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The Faraday effect in quantum cylinder with finite thickness

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The interband Faraday rotation (FR) is theoretically studied for semiconductor hollow quantum cylinder with finite thickness.The FR angle is calculated as a function of an incident light energy for different values of the thickness of quantum cylinder. It is shown that the resonance peaks appear on FR curve. The selection rules are obtained. Numerical results are presented for a GaAs/AlGaAs hollow cylinder.

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1. Introduction

Science and technology on the nanometer scale have attracted intense interest over recent years as they exhibit new and exciting properties compared with their bulk counterparts [1-3]. Such materials come in a wide range of forms, such as dots , wires and wells.

Progress in nanotechnology has made it possible to produce surfaces of different curvature from the layers of heterostructures [4], and particularly cylindrical surfaces, which physical properties show interesting features [5,6]. These structures can be described using a parabolic potential model. In quantum nanostructures, which confinement can be modelled by parabolic potential according to the Kohn generalized theorem electron-electron interaction as a rule does not influance on the optical properties of system. In [7] it is shown that the parabolic potential is equivalent to the potential of infinite layer with positive charge distributed uniformly and in this case the optical properties of system will not depend on both elctron-electron interaction and a number of electrons in layer.The model of parabolic potential is suitable for discription of the physical properties of quantum wires. Moreover, the technology producing nearly ideal quantum wires with parabolic confinement potential has now been elaborated [8].

The investigation of the Faraday effect in a resonant medium is of considerable interest for nonlinear optical spectroscopy, since it provides important information about the level structure of the quantum system, including the set of closely lying sublevels, due to the Zeeman splitting in an external magnetic field. The resonant Faraday effect occurs in semiconductor quantum systems whenever the energy of the excitation light corresponds to the difference in energies of one pair of the conduction and valence Zeeman-split subbands of the quantum systems.

The Faraday effect in low-dimensional semiconductor systems is the subject of many theoretical and experimental investigations [9-12]. The Faraday rotation angle from the eigenstates of the quantum–dot quantum well obtained with $\vec{k} \cdot \vec{p}$ theory is calculated in Ref.10. In [11] the quantum Faraday effect is theoretically studied in a quasi-two-dimensional electron gas.

In the present work, we have investigated the interband Faraday effect in a semiconductor hollow quantum cylinder with finite thickness of walls. This is a quantum system –nanostructure, which confinement can be modelled by parabolic potential. An analytical expression for the rotation angle as a function of an incident light energy and magnetic field is obtained. The selection rules are derived. Numerical results for the FR are presented for GaAs/AlGaAs hollow quantum cylinder.

2. Wave functions and energy eigenvalues

We will model a quantum cylinder with thin walls using the following approach.We consider 2D-electron gas in a quantum film with parabolic confinement potential.If we impose periodic boundary condition on an electron wave function in one of directions

$$\psi(x, y) = \psi(x + L_x, y), \qquad (1)$$

where L_x is the transverse length of the film, then we obtain a model of hollow quantum cylinder with finite thickness. Here $x \in [-\frac{L_x}{2}, \frac{L_x}{2}]$ is the circumferential coordinate, which can be expressed as an angular variable $\varphi = 2\pi x/L_x$ .At the same time the effective thickness of the quantum cylinder will coincide with the film thickness L_y.

The effective mass Schrödinger equation for an electron in a conduction band is,

$$\left[\frac{1}{2m_{0c}}(\vec{p}-\frac{e}{c}\vec{A})^2+\frac{m_{0c}\omega_{0c}^2y^2}{2}+g_c\mu_B Bm_s\right]\psi(\vec{r})=E\psi(\vec{r}), \quad (2)$$

where ω_{0c} is the frequency corresponding to the confinement potential along the y axis, m_{0c} is the electron effective mass , $\vec{p}$ is the momentun of an electron. The term $g_c\mu_B Bm_s$ takes into account the electron spin degeneracy. g_c is the effective g factor of the conduction band , m_s is the spin quantum number, $\mu_B=\frac{e\hbar}{2m_0}$ is the Bohr magneton.The vector potential is chosen in the form :

$$\vec{A}=(-By,0,0). \quad (3)$$

Taking into account the periodic boundary condition (1) we look for the solutions in the form

$$\psi(x,y,z)=\frac{e^{im\frac{x}{R}}}{\sqrt{2\pi R}}\frac{e^{ik_z z}}{\sqrt{L_z}}\phi(y) \quad (4)$$

$L_x=2\pi R$ is the quantum cylinder circumference, and R is the average radius of the quantum cylinder, $m=0,\pm1,...$ is the quantum number related to the projection of the angular momentum on z direction, k_z is the wave number along z direction.

Taking into account (3),(4) in (2) we obtain the energy eigenvalues for the hollow quantum cylinder in the form

$$E_c=\hbar\Omega_c(n+\frac{1}{2})+\lambda_c m^2+\frac{\hbar^2k_z^2}{2m_{0c}}+g_c\mu_B Bm_s, \quad (5)$$

where $\Omega_c=\sqrt{\omega_c^2+\omega_{oc}^2}$, $\omega_c=\frac{eB}{m_{0c}}$ is the cyclotron frequency , $\lambda_c=\frac{2\pi^2\hbar^2\omega_{oc}^2}{m_{0c}L_x^2\Omega_c^2}$ is the geometric confinement energy, n =0,1,2,... is the principal quantum number.

The corresponding wave functions can be written as

$$\psi_c=\frac{e^{ik_z z}}{\sqrt{L_z}}\frac{e^{im\frac{x}{R}}}{\sqrt{2\pi R}}\phi_n\left[\frac{1}{l_c}\left(y+y_{oc}\right)\right], \quad (6)$$

where $l_c=\sqrt{\frac{\hbar}{m_{0c}\Omega_c}}$, $y_{oc}=\frac{\omega_c l_c^2}{\Omega_c R}m$ is the oscillator center. $\phi_n\left[\frac{1}{l_c}\left(y+y_{0c}\right)\right]$ are the oscillator functions, which determined in the form

$$\phi_n\left[\frac{1}{l_c}\left(y+y_{0c}\right)\right]=\frac{1}{\pi^{1/4}l_c^{1/2}\sqrt{2^n n!}}$$

$$\times\exp\left(-\frac{(y+y_{0c})^2}{2l_c^2}\right)H_n\left(\frac{y+y_{0c}}{l_c}\right). \quad (7)$$

H_n are Hermite polynomials.

The energy eigenvalues and wave functions for electrons in valence band are as follows

$$E_v=-E_g-\hbar\Omega_v(n'+\frac{1}{2})-\lambda_v m'^2-\frac{\hbar^2k_z'^2}{2m_{0v}}+g_v\mu_B Bm_s', \quad (8)$$

$$\psi_v=\frac{e^{ik_z' z}}{\sqrt{L_z}}\frac{e^{im'\frac{x}{R}}}{\sqrt{2\pi R}}\phi_{n'}\left[\frac{1}{l_v}\left(y+y_{ov}\right)\right], \quad (9)$$

where E_g is the energy gap in bulk material.

3. Faraday rotation angle

The general expression for the FR angle is given by [13]

$$\Theta=-CE^2\sum_{c,v}\sum_{+}^{-}(\pm)\frac{(E_c-E_v)_\pm^2-E^2}{\left[(E_c-E_v)_\pm^2-E^2\right]^2+4E^2\Gamma^2}\frac{1}{(E_c-E_v)_\pm^2} \quad (10)$$
$$\times\left|\langle c|\vec{e}_\pm\vec{P}|v\rangle\right|^2$$

The inner sum is over the right and left-hand circular polarized light. $E=\hbar\omega$ is the incident light energy, E_c and E_v are the energy eigenvalues for carriers in the conduction and velence –band, respectevely. Γ is the half-width of the spectral band of the $v\rightarrow c$ transitions. $\vec{e}_\pm=\frac{1}{\sqrt{2}}(\vec{e}_x\pm i\vec{e}_y)$ correspond to right and left polarization. $C=\frac{\hbar k^{1/2}e^2L_z}{cn\varepsilon_0 m_0^2V}$,

$V=\pi(R_{out}^2-R_{in}^2)L_z$ is the normalization volume of the hollow quantum cylinder and the other symbols have the usual meaning.

An expression for the interband matrix elements $\langle c|\vec{e}_{\pm}\vec{P}|v\rangle$ can be written in the form [2]

$$\langle c|\vec{e}_{\pm}\vec{P}|v\rangle = \langle \psi_c|\psi_v\rangle \cdot \langle u_{c0}|\vec{e}_{\pm}\vec{P}|u_{v0}\rangle . \quad (11)$$

Taking into account the wave functions (6) ,(9) and the electron spin in (11) we obtain the selection rules in the form

$\Delta k_z = 0$, $|\Delta n| = 0,1,2,..$, $\Delta m = 0$. $\Delta m_s = \pm 1$

$$(m_s = \pm \frac{1}{2}). \quad (12)$$

The $\pm$ signs correspond to the right- and left-hand circular polarized light.

$\langle c|\vec{e}_{\pm}\vec{P}|v\rangle$ can be written as

$$\langle c|\vec{e}_{\pm}\vec{P}|v\rangle = |\vec{P}_{cv}| I_{nm,n'm'} \delta_{k_z k_z'} \delta_{m,m'} \quad (13)$$

$\vec{P}_{cv} = \langle S|\vec{P}_z|Z\rangle$ is the momentum matrix element between bulk Bloch functions at the zone centre,

$$I_{n\ m;n'm'} = \left(\frac{\alpha}{\pi}\right)^{\frac{1}{2}} \frac{1}{\sqrt{2^{n+n'} n! n'!}}$$
$$\times \int_{-\infty}^{\infty} \exp[-\frac{1}{2}\eta^2 - \frac{1}{2}(\eta+Y)^2\alpha^2] H_n(\eta) H_{n'}((\eta+Y)\alpha) d\eta \quad (14)$$

$$\alpha = \frac{l_c}{l_v} , \eta = \frac{y + y_{0c}}{l_c} , Y = \frac{y_{0v} - y_{0c}}{l_c} .$$

In (10) after integrating over all k_z and taking into account (13) we have

$$\theta = -C'E^2 \sum_{n,m} \sum_{n',m'} \left(F^{+}_{nm;n'm'} - F^{-}_{nm;n'm'} \right) \times \left| I_{nm;n'm'} \cdot \delta_{k_z k_z'} \cdot \delta_{m,m'} \right|^2 \quad (15)$$

$$C' = \frac{k^{1/2} e^2 |\vec{P}_{cv}|^2 \sqrt{2\mu} L_z}{4\pi^2 c n \varepsilon_0 m_0^2 (R_{out}^2 - R_{in}^2)} , \quad (16)$$

$$\mu = \frac{m_{0c} m_{0v}}{m_{0c} + m_{0v}} ,$$

$$F^{\pm}_{nm;n'm'} = \int_0^{\infty} \frac{(E^{\pm}_{nm;n'm'} + E_z)^2}{[(E^{\pm}_{nm;n'm'} + E_z)^2 - E^2]^2 + 4\Gamma^2 E^2}$$
$$\times \frac{dE_z}{(E^{\pm}_{nm;n'm'} + E_z)^2 \sqrt{E_z}} , \quad (17)$$

$$E^{\pm}_{nm;n'm'} = E_g + E_{nm} + E_{n'm'} \pm \frac{1}{2}(g_c + g_v)\mu_B B \quad (18)$$

The ($\pm$) signs correspond to the right- and left-hand circular polarized light

$$E_{nm} = \hbar\Omega_c (n + \frac{1}{2}) + \lambda_c m^2 , \quad (19)$$

$$E_{n'm'} = \hbar\Omega_v (n' + \frac{1}{2}) + \lambda_v m'^2 , \quad (20)$$

$$E_z = \frac{\hbar^2 k_z^2}{2\mu} , \quad (21)$$

$$\Omega_{c(v)} = \sqrt{\omega^2_{c(v)} + \omega^2_{0c(v)}} . \quad (22)$$

The oscillation frequencies $\omega_{0c(v)}$ are determined as

$$\omega_{0c(v)} = \frac{1}{L_y} \sqrt{\frac{2\Delta_{c(v)}}{m_{c(v)}}} , \quad (23)$$

where $\Delta_{c(v)}$ are the conduction and valence barrier heights.

We have used the following physical parameters for analyze the interband FR in GaAs/AlGaAs hollow cylynder .E_g=1.5eV, m_{0c}=0.067m_0, and m_{0v}=0.45m_0 (for heavy holes), the level broadening factor Γ=40 meV .The heights of barriers for electrons and holes are Δ_c= 255meV and Δ_v=170meV [14-15].The average radius of the hollow quantum cylinder is R=1500 $\overset{0}{A}$. g_c = 0.32, g_v = -2.42 [16].

According to our notation, an interband transition from the initial hole state, characterized by quantum numbers (n`, m`, m_s`), to the final electron state defined by (n, m, $m_s$), is represented as (n, m, $m_s$, n`, m`, $m_s$`). We take for the quantum numbers n=1,2,3,4; m=0; $m_s = \frac{1}{2}(-\frac{1}{2})$, n`=1,2,3,4; m`=0, $m`_s = -\frac{1}{2}(\frac{1}{2})$. There are 8 interband transitions with selection rules $|\Delta n| = 0$, $\Delta m = 0$, $\Delta m_s = +1(-1)$ in the field of right (left) circularly polarized light, and the FR curve is a result of the addition of all 8 possible contributions. When $E = E^{\pm}_{nm;n'm'}$ the resonance peaks appear on the FR curve. On FR curve there are 4 maxima and 4 minima. The maxima are related to the transitions which occur at $E = E^{+}_{n_r m;n_r' m'}$ with the selection rule $\Delta m_s = +1$, and minima are related to the transitions which occur at $E = E^{-}_{n_r m;n_r' m}$ with selection rule $\Delta m_s = -1$ .The distance between maxima and minima is $(g_c + g_v)\mu_B B$.

Peaks contributing to the FR angle are as folllows:

$$1,\ (1,0,\frac{1}{2},1,0,-\frac{1}{2})\ ;\ (1,0,-\frac{1}{2},1,0,\frac{1}{2})\ ;$$

$$2,\ (2,0,\frac{1}{2},2,0,-\frac{1}{2})\ ;\ (2,0,-\frac{1}{2},2,0,\frac{1}{2})\ ;$$

$$3,\ (3,0,\frac{1}{2},3,0,-\frac{1}{2})\ ;\ (3,0,-\frac{1}{2},3,0,\frac{1}{2})\ ;$$

$$4,\ (4,0,\frac{1}{2},4,0,-\frac{1}{2})\ ;\ (4,0,-\frac{1}{2},4,0,\frac{1}{2})\ .$$

The maxima are located on distance $\hbar(\Omega_c + \Omega_v)$ from each other at the same value of azimuthal quantum number m and $|\Delta n| = 0$ for each maximum. But for the cases $|\Delta n| = 1,2,..$ the distances between maxima are not equidistant .Such behaviour of peaks is realated to an external magnetic field.

The amplitudes of the peaks and the distance between them increase and their positions move toward higher photon energies with decreasing the thickness of the wall. This is due to the increase of the distance between the discrete energy levels.

In Figs. 1 and 2 the FR angle is displayed as a function of the incident light energy for two different values of the thickness of the wall of quantum cylinder $L_y=100\ \overset{0}{A}$ and $L_y=50\ \overset{0}{A}$ for the same value of magnetic field B=0.5 T.

In Fig. 3, the dependences of overlap integral on quantum number n are shown for two different values of the thickness of the wall of quantum cylinder.

In Fig. 4, the dependence of the transition energies corresponding to the right and left circularly polarized light on magnetic field are shown.

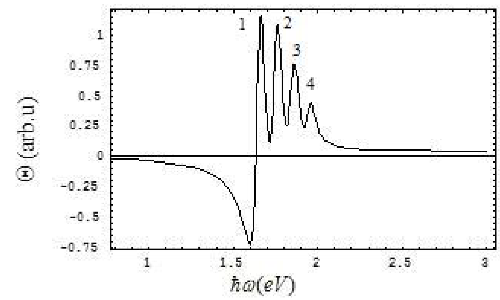

Fig. 1. Plot of the FR angle as a function of photon energy calculated from Eq.15 for $L_y = 100\ \overset{0}{A}$, B=0.5 T.

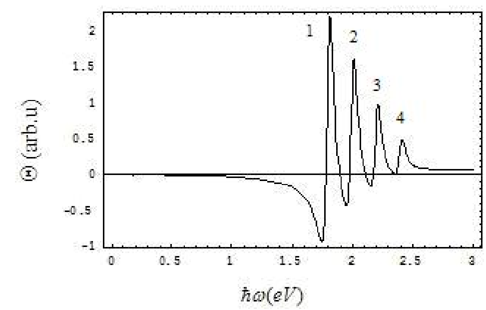

Fig. 2. Same that in Fig.1. for $L_y = 50\ \overset{0}{A}$.

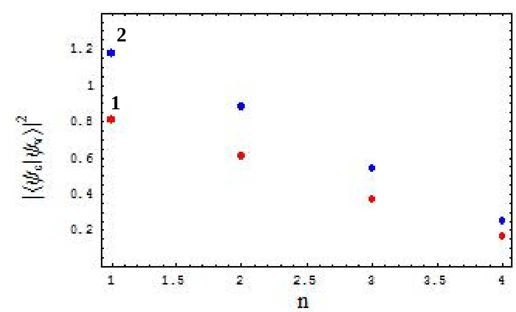

Fig. 3. Plot of the overlap integral as a function of quantum number n for $L_y = 100$ Å (1) and $L_y = 50$ Å (2).

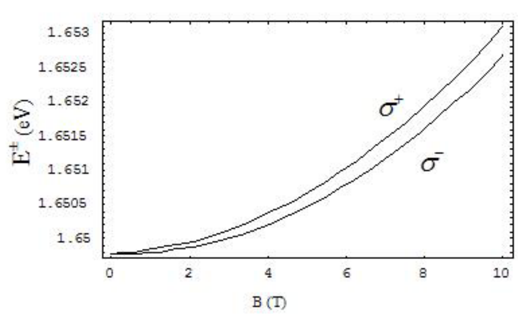

Fig. 4. Plotof the transition energies as a function of magnetic field for right (σ^{+}) and left (σ^{-}) circular polarization.

4. Conclusions

In conclusion, we studied the interband FR in a semiconductor hollow quantum cylinder with parabolic confinement potential. The wave functions and energy spectra have been obtained for hollow quantum cylinder with finite thikcness in longitudinal magnetic field.The selection rules for the interband transitions in the fields of right and left circularly polarized waves and analytically formula for the FR angle have been derived. It is shown the dependences of the FR angle as a function of incident light energy for different values of the thikcness of the hollow cylinder. It is shown that the resonance peaks appear on FR curve. The amplitudes of the peaks and the distance between them increase and their positions move toward higher photon energies with decreasing the thickness of the wall of hollow quantum cylinder.

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