Thermal effects analysis in the generation of squeezed vacuum on the alkali metal atomic transitions

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We theoretically investigate several thermally-induced limitations in the semi-monolithic optical parametric oscillator (OPO) that prepare the squeezed vacuum on alkali metal atomic transitions. Due to the severe pump beam absorption, the inevitable temperature fluctuations cause the squeezed quadrature angle rotation, and even a slight change can decrease the squeezing level significantly, especially at the lower measurement frequencies; in the meantime, the thermal lensing is shown to be also important constraint and must be taken into account. In order to reduce the deleterious effects of thermal effect, one should utilize OPO eigenmode waist radius of more than 45 μm in practice.

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1. Introduction

Squeezed light on alkali metal atomic transition lines have great application potential in the quantum optics fields, such as quantum information network, quantum storage, interaction between light and atoms, entanglement between atomic ensembles, and precision measurement [1, 2]. In order to improve the fidelity of information storage and the sensitivity of measurement, it is necessary to generate highly squeezed light. At present, parametric down conversion process in the sub-threshold OPO is the excellent approach to obtain continuous wave squeezed vacuum. In 1986, Wu et al. obtained a vacuum squeezed light of 63% below the standard quantum limit based on the crystal of MgO: $LiNbO_3$ placed in the ring cavity OPO [3]. In recent years, a great effort has been dedicated to improvement the squeezing degree of the vacuum squeezing. R. Schnabel's group obtained 12.7 dB squeezed light at 1064 nm by an OPO with periodically poled $KTiOPO_4$ (PPKTP) in 2010, and 12.3 dB squeezed state at 1550 nm in 2011 [4, 5]. As a result of reducing all losses in the experimental system, the vacuum squeezed light of 15 dB at 1064 nm has been obtained with a semi-Monolithic OPO in 2016 [6], and K. C. Peng's group also detected 12.6 dB noise reduction of surpassing the standard quantum limit at the same wavelength by optimizing the experimental system in 2017 [7]. However, these results were observed at 1550 nm and 1064 nm, which are far from those of the alkali metal atomic transitions. To obtain squeezed vacuum source on atomic lines, Y. Takeno et al. measured 9 dB quardrature squeezed light at 860 nm, corresponding to the wavelength close to the cesium D2 line in 2007 [8]. In 2009, Burks et al. obtained about 3 dB of squeezing for the measured frequency down to 50 kHz at 852 nm [9]. In 2016, results of 5.6 dB have been obtained on the rubidium D1 line with PPKTP [10].

In order to obtain vacuum squeezed on atomic lines more than 10 dB, it is necessary to further reduce the experimental system loss and improve the stability of squeezing quardrature angle. Firstly, in order to reduce the intracavity loss of OPO, one can employ a semi-monolithic standing wave cavity configuration with lower intracavity loss and highly stable as well as controllable, as shown in Fig. 1, which is the most efficient optical parametric down-conversion cavity at present [11]. Secondly, in order to reduce phase noise due to the pump beam absorption-induced thermal effects, one can design OPO as the single resonance cavity and the pump light single passes through it. In addition, the significant alkali atomic transitions are mostly near-infrared, and corresponding pump light are in blue or even ultra-violet regimes, which suffers the strong absorption in many nonlinear crystals and will generate significant absorption heat. The light absorption-induced thermal effects will reduce the OPO performance, which will produce strong limitation on the squeezing level. Therefore, people have to minimize the detrimental factors of crystal thermal effects in practice. However, very few papers discussed how the thermal effects limit the squeezing level at those wavelengths. In this paper, as an example of vacuum squeezed light resonant on the Cs D2 line generated from a semi-monolithic OPO with PPKTP crystal, we have numerically calculated the crystal temperature fluctuation and the thermal lensing effect induced by the pump light absorption and estimated the influence on the noise reduced degree of the squeezed vacuum state on atomic transitions.

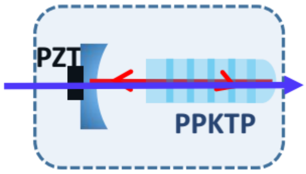

Fig. 1. Schematic of the single resonance semi-monolithic OPO

2. Theory and model

2.1. Squeezed noise power

In 1992, Polzik et al. theoretically calculated the detected maximum squeezing level for the first time generated from OPO in practical experimental system [12]. In this paper, we make use of the same method to calculate and analyze the limiting factors of the squeezed vacuum field. When OPO is on resonance and phase matched, the measurable maximum squeezed quadrature level can be expressed as [13]:

$$V_{Sq}=1+4\eta_{det}\eta_{esc}[\frac{\sin^2\theta}{(1-x)^2+\Omega^2}-\frac{\cos^2\theta}{(1+x)^2+\Omega^2}] \qquad (1)$$

$$\eta_{det}=\eta_{tr}\times\eta_{vis}^2\times\eta_{qu} \qquad (2)$$

where, V_{Sq} is the measurable squeezing level by the detection system, the normalized interaction strength $x=\sqrt{P_2/P_{2,t}}$, the detection efficiency of homodyne system includes the propagation efficiency η_{tr}, the interference efficiency η_{vis}^2 and the quantum efficiency η_{qu}, θ is the squeezed quadrature angle fluctuation. The escape efficiency of OPO is $\eta_{esc}=T/(T+L)$, $\Omega=2\pi f/k$ is normalized measurement frequency, and the decay rate of the cavity can be defined as $k=c(T+L_{loss})/l_{cav}$, l_{cav} is OPO cavity length, L_{loss} is intracavity loss of OPO, and c is the speed of light.

In the experiment process, any variations in crystal temperature and cavity length due to light absorption-induced thermal effects manifest OPO cavity detuning and imperfect phase matching, this will eventually cause the rotation of the squeezing angle and become a limitation to the measured squeezing level.

2.2. Temperature distribution inside the crystal

Let's investigate the heat conduction equation here. The equation governing heat conduction inside the rectangular PPKTP crystal is given by [14]:

$$c\rho\frac{\partial u}{\partial t}=K_x\frac{\partial^2u}{\partial x^2}+K_y\frac{\partial^2u}{\partial y^2}+K_z\frac{\partial^2u}{\partial z^2}+q_v \qquad (3)$$

where ρ is the crystal density, c is the specific heat, K_x, K_y and K_z are the thermal conductivity in the three dimensional direction of the crystal, q_v is the heat source density induced by pump light absorption. During the experiment, squeezed light is produced only when the crystal temperature is heated to its phase matching temperature by a temperature controllor. Therefore, for four lateral faces we can keep constant temperature condition (the phase matching temperature of the crystal); for two incident and transmission end faces, the convection condition is used, so the longitudinal heat dissipation can be omitted, Eqs. (3) can simplify the two dimensional heat conduction equation:

$$c\rho\frac{\partial u}{\partial t}=K_x\frac{\partial^2u}{\partial x^2}+K_y\frac{\partial^2u}{\partial y^2}+q_v \qquad (4)$$

when the pump Gaussian beam incident, the heat source density can be expressed as:

$$q_v=\frac{\alpha_2P_2}{\pi\omega'(z)^2}\bullet\exp[-\frac{2(x^2+y^2)}{\omega'(z)^2}]\bullet\exp(-\alpha_2z) \qquad (5)$$

$$\omega'(z)=\omega_0'\sqrt{1+\left(\frac{z}{z_0}\right)^2} \qquad (6)$$

where $\omega'(z)$ is $1/e$ Gaussian radius of the pump beam, ω_0' represents the pump beam waist radius at the crystal center, the beam Rayleigh length $z_0=\pi\omega_0'^2/\lambda_2$. The steady state heat conduction equation can be written as:

$$K_x\frac{\partial^2u(x,y,z)}{\partial x^2}+K_y\frac{\partial^2u(x,y,z)}{\partial y^2}+q_v=0 \qquad (7)$$

As previously described above, the corresponding boundary conditions for six faces of the crystal can be given by:

$$u(0,y,z)=320 \quad u(a,y,z)=320$$
$$u(x,0,z)=320 \quad u(x,b,z)=320$$
$$\left.\frac{\partial u(x,y,z)}{\partial z}\right|_{z=0}=0 \quad \left.\frac{\partial u(x,y,z)}{\partial z}\right|_{z=l}=0 \qquad (8)$$

where the phase matching temperature of the crystal T_0=320K. *a*, *b*, and *l* are the dimensions of crystal as *a*=2mm, *b*=1mm, *l*=10mm. $\alpha_2=10\%\mathrm{cm}^{-1}$ is the linear absorption coefficient, the pump light power $P_2=150\mathrm{mW}$, and the thermal conductivity $K_x=K_y=K_z=3.3\,\mathrm{W}/\mathrm{m}\,^{\circ}\mathrm{C}$. During the preparation of squeezed vacuum, one can achieve the quasi-phase matching by tuning the operating temperature of the crystal, the pump light absorbtion-induced crystal temperature fluctuation results in the change of the cavity length that are on resonance. Here, the thermal expansion coefficient of the PPKTP $\alpha=\partial l/(l\cdot\partial t)=9\times10^{-6}\,^{\circ}\mathrm{C}^{-1}$, so at temperature T, the crystal length can be calculated by the following relation [15]:

$$l(t)=l_0\exp\left[1+\alpha\left(T-T_0\right)\right] \qquad (9)$$

A small change δL_{cav} in the OPO cavity length away from resonance will result in detuning by $\Delta\, a=\omega\delta L_{cav}/L_{cav}$, where ω is resonance wavelength and L_{cav} is the cavity length on resonance. Taking the influence of both cavity detuning and imperfect phase matching into account, similar to reference [13], we can estimate the linear coupling of the temperature fluctuation and the squeezed quardrature angle by the following method [13]:

$$\frac{d\theta_{sqz}}{d\delta L_{cav}}=\frac{1}{2}\left.\frac{dV/d\delta L_{cav}}{dV/d\theta_b}\right|_{\theta_B=\pi/2,\delta L_{cav}=0}=\frac{\omega}{L_{cav}}\left(\frac{1}{k\left(1+x^2\right)}\right) \qquad (10)$$

where *k* is the cavity decay rate, the wavelength of the incident pump laser is 426 nm, and the round-trip cavity length of the semi-monolithic OPO is 80.8 mm.

2.3. Thermal lensing

The severe light absorption-induced thermal effects form a thermal gradient inside the crystal in the process of preparation of squeezed vacuum on alkali atomic transitions, and the majority is usually the thermal lensing, which changes the eigenmode of OPO and the nonlinear conversion coefficient of crystal, and then affects the quality of the noise reduction. The thermal lensing focal length in the middle of the crystal can be given by [16, 17]:

$$f_{th}=\frac{\pi K\,\omega_0^2}{P_2\left(dn/dT\right)}\left(\frac{1}{1-\exp\left(-\alpha_2 L\right)}\right), \qquad (11)$$

where ω_0 is waist radius of OPO eigenmode, $dn_1/dT=15.3\times10^{-6}\,\mathrm{K}^{-1}$ is the refractive index change with temperature and the thermal conductivity K_c is $3.3\times10^3\,\mathrm{W}/{}^0\mathrm{C}$ for the PPKTP crystal.

3. Results and discussions

3.1. Influence of temperature fluctuation on the squeezing level

Generally, the finite difference scheme is adopted to obtain the more precise numerical solution of the heat conduction equation (3), nevertheless, for the case of generation of the continuous variable squeezed vacuum using the rectangular nonlinear crystal under constant boundary conditions mentioned in the previous section, it can use Matlab to solve partial differential equation of heat transfer, calculate and display the temperature distribution situations of section of the crystal [18]. Fig. 2(a) and 2(b) show the temperature distributions within the crystal central cross section, the corresponding eigenmode waist radius of OPO are 50 μm and 25 μm, respectively. It can be seen from the figure that the maximum temperature of the crystal central cross section increases about 18 mk and 35 mk, respectively. The smaller the waist radius is, the more serious the temperature fluctuation is. This is because the pump beam absorbtion increases with the reduction of the waist radius and the rise of the incident pump light power density. On the other hand, if the crystal central temperature is too high, a large temperature difference will be formed inside the crystal, which will be harmful to the crystal.

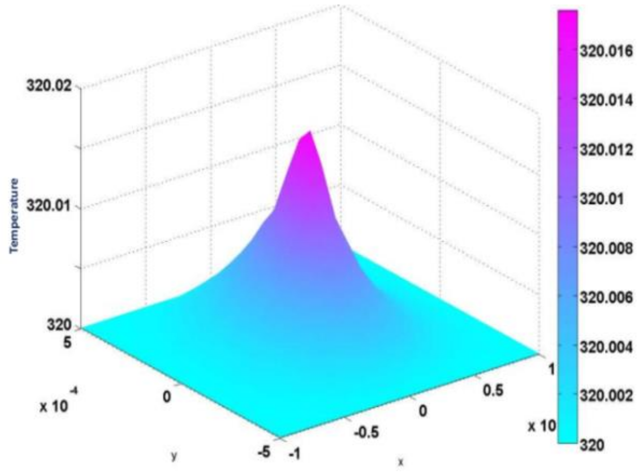

Fig. 2(a). Temperature distribution within the crystal central cross section, OPO eigenmode waist radius is 50 μm

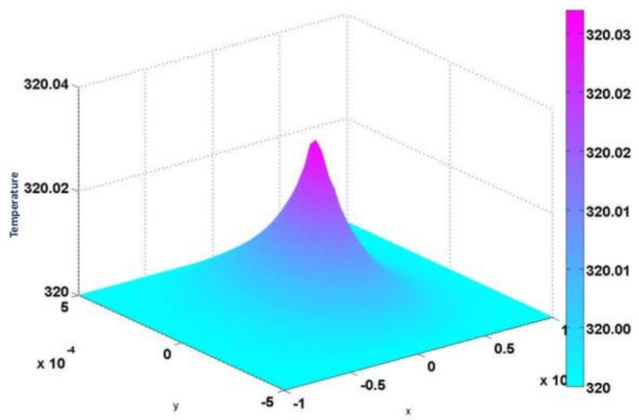

Fig. 2(b). Temperature distribution within the crystal central cross section, OPO eigenmode waist radius is 25 μm

Fluctuation in the temperature of the crystal leads to a certain distribution of the refractive index and the crystal length, which results in the squeezed quadrature phase noise. According to the analysis of literature [13], we can estimate the linear coupling about temperature fluctuation and the squeezed angle rotation by Eq. (10). It can be obtained that, operated with the nonlinear gain of 10, the per nanometer crystal length change results in 47.8 mrad squeezed quadrature rotation. It can be seen that even the tiny change in crystal temperature and length will obviously affect the fluctuation of squeezed angle. According to the results for the simulation, when the OPO eigenmode waist radius are 25 μm and 50 μm, the squeezed quadrature angle fluctuation are 4.3°and 2.2°, respectively.

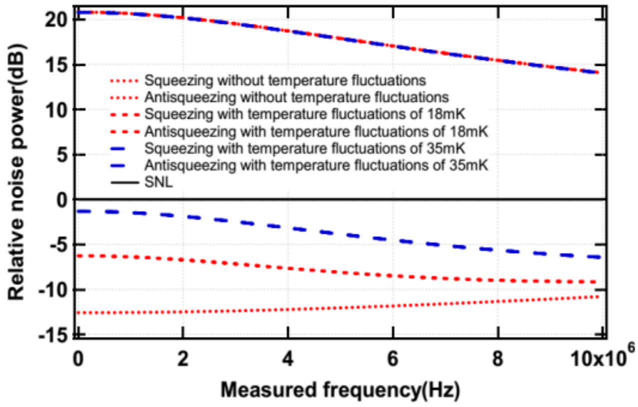

Fig. 3. Squeezing and anti-squeezing amount as a function of measurement frequency for different temperature fluctuations. SNL: shot noise limit

As shown in Fig. 3, under different temperature fluctuation conditions, the squeezing degree changes with the measurement frequency, assuming that the overall detection efficiency η=100% (without regard to the detection loss). Under without temperature fluctuation condition, the obtained squeezed vacuum can be more than 10 dB. However, when there is the temperature fluctuation, it leads to the drift of squeezed angle, although the antisqueezing is almost unaffected, the squeezing level will decrease seriously. Even if the temperature fluctuation is very small, the squeezing level will decrease significantly. Especially when the OPO waist radius is 25 μm, the temperature fluctuation of 35 mK causes the squeezing level decrease dramatically, and the lower the measurement frequency is, the more serious the impact is. These features may be understood as follows: temperature fluctuations result in a rotation of the squeezed angle, which results in a suboptimal detection angle for the homodyne detector setup. If the temperature fluctuations are slow compared to the detection time, the measured degree of squeezing will fluctuate all the time, causing a nonstationary noise spectrum; if the fluctuations are faster than the measurement time, the only result will be a lower squeezing level.

3.2. Influence of thermal lensing effect on the squeezing level

Employing Eq. (11), Fig. 4 shows that the thermal lensing power changes with the waist radius of OPO at the input pump power of 150 mW. It is clear that the thermal lensing is sensitive to the beam waist size in the range of $16\ \mu m \le \omega_0 \le 70\ \mu m$. Then the blue dashed and red solid lines in Fig. 5 displays the available squeezing level versus OPO waist radius without and with considering thermal lensing effect, respectively. It clearly shows that, when the thermal lensing effect is not considered, the squeezing level undergoes significant change over the range of $16\ \mu m \le \omega_0 \le 70\ \mu m$. On the focusing condition of

$\omega_0 = 16\mu m$, the squeezing degree reaches a maximum value of 13.5 dB, whereas it decreases to about 6dB at $\omega_0 = 70\ \mu m$, which tells that the squeezing level decreases as the waist radius of OPO increases. However, the inevitable light absorption-induced thermal lensing effect induces serious pump light phase matching losses, reduces the conversion efficiency of OPO, and the output squeezed light changes significantly, as shown in the red solid curve in Fig. 5. At the loose focusing regime $45\ \mu m \le \omega_0 \le 60\ \mu m$, the squeezing level is close to 14 dB, while it is reduced to about 8 dB for the tight focusing regime. In general, under without thermal effect conditions, the conversion efficiency of OPO increases with pump power density increasing, which requires the waist radius of the incidence pump light as small as possible. But the smaller the waist radius is, the more obvious the thermal lensing effect is, in this situation, the output of OPO will be influenced severely. Therefore, it is necessary to choose the proper cavity waist radius in the range of more than $45\ \mu m$ for the higher squeezing degree. In addition, applying the single-pass optical parametric process based on the single-spatial-mode optical waveguide is a promising means to slightly reduce the thermal lensing effect and improve the performance of OPO in the continuous variable squeezed vacuum system [19].

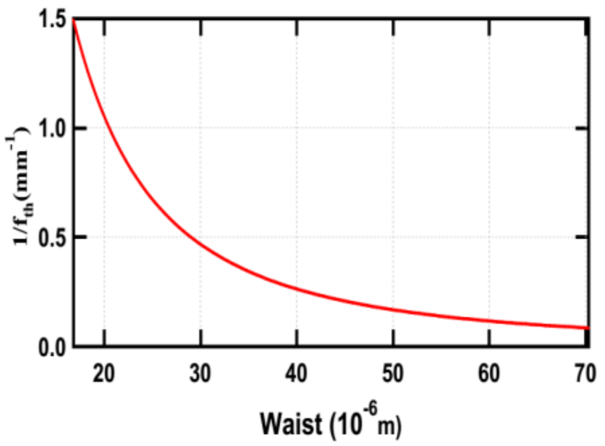

Fig. 4. Thermal lens power versus OPO eigenmode waist radius

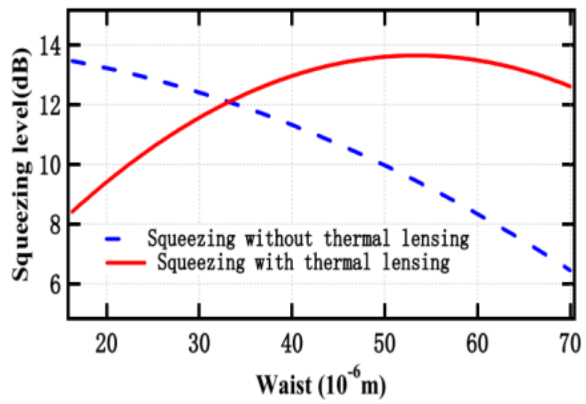

Fig. 5. Squeezing level versus OPO eigenmode waist radius without (blue dashed line) and with thermal lensing (red solid line)

4. Conclusions

In this paper we have theoretically investigated thermal effects in singly resonant semi-monolithic OPO with different waist sizes that provide squeezed vacuum on alkali metal atomic transitions. The squeezing degree is affected by the temperature fluctuation and thermal lensing effect arising from pump light absorption in nonlinear crystals. Temperature fluctuations of dozens of mK inside the crystal causes the squeezed phase instability, and the obtained squeezing level decreases dramatically, Especially at the lower measurement frequency, its influence is more serious. At the same time, the light absorption induced thermal lensing effect destroys the OPO previous focusing conditions, thus affects the matching efficiency of the pump light and results in decreases of the output squeezing degree. We analyzed the effect of the eigenmode waist radius and confirmed that the weak focusing can reduce the thermal effect and improve the performance of OPO. The theoretical analysis results can provide theoretical guidance for the optimization design of the preparation of squeezed vacuum on alkali atomic transitions.

Acknowledgments

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References

[1] J. Hald, J. L. Sorensen, C. Schori, E. S. Polzik, Phys. Rev. Lett. **83**, 1319 (1999).
[2] R. E. Slusher, L. W. Hollberg, B. Yurke, J. C. Mertz, J. F. Valley, Phys. Rev. Lett. **55**, 2409 (1985).
[3] L. A. Wu, H. J. Kimble, J. L. Hall, H. F. Wu, Phys. Rev. Lett. **57**, 2520 (1986).
[4] T. Eberle, S. Steinlechner, J. Bauchrowitz, V. Händchen, H. Vahlbruch, M. Mehmet, H. Müller-Ebhardt, R. Schnabel, Phys. Rev. Lett. **104**, 251102 (2010).
[5] M. Mehmet, S. Ast, T. Eberle, S. Steinlechner, H. Vahlbruch, R. Schnabel, Opt. Express **19**, 25763 (2011).
[6] H. Vahlbruch, M. Mehmet, K. Danzmann, and R. Schnabel, Phys. Rev. Lett. **117**, 110801(2016).
[7] W. H. Yang, S. P. Shi,Y. J. Wang, W. G. Ma, Y. H. Zheng, K. C. Peng, Optics Letter **42**, 4553(2017).
[8] Y. S. Takeno, M. Yukawa, H. Yonezawa, A. Furusawa, Opt. Express **15**, 4321 (2007).
[9] S. Burks, J. Ortalo, A. Chiummo, X. Jia, F. Villa, A. Bramati, J. Laurat, E. Giacobino, Opt. Express **17**, 3777 (2009).
[10] Y. S. Han, X. Wen, J. He, B. D. Yang, Y. H. Wang, J. M. Wang, Opt. Express **24**, 002350 (2016).

[11] J. F. Tian, G. H. Zuo, Y. C. Zhang, G. Li, P. F. Zhang, T. C. Zhang, Acta Optica Sinica **37**, 0414002 (2017) (in Chinese).

[12] E. S. Polzik, J. Carri, H. J. Kimble, Appl. Phys. B **55**, 279 (1992).

[13] S. Dwyer, L. Barsotti, S. S. Y. Chua, S. Dwyer, L. Barsotti, S. S. Y. Chua, M. Evans, M. Factourovich, D. Gustafson, T. Isogai, K. Kawabe, A. Khalaidovski, P. K. Lam, M. Landry, N. Mavalvala, D. E. McClelland, G. D. Meadors, C. M. Mow-Lowry, R. Schnabel, R. M. S. Schofield, N. Smith-Lefebvre, M. Stefszky, C. Vorvick, D. Sigg, Opt. Express **21**, 19047 (2013).

[14] J. L. Liu, X. Y. Chen, Y. J. Yu, C. T. Wu, F. Bai, G. Y. Jin, Physical Review A **93**, 013854 (2016).

[15] J. G. Ren, H. Zhang, X. D. Cai, J. Yin, F. Zhou, C. Z. Peng, Acta Phys. Sin. **58**, 5169 (2009) (in Chinese).

[16] J. F. Tian, C. Yang, J. Xue, Y. C. Zhang, G. Li, T. C. Zhang, Journal of Optics **18**, 055506 (2016).

[17] R. L. Targat, J. J. Zondy, P. Lemonde, Opt. Commun. **247**, 471 (2005).

[18] O. A. Louchev, N. E. Yu, S. Kurimura, K. Kitamura, Applied Physics Letters **87**, 131101 (2005).

[19] T. Kashiwazaki, N. Takanashi, T. Yamashima, T. Kazama, K. Enbutsu, R. Kasahara, T. Umeki, A. Furusawa, APL Photonics **5**, 036104 (2020).

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