section for erbium doped alumino-germanosilicate glass fiber amplifier, calculated for different values of temperature.

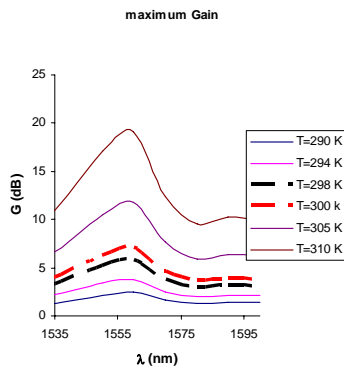


Fig. 2. The maximum gain spectrum for erbium doped alumino-germano-silicate glass fiber amplifier, calculated for different values of temperatures.

The maximum gain values in dB for the erbium doped alumino-germanosilicate glass fiber amplifier are plotted against wavelength range from 1530 to 1560 nm at input power 64 mw at temperature range 290-310 °K as shown in Fig. 2. Also the gain is plotted with length at different values of wavelength in Fig. 3, where the relation is found to be linear between the gain and the length. The Optimum length for the erbium doped alumino-germanosilicate glass fiber amplifier is calculated for fixed pump power and the maximum gain of the amplifier studied before. Since the maximum gain studied at different values of temperature, also the optimum length is studied at different values of temperature in Fig. 4.

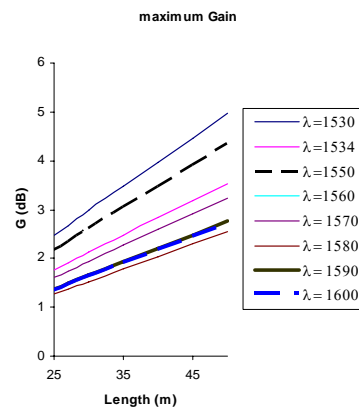


Fig. 3. The maximum gain for erbium doped alumino-germano-silicate glass fiber amplifier, calculated for different values of maximum wavelength at room temperature.

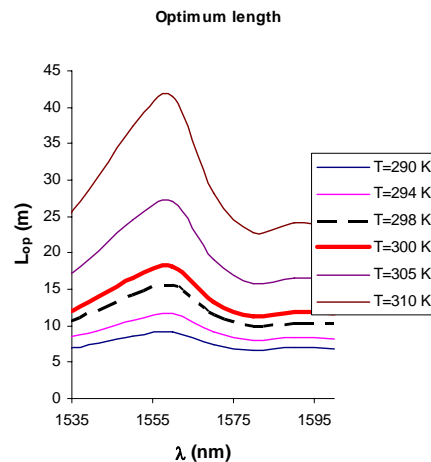


Fig. 4. The Optimum length variation for erbium doped alumino-germano-silicate glass fiber amplifier, calculated for different values of temperature at maximum gain of the amplifier.

Fig. 5 and Fig. 9 represents the normalized values of the emission cross section of Er-doped Al<sub>2</sub>O<sub>3</sub> fiber amplifier and yttria-Alumina-Silicate Erbium Doped Fiber

Amplifier as calculated using McCumber theory at different temperatures, from 290 to 310 °K respectively. It is clear that, the emission cross section increases with temperature. The maximum gain values in dB for the Er-doped Al<sub>2</sub>O<sub>3</sub> fiber amplifier and yttria-Alumina-Silicate Erbium Doped Fiber Amplifier are plotted against wavelength range from 1530 to 1560 nm at input power 64 mw at temperature range 290-310 °K respectively as shown in Fig. 6 and Fig. 10. Also the gain for the Er-doped Al<sub>2</sub>O<sub>3</sub> fiber amplifier and yttria-Alumina-Silicate Erbium Doped Fiber Amplifier respectively is plotted with length at different values of wavelength in Fig. 7 and Fig. 11, where the relation is found to be linear between the gain and the length. The Optimum length for the Er-doped Al<sub>2</sub>O<sub>3</sub> fiber amplifier and yttria-Alumina-Silicate Erbium Doped Fiber Amplifier is calculated for fixed pump power and the maximum gain of the amplifier studied before. Since the maximum gain studied at different values of temperature, also the optimum length is studied at different values of temperature in Fig. 8 and Fig. 12 respectively.

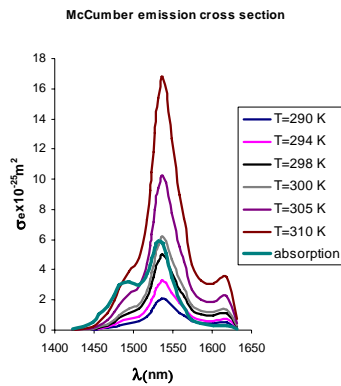


Fig. 5. The normalized values of the emission cross section of Er-doped Al<sub>2</sub>O<sub>3</sub> fiber Amplifier as calculated using Mc Cumber theory at different temperatures.

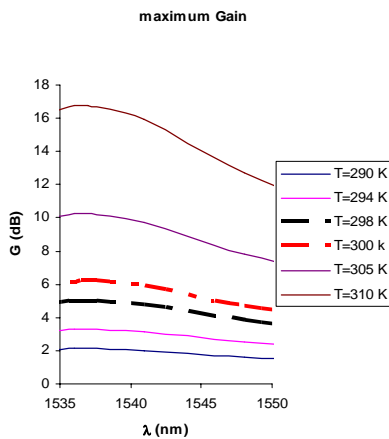


Fig.6 The maximum gain spectrum for erbium doped Al<sub>2</sub>O<sub>3</sub> glass fiber amplifier, calculated for different values of temperatures.

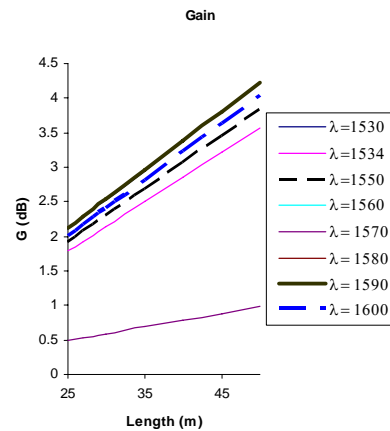


Fig. 7. The maximum gain erbium doped Al<sub>2</sub>O<sub>3</sub> glass fiber amplifier, calculated for different values of maximum wavelength at room temperature.

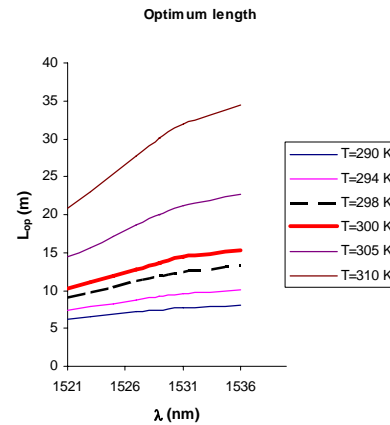


Fig. 8. The Optimum length variation for erbium doped Al<sub>2</sub>O<sub>3</sub> glass fiber amplifier, calculated for different values of temperature at maximum gain of the amplifier.

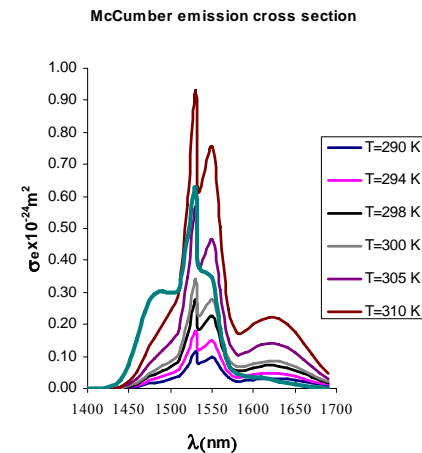


Fig. 9. The normalized values of the emission cross section of yttria-Alumina-Silicate Erbium Doped Fiber Amplifier as calculated using Mc Cumber theory at different temperatures.

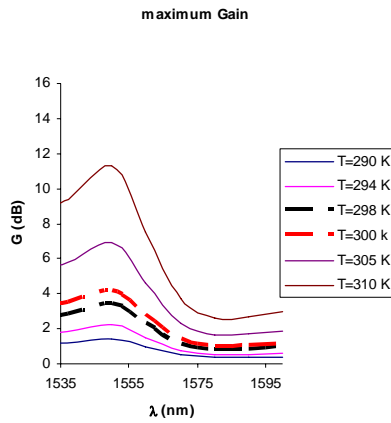


Fig. 10. The maximum gain spectrum for yttria-Alumina-Silicate Erbium Doped Fiber Amplifier, calculated for different values of amplifier length at room temperature.

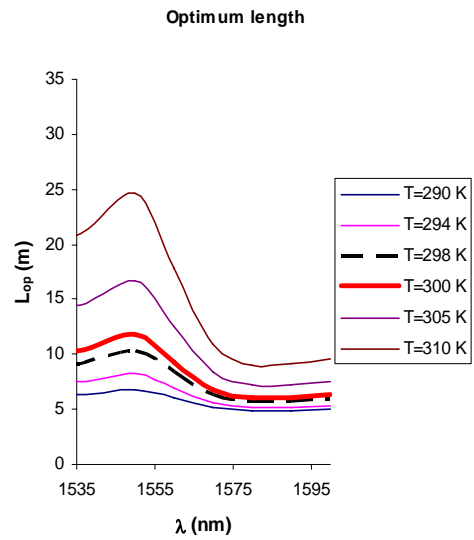


Fig. 12. The Optimum length variation for yttria-Alumina-Silicate Erbium Doped Fiber Amplifier, calculated for different gain values of temperature at maximum gain of the amplifier.

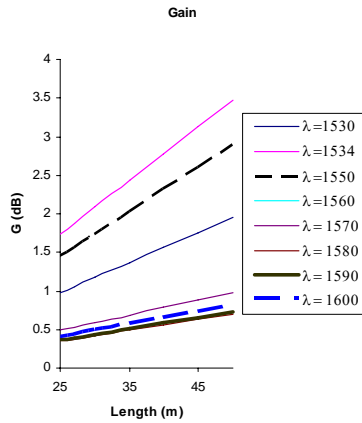


Fig. 11. The maximum gain for yttria-Alumina-Silicate Erbium Doped Fiber Amplifier, calculated for different values of maximum wavelength at room temperature.

#### 4. Conclusions

We can summarize the features of the erbium doped fiber amplifier with the three different hosts material in temperature range 290 - 310 °K and the optimum length of the amplifier to improve the performance of the optical amplifier from the results given before in the Tables 4 and 5.

Table 4. Features of the EDFA with three different hosts.

Host	$\lambda_o$ (nm)	$G_{max}$ (dB)						
		T(°K)	290	294	298	300	305	310
Alumino-grmanosilicate	1560		2.48	3.82	5.81	7.13	11.8	19
Al <sub>2</sub> O <sub>3</sub>	1537		2.11	3.27	5.01	6.17	10.3	16.8
yttria-Alumina-Silicate	1550		1.45	2.24	3.41	4.2	6.96	11.3

Table 5. Features of the EDFA with three different hosts.

Host	$\lambda_o$ (nm)	$L_{opt}$ (m)						
		T(°K)	290	294	298	300	305	310
Alumino-grmanosilicate	1560		9	11.6	15.5	18.1	27.1	41.4
Al <sub>2</sub> O <sub>3</sub>	1537		8	10.1	13.2	15.3	22.7	34.4
yttria-Alumina-Silicate	1550		6.8	8.2	10.3	11.8	16.7	24.6

It is found that the erbium doped almino-germanosilicate fiber amplifier exhibits large value of gain 7.13 dB at center wavelength 1560 nm and temperature 300 °K relative to erbium doped aluminum-oxide fiber amplifier which exhibits a value of gain 6.17 at center wavelength 1537 nm and temperature 300 °K and for erbium doped yatria-silicate fiber amplifier which exhibits a value of gain 4.2 at center wavelength 1550 nm and temperature 300 °K.

Also it is found that the erbium doped almino-germanosilicate fiber amplifier exhibits a more broadening in the gain curve (= 40 nm). The broadening in gain is required in amplification of the signal at large range of wavelengths.

For the optimum fiber length it is found that the erbium doped almino-germanosilicate fiber amplifier exhibits value of 18.1 m at center wavelength 1560 nm and temperature 300 °K relative to erbium doped aluminum-oxide fiber amplifier which exhibits a value of gain 15.3 at center wavelength 1537 nm and temperature 300 °K and for erbium doped yttria-silicate fiber amplifier which exhibits a value of gain 11.8 at center wavelength 1550 nm and temperature 300 °K.

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